The Combinatorics of Topology-Transparent Scheduling

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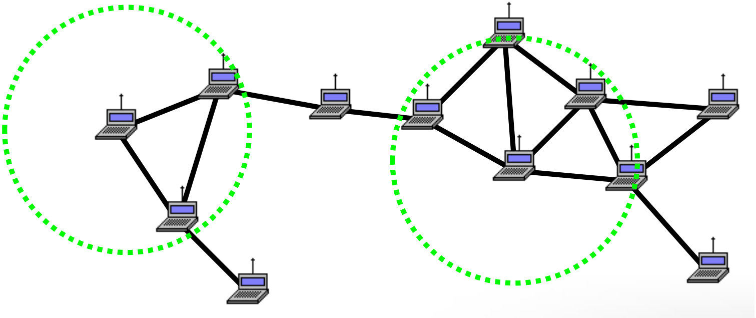
Medium Access Control (MAC) Protocols

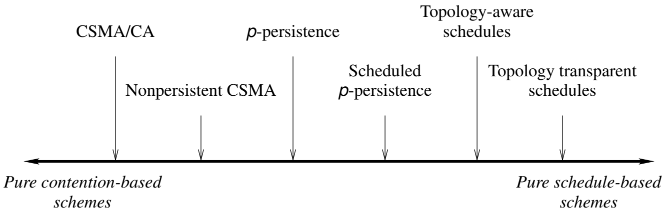
- Many networks use a broadcast channel (medium).
  - *e.g.*, WiFi, satellite, radio, optical, sensor.
- The MAC protocol coordinates all packet transmissions.
- The MAC protocol has a fundamental impact on overall network performance.

Mobile Ad Hoc Wireless Networks (MANETs)

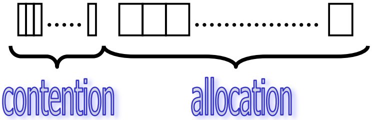
- A mobile ad hoc network is a self-organizing collection of mobile wireless nodes.
  - It has no centralized control or wired infrastructure.
- The network is multi-hop, and allows spatial re-use.
  - The simplest way to model a MANET is to use a unit disk graph.



- There is a spectrum of approaches to medium access control.
  - Contention-based protocols:
    - Pros: agile and adapt quickly to changes in perceived contention.
    - Cons: short-term unfair, large variations in delay, and poor performance at high load.
  - Schedule-based protocols:
    - Pros: stable persistences, low variation in delay and throughput, can sometimes bound maximum delay.
    - Cons: adapt slowly, if at all, to changes in the network.



- The topology of a MANET is dynamic, due to node mobility and physical characteristics of radio transmission.
- Topology-dependent approaches to cope topology change:
  - Recompute the schedule when the topology changes.



- Topology-transparent approaches to cope with topology change:
  - Schedules are independent of topology change, *i.e.*, each node's schedule is fixed at initialization and does not change.
  - Constructions use two design parameters: N the number of nodes, and D_{max} the maximum neighbourhood size.

Combinatorial Characterization of TT Scheduling

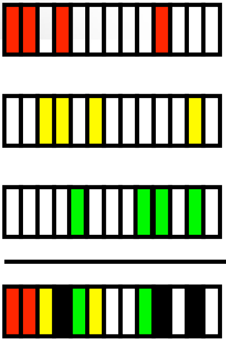
- The combinatorial problem asks for each node $i \in \{0, \ldots, N-1\}$ to be given a subset S_i of $\{0, 1, \ldots, n-1\}$ slots with the property that the union of $D_{\max}$ or fewer other subsets cannot contain S_i.

- This may be expressed mathematically by requiring that

$$\left(\bigcup_{j \in X} S_j \right) \not\supset S_i,$$

  where $X \subseteq \{0, \ldots, N-1\} \setminus \{i\}$ with $|X| \leq D_{max}$.
- In the language of sets this is precisely a cover-free family.
  - These are equivalent to disjunct matrices and to certain superimposed codes.

A Very Small Example from S(2,4,13)



$\{0,1,3,9\}$

$\{2,3,5,11\}$

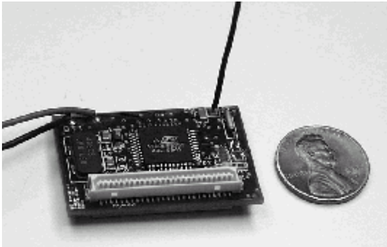
$\{4,8,9,11\}$

$\{0,1,2,3,4,5,8,9,11\}$

- This example is from a Steiner system, S(2,4,13); it can support $N = 13$ nodes and $D_{\text{max}} = 3$.
- While there are collisions (≥ 2 nodes transmit at the same time; in black), each node has 2 successful slots!

Sensor Networks

- Most constructions consider two slot states: transmit and receive.

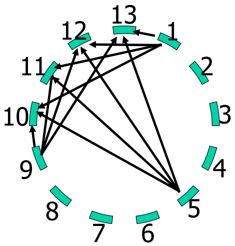


Radio	Transmit	Receive	Idle
1	15 W	11 W	0.05 W
2	5.76 W	2.88 W	0.35 W

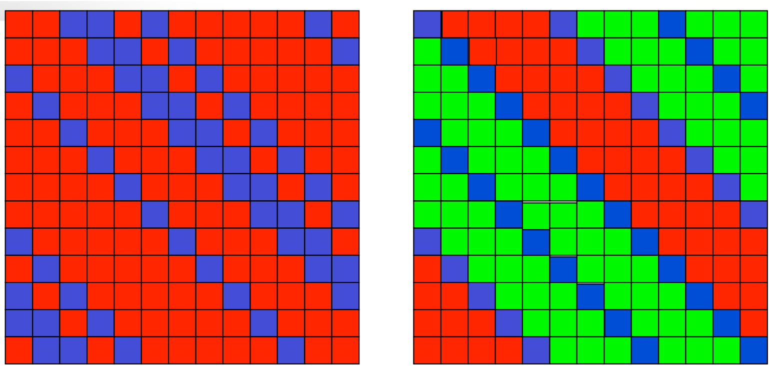
- Listening is expensive!

Extension to Three States

- Introduce a third slot state — sleep — for energy efficiency.
- The cover-free requirements are more complex.
- For each time slot, we need a slot schedule, *i.e.*, a partition $[T, R, S]$ of the N nodes into nodes T that can transmit, nodes R that are eligible to receive, and nodes in S that are asleep.
  - Indirect (recursive) constructions include dual cover-free families and packcovers.
  - Direct constructions include addition sets, and computational methods (*e.g.*, hill climbing).
  - It is still possible to bound delay!

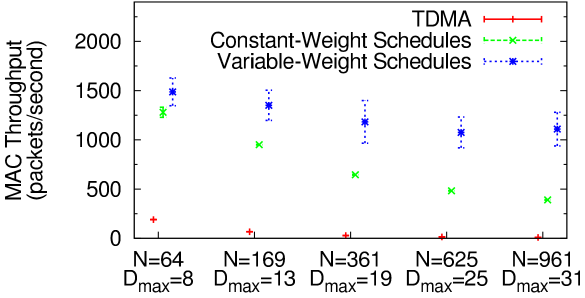


Example of Binary vs. Ternary TT Schedules



- Receive ρ, Transmit τ, Sleep σ
- Energy budget: cost of $4\tau + 9\rho$ per slot vs. $3\tau + 3\rho + 6\sigma$ per slot

Constant- vs. Variable-Weight Schedules



- TDMA schedules are trivially topology transparent.
- Constant-weight schedules place an unnecessary constraint on throughput in neighbourhoods smaller than $D_{\max}$.
- Variable-weight schedules have the potential to recover throughput lost to constant-weight schedules.

Requirements of Variable-Weight TT Schedules

- The variable-weight topology-transparent schedule design problem:
  - Suppose there are N nodes, m schedule weights, and a frame length of n.
  - Let $W_{\max}$ be a fixed fraction of n.
  - We are to form schedules $S_i = \{S_{i,j} : 0 \le i < N, 0 \le j < m\}$ so that the weight $\mathrm{wt}(S_{i,j})$ is w_j; and whenever $\{i_0, \ldots, i_D\} \subseteq \{0, \ldots, N-1\}$ and $j_\ell \in \{0, \ldots, m-1\}$ for $0 \le \ell \le D$,

$$\left(\bigcup_{\ell=1}^{D} S_{i_\ell j_\ell} \right) \not\supset S_{i_0 j_0} \quad \text{(the \textit{cover-free condition})}$$

    whenever

$$\sum_{\ell=1}^{D} \mathrm{wt}(S_{i_\ell j_\ell}) \le W_{\max} \quad \text{(the \textit{weight condition})}.$$

Variable-Weight Schedules from a TD(t+1,v,v)

- We construct a set of variable-weight topology-transparent schedules for each node in the network from the blocks of a transversal design, $\text{TD}(t+1, v, v)$.
  - It supports a maximum of $N = v^t$ nodes, each with $m = v$ schedules of length $n = v^t$.
  - That is, each node i has a collection of m schedules $S_i = \{S_{i,j} : 0 \leq i < N, 0 \leq j < m\}$ where the weight $\text{wt}(S_{i,j}) = w_j = (j+1)t - 1$.

- The weight w_j is an upper bound on the number of collisions node i operating with schedule $S_{i,j}$ can experience while still satisfying the cover-free condition.
  - For a $\text{TD}(t+1, v, v)$, this is $W = \{t-1, 2t-1, \ldots, mt-1\}$, and $W_{\max} = v$.

Weakening the Assumption on Synchronization

- Most known topology-transparent schedules assume synchronization on frame boundaries.
- Constructions are generalized for synchronization on slot boundaries and the asynchronous case.
- Idea: For slot synchronization, give a node a schedule and all its cyclic shifts.
    - Constructions from cyclic superimposed codes and optical orthogonal codes exist.
- It is somewhat of a surprise that the construction for the asynchronous model is achieved by a simple variant of the construction for the slot synchronized model.
- A substantial loss in the delay guarantee results each time the synchronization model is weakened.

Summary

- We introduced the combinatorial requirements on topology-transparent scheduling in MANETs.
- We provided extensions from binary to ternary schedules for energy-efficiency.
- We provided extensions from constant-weight to variable-weight to accommodate changes in network load.
- Finally, we weakened the assumption on frame synchronization to slot synchronization and the asynchronous case.

Thanks!

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