Standard PID Tuning Methods
(tbc0 2/17/2012)

I. Cohen-Coon Method (Open-loop Test)

Step 1: Perform a step test to obtain the parameters of a FOPTD (first order plus time delay) model
   i. Make sure the process is at an initial steady state
   ii. Introduce a step change in the manipulated variable
   iii. Wait until the process settles at a new steady state

Step 2: Calculate process parameters: \( t_1, \tau, \tau_{del}, K, r \) as follows

\[
t_1 = \frac{t_2 - (\ln(2))t_3}{1 - \ln(2)}
\]
\[
\tau = t_3 - t_1
\]
\[
\tau_{del} = t_1 - t_0
\]
\[
K = \frac{B}{A}
\]
\[
r = \frac{\tau_{del}}{\tau}
\]

Step 3: Using the process parameters, use the prescribed values given by Cohen and Coon.
### Table 1. Cohen-Coon Tuning Rules

<table>
<thead>
<tr>
<th></th>
<th>$K_c$</th>
<th>$\tau_{int}$</th>
<th>$\tau_{der}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>$\frac{1}{rK} \left(1 + \frac{r}{3}\right)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PI</td>
<td>$\frac{1}{rK} \left(0.9 + \frac{r}{12}\right)$</td>
<td>$\frac{30 + 3r}{9 + 20r}$</td>
<td></td>
</tr>
<tr>
<td>PID</td>
<td>$\frac{1}{rK} \left(\frac{4}{3} + \frac{r}{4}\right)$</td>
<td>$\frac{32 + 6r}{13 + 8r}$</td>
<td>$\frac{4}{11 + 2r}$</td>
</tr>
</tbody>
</table>

### II. Ziegler-Nichols Method (Closed-loop P-Control Test)

1. Determine the sign of process gain (e.g. open loop test as in Cohen-Coon).
2. Implement a proportional control and introducing a new set-point.
3. Increase proportional gain until sustained periodic oscillation.
4. Record ultimate gain and ultimate period: $K_u$ and $P_u$.
5. Evaluate control parameters as prescribed by Ziegler and Nichols

### Table 2. Ziegler Nichols Tuning Rules

<table>
<thead>
<tr>
<th></th>
<th>$K_c$</th>
<th>$\tau_{int}$</th>
<th>$\tau_{der}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>$\frac{K_u}{2}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PI</td>
<td>$\frac{K_u}{2.2}$</td>
<td>$\frac{P_u}{1.2}$</td>
<td></td>
</tr>
<tr>
<td>PID</td>
<td>$\frac{K_u}{1.7}$</td>
<td>$\frac{P_u}{2}$</td>
<td>$\frac{P_u}{8}$</td>
</tr>
</tbody>
</table>
III. Tyreus-Luyben Method (Closed-loop P-Control test)

Step 1-4: Same as steps 1 to 4 of Ziegler-Nichols method above
Step 5: Evaluate control parameters as prescribed by Tyreus and Luyben

<table>
<thead>
<tr>
<th></th>
<th>$K_c$</th>
<th>$\tau_{int}$</th>
<th>$\tau_{Der}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>PI</td>
<td>$\frac{K_u}{3.2}$</td>
<td>2.2$P_u$</td>
<td></td>
</tr>
<tr>
<td>PID</td>
<td>$\frac{K_u}{2.2}$</td>
<td>2.2$P_u$</td>
<td>$\frac{P_u}{6.3}$</td>
</tr>
</tbody>
</table>

IV. Autotune Method (Closed-loop On-Off test)

Step 1: Let process settle to a steady state
Step 2: Move the setpoint to the current steady state
Step 3: Implement an on-off (relay) controller

If process gain is positive, $u = \begin{cases} 
  u_0 + h & \text{if } e \geq 0 \\
  u_0 - h & \text{if } e < 0 
\end{cases}$

If process gain is negative, $u = \begin{cases} 
  u_0 - h & \text{if } e \geq 0 \\
  u_0 + h & \text{if } e < 0 
\end{cases}$

Step 4: Let the process settle to a sustained periodic oscillation
Step 5: Evaluate ultimate gain using autotune formulas ($P_u$ can be obtain from the plots)

$$K_u = \frac{4h}{\pi a}$$

Step 6: Use either Ziegler-Nichols or Tyreus-Luyben prescribed tunings