Algebraic Combinatorics and Applications

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Abstracts

Spherical embeddings of strongly regular graphs

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In their seminal work Delsarte, Goethals and Seidel described, among many other things, certain spherical embeddings of strongly regular graphs. In this talk we will characterize all spherical embeddings of strongly regular graphs via the embeddings of Delsarte, Goethals and Siedel, show that only their embeddings form 2-designs, and use this to complete the description of two-distance tight frames.

Keywords: strongly regular graphs, s-distance sets, spherical designs, tight frames

On regular difference systems of sets from cyclotomic classes

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A *difference system of sets (DSS)* is a collection $\mathcal{F}$ of t disjoint subsets (called blocks) $Q_i \subseteq \{0, 1, \ldots, n-1\},\ |Q_i| = \tau_i,\ 0 \leq i \leq t-1$, such that the multiset

$$\triangle\mathcal{F} = \{a - b \pmod n) | a \in Q_i,\ b \in Q_j,\ 0 \leq i, j \leq t-1,\ i \neq j\} \tag{1}$$

contains every element $i,\ 1 \leq i \leq n-1$, at least ρ times. A DSS is *perfect* if every element $i,\ 1 \leq i \leq n-1$, is contained exactly ρ times in the multiset (1). A DSS is *regular* if all blocks Q_i are of the same size ($\tau_0 = \tau_1 = \cdots = \tau_{t-1} = m$). A regular DSS on n points with t blocks of size m is denoted by (n, m, t, ρ)-DSS. If a DSS has the minimum redundancy $r = \sum_{i=0}^{t-1} \tau_i$ for the given parameters n, t, and ρ, a DSS is *optimal.*
Let $p = ef + 1$ be an odd prime, and let α be a primitive element of a finite field of order p, $\mathbb{F}_p$. We consider a collection $\mathcal{F}$ of f subsets $Q_0, Q_1, \ldots, Q_{f-1}$ of $\mathbb{F}_p$, for $f \geq 2$, defined as

$$Q_0 = \{a_1, a_2, \ldots, a_m\} \text{ and } \ Q_i = \alpha^{ie} Q_0,\ 1 \leq i \leq f-1,$$

where $a_k \in \mathbb{F}_p,\ a_k \neq a_l,\ 1 \leq k, l \leq m$.

In this talk, we discuss a condition for which a collection $\mathcal{F}$ forms a regular (p, m, f, ρ)-DSS and show that a lower bound on the parameter ρ of the DSS is given by cyclotomic numbers. When $m = 2$ and 3, we present a condition for an optimal DSS with $p = 3f + 1, 4f + 1$ and $6f + 1$. When $m \geq 4$, some numerical results of optimal DSSs are given.

Keywords: difference systems of sets, cyclotomic classes, cyclotomic numbers

Fault Location and Resolvable Set Systems

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Let N be a positive integer and let R be a set of size N. A *set system* is a pair $(R, \mathcal{B})$ with $\mathcal{B} \subseteq 2^R$. A *parallel class* in the set system is a subset $\{B_1, \ldots, B_\ell\} \subseteq \mathcal{B}$ for which $B_i \cap B_j = \emptyset$ unless $i = j$ and $\bigcup_{i=1}^{\ell} B_i = R$. A *resolution* of the set system is a partition of the set system into parallel classes, and the set system is *resolvable* when it has a resolution. When v is a positive integer, a resolution is a *v-resolution* when every parallel class contains exactly v sets of the set system.

The question that we address is: *Given R and v, what is the largest number of parallel classes in a v-resolution of a set system on R elements?* At first, this problem appears to be quite contrived. However, we describe how it addresses an important problem in testing complex component-based systems; we also outline how it relates to covering arrays, locating arrays, detecting arrays, and Sperner partition systems.

We determine an upper bound on the number of v-resolutions via linear programming. We then show that the best upper bound so obtained is exact, by generalizing Baranyai's theorem.

Keywords: covering array, orthogonal array, detecting array, locating array, Sperner partition system, Baranyai's theorem.

The Combinatorics of Topology Transparent Scheduling

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A mobile ad hoc network (MANET) is a collection of mobile wireless nodes that communicate without any fixed infrastructure or centralized control. Medium access control (MAC) is a fundamental problem in all networks that have a broadcast channel as a basis of communication. The MAC protocol determines which nodes transmit and when. In an effort to maximize spatial reuse and support bounded delay, scheduling schemes that are transparent to changes in the MANET topology have been developed. Although they did not know it, the initial schemes were based on orthogonal arrays. We showed the general combinatorial requirement of the schedules is met by a cover-free family and that Steiner systems support the largest number of nodes for a given schedule length. These approaches assume that all the nodes are synchronized on frame boundaries. If the synchronization requirements are weakened, topology transparent schedules based on cyclic superimposed codes exist. If in addition to transmitting and receiving, nodes are allowed to sleep to conserve energy, then certain graph decompositions are a model for ternary topology transparent schedules. Most recently, extensions from constant-weight to variable-weight schedules based on transversal designs from the finite field can be used to accommodate nodes with differing traffic loads.

Keywords: covering array, orthogonal array, detecting array, locating array, Sperner partition, Baranyai's theorem.

On some incidence structures constructed from groups and related codes

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We describe a construction of block designs and strongly regular graphs determined by a transitive action of a finite group. Further, we discuss linear codes obtained from the

constructed combinatorial structures and their orbit matrices. The obtained codes usually have large automorphism groups, hence they are suitable for permutation decoding.

Keywords: block design, strongly regular graph, automorphism group, linear code.

Codes from orbit matrices and extended orbit matrices of symmetric designs

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We study codes spanned by the rows of an orbit matrix of a symmetric design with respect to an automorphism group that acts with all orbits of the same length. The dimension of such codes is discussed. We define an extended orbit matrix and show that under some condition the rows of the extended orbit matrix span a code that is self-dual with respect to a certain scalar product.

Keywords: symmetric design, orbit matrix, linear code, self-dual code

Laplacian Eigenvalues of a Graph

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The Laplacian matrix of a graph is the difference of the diagonal matrix of vertex degree and the adjacency matrix. In this talk, we present various aspects of eigen values of the laplacian matrix. Eigenvalue methods has been used in combinatorics, graph theory and combinatorial optimization. We present some of this application and their results. Specially they have been used in combinatorial optimization problems such as using eigenvalue method for quadratic assignment problem and general graph partition problem, max-cut problem and also labeling problem. Also we can use eigenvalues to convert combinatorial optimization

problems to continuous optimization problems such as bisection problem, max-cut problem, generalized partition problem and the theta function. After introducing the Laplace matrix of graph and its property, relations between Laplace eigenvalues and separation properties of graphs are presented. **Keywords:** Laplacian, Eigen Value, Graph Separation, Combinatorial Optimization

Weak isometries of Hamming spaces

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Weak isometries of a metric space are permutations of the elements of the metric space that preserve a given distance p. A natural, but in general very difficult, question asks for conditions that guarantee a weak isometry will automatically be an isometry. Here we discuss this question for the Hamming spaces, both from a linear algebraic and purely combinatorial point of view.

Keywords: Hamming space, eigenvalue collapsing, weak isometry

Automorphisms of strongly regular graphs and PDS in Abelian groups

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Recently we generalized a theorem of Benson for generalized quadrangles to strongly regular graphs, deriving numerical restrictions on the number of fixed vertices and the number of vertices mapped to adjacent vertices under an automorphism. We then used this result to develop a few new techniques to study regular partial difference sets in Abelian groups. In 1994 S.L. Ma provided a list of parameter sets of regular partial difference sets with $k \leq 100$

in Abelian groups for which existence was known or had not been excluded. In particular there were 32 parameter sets for which existence was not known. In 1997 S.L. Ma excluded 13 of these parameter sets. As an application of our results we excluded the existence of a regular partial difference set for all but two of the undecided upon parameter sets from Ma's list.

Keywords: Strongly regular graph, Benson's theorem, Partial difference set, Multiplier theorem

Cyclic $(v; k_1, k_2, k_3; \lambda)$ difference families with $v \equiv 3 \pmod 4$ a prime

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We construct several new cyclic difference families $(v; k_1, k_2, k_3; \lambda)$ with $v \equiv 3 \pmod 4$ a prime and $\lambda = k_1 + k_2 + k_3 - (3v - 1)/4$. The construction is based on the method of orbits, together with an efficient algorithm to solve a corresponding 3-way matching problem. Such families can be used in conjunction with the well-known Paley-Todd difference sets to construct Hadamard and skew Hadamard matrices of order $4v$. In particular, we construct the first example of a skew Hadamard matrix of order $4 \cdot 239$.

Keywords: Difference families, PSD test, matching algorithms.

Domination Parameters of Total Graph of Finite Rings

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Let R be a commutative ring the total graph of R, $T_{\Gamma(R)}$ was introduced by Anderson and Badawi. In this talk we discuss about various number of domination parameters including the dominating number, independent dominating number, clique dominating number, connected dominating number, strong dominating number and weak dominating number of total graph of $\mathbb{Z}_n \times \mathbb{Z}_m$.

Keywords: Commutative Rings, Total Graph, Domination.

On the free Lie algebra with multiple brackets

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It is a classical result that the multilinear component of the free Lie algebra is isomorphic (as a representation of the symmetric group) to the top (co)homology of the proper part of the poset of partitions Π_n tensored with the sign representation. We generalize this result in order to study the multilinear component of the free Lie algebra with multiple compatible Lie brackets. We introduce a new poset of weighted partitions Π_n^k that allows us to generalize the result. The new poset is a generalization of Π_n and of the poset of weighted partitions Π_n^w introduced by Dotsenko and Khoroshkin and studied by the author and Wachs for the case of two compatible brackets. We prove that the poset Π_n^k with a top element added is EL-shellable and hence Cohen-Macaulay. This and other properties of Π_n^k enable us to answer questions posed by Liu on free multibracketed Lie algebras.

Keywords: Poset of partitions, Poset topology, Poset cohomology, Free Lie algebra, Shellability

Covering Some Groups with Proper Subgroups

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Any group with a finite noncyclic homomorphic image is a finite union of proper subgroups. Given such a group G, we define the covering number of G to be the least positive integer m such that G is the union of m proper subgroups. The aim of this talk is to present recent results on the determination of the covering numbers of the alternating groups on nine and eleven letters, as well as the Mathieu group M_{24}.

Keywords: group, covering number of a group.

Handicap incomplete tournaments of odd order

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A graph G with the vertex set $V(G)$, edge set $E(G)$ and $|V(G)| = n$ is called *distance magic* if there exists a bijection

$$f : V \to \{1, 2, \ldots, n\}$$

such that the *weight* of each vertex x, defined as

$$w(x) = \sum_{xy \in E(G)} f(y),$$

is equal to the same constant μ, called the *magic constant.* The labeling is called a *distance magic labeling.*
A *handicap distance antimagic labeling* of a graph $G(V, E)$ with n vertices is a bijection $\vec{f} : V \to \{1, 2, \ldots, n\}$ with the property that $\vec{f}(x_i) = i$ and the sequence of the weights $w(x_1), w(x_2), \ldots, w(x_n)$ forms an increasing arithmetic progression with difference one. A graph G is a *handicap distance antimagic graph* if it allows a handicap distance antimagic labeling.

The notions of distance magic and handicap distance antimagic labelings are closely related to *fair* and *handicap* incomplete round robin tournaments.

The spectrum of all pairs (n, r) for which there exists an r-regular handicap distance antimagic graph with n vertices has been completely determined for even n by Froncek, Kovar, Kovarova, and Shepanik. For odd n, some sporadic results were known due to the first author. We will present some new classes of handicap distance antimagic graphs with odd number of vertices.

Keywords: Distance magic labeling, handicap labeling, handicap incomplete tournaments

Perfect Hash Families with Strength Three with Three Rows

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A *perfect hash family* $\mathrm{PHF}(N; k, v, t)$ is an $N \times k$ array on v symbols with $v \geq t$ in which every $N \times t$ subarray (t is called the *strength*) contains at least one row comprised of distinct symbols. Perfect hash families have applications in information retrieval, cryptographic key distribution, secure frameproof codes, software testing, and so on. The most basic non-trivial case is PHFs with $N = t = 3$. R. A. Walker II and C. J. Colbourn [1] listed a table of existing PHFs for the case of $N = t = 3$. We are interested in constructing PHS$(3; k, v, 3)$ with k as large as possible. Here we like to show constructions of PHF$(3; q^2(q+1), q^2, 3)$ and PHF$(3; q^5, q^3, 3)$ for q a prime power . The second construction claims $k = v^{5/3}$ for $v = q^3$, which exceeds all k in the table of [1]. For the constructions of these PHFs, Quadrics Q$(4, q)$ in PG$(4, q)$ and Hermitian Varieties H$(3, q^2)$ in PG$(4, q^2)$ known as classical generalized quadrangles, see [2], are effectively used.

References

[1] R. A. Walker II and C. J. Colbourn, Perfect Hash Families: Constructions and Existence, J. Math. Crypto. (2007), 12–37.

[2] K. Thas, Symmetry in Finite Generalized Quadrangles, Birkhäuser Verlag, 2004.

Keywords: Perfect hash families, Quadrics, Hermitian varieties.

What if my syndrome is not reliable either? Coding theory for somewhat fault-tolerant quantum computing

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We consider error correction based on syndrome decoding in a situation where the extracted syndrome may also be erroneous. The simplest toy model is faulty decoding with a binary $[n, k]$ linear code $\mathcal{C}$ with its $m \times n$ parity-check matrix H over a binary symmetric channel, where the extracted syndrome $\boldsymbol{s}$ of a received vector $\boldsymbol{y} = \boldsymbol{c} + \boldsymbol{e}$ with $\boldsymbol{c} \in \mathcal{C}$ and $\boldsymbol{e} \in \mathbb{F}_2^n$ is the sum $\boldsymbol{s} = H\boldsymbol{y}^T + \boldsymbol{f}$ of the correct syndrome $H\boldsymbol{y}^T$ and a random binary vector $\boldsymbol{f} \in \mathbb{F}_2^m$ chosen from some probability space. In other words, the channel produces noise $\boldsymbol{f} \in \mathbb{F}_2^m$ on the syndrome bits on top of the usual noise $\boldsymbol{e} \in \mathbb{F}_2^n$ on the transmitted codeword. This talk focuses on how this simple scenario is related to the *phenomenological error model* in the context of fault-tolerant quantum information processing and what we can do with combinatorics.

Keywords: Quantum error correction, stabilizer code, linear code, syndrome.

Constructing the shaddow geometry of the new $W(23, 16)$

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A weighting matrix of length n and weight k denoted W(n,k) is an orthogonal $\{0, \pm 1\}$ matrix with rows of length $\sqrt{k}$. I would like to report on a new weighting matrix W(23,16), the smallest pair of numbers where the existense of the weighting matrix is currently not known. I will survey the details of the paper by the authors. This matrix was found in two stages: the one called finding the geometry which is equivalent to finding W modulu 2, and the whole matrix from the geometry called colouring. In this note we concentrate on the geometry stage.
Define [G] a (n, k) *shaddow geomtry* to be a set G of size n together with a family $\mathcal{L}$, $|\mathcal{L}| = n$, of susbstes called *lines* such that all lines have cardinality $n - k$, every two lines intersect in $n \mod 2$ points, and the dual statments. Define a shaddow matrix S for W, where each $\pm 1 \in$W is replaced by 0$\in$S and each 0$\in$W is replaced by 1$\in$S. In [G], each row of S is thought of as a line, and each column of S as a point, and S is the incidence matrix of the resulting shaddow geometry. For $(n, k) = (23, 16)$, we show that more that 50% of the pairs intersect at a single point. Therefore, finding a $(23, 16)$ geometry came as a surprise.
A local shaddow geomtry is given by a rectangular incidence matrix, satisfying an axiom system which is a natural restriction of the axiom system for the (full) shaddow geometry. Singling out a line ℓ in a shaddow geometry and restricting to the points of that line, one obtaines a local geometry. Let z_i be the number of lines $\Updownarrow$ with $|\Updownarrow \cap \ell| = i$. Then one can obtain linear equations for the $\{z_i\}$. For $(n, k) = (23, 16)$ there are 14 possible 4-tuples (z_7, z_5, z_3, z_1), from which we can study the set of all local geometries. We worked with a local geometry of the type $(z_7, z_5, z_3, z_1) = (2, 0, 4, 16)$ and extended it to a full geometry. The core of our construction is a 16×16 $\{0, 1\}$-matrix, enveloped by that local geometry and its dual. The 16×16 matrix has a 4×4 block structure with blocks coming from a small family of 512 blocks. We used linear algebra modulo 2 and branch cutting to find the complete geometry.

References

[G] A. Goldberger, On the Finite Geometry of $W(23, 16)$, http://arxiv.org/abs/1507.02063

Keywords: Finite Geometries, Weighing Matrices

The weight distribution of the self-dual $[128, 64]$ polarity design code

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The weight distribution of the binary self-dual $[128, 64]$ code being the extended code C^* of the code spanned by the incidence vectors of the blocks of the polarity design in $PG(6, 2)$ [1], [2] is computed. The code C^* has the same weight distribution as the 3rd order Reed-Muller code of length 128.

References

[1] D. Clark, D. Jungnickel, and V. D. Tonchev, Affine geometry designs, polarities, and Hamada's conjecture, *J. Combin. Theory Ser. A* **118** (2011), 231–239.

[2] D. Jungnickel and V. D. Tonchev, Polarities, quasi-symmetric designs, and Hamada's conjecture, *Des. Codes Cryptogr.* **51** (2009), 131–140.

Keywords: self-dual code, Reed-Muller code, weight distribution, polarity design.

Some New Large Sets of Geometric Designs of type $LS[3][2,3,2^8]$

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Let V be an n-dimensional vector space over the field of q elements. By a *geometric* $t-[q^n,k,\lambda]$ design we mean a collection $\mathcal{D}$ of k-dimensional subspaces of V, called blocks, such that every t-dimensional subspace T of V appears in exactly λ blocks in $\mathcal{D}$. Prior to a recent article by M. Braun, A. Kohnert, P. Östergard, and A. Wasserman, only large sets of geometric 1-designs were known to exist. In their article, the authors construct the world's first large set of geometric 2−designs LS[N][2,k,q^n], namely a LS[3][2,3,2^8], under the action of the Singer group G of order 255. In this work we construct 9, pairwise non-isomorphic, large sets LS[3][2,3,2^8], using the appropriate Kramer-Mesner matrices, and a new optimization technique based on the L^3 algorithm for lattice basis-reduction.

Keywords: geometric t-designs, large sets, Kramer-Mesner matrices, lattice basis reduction.

Resolvability of a cyclic orbit of a subset of $\mathbb{Z}_v$ and a spread decomposition of a Singer cycle of a projective line

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For an integer k, let $v(>k)$ be a multiple of k and A be a k-subset of $\mathbb{Z}_v$. In this talk, firstly, we consider the condition of A that $\mathrm{Orb}_{\mathbb{Z}_v}(A)$ is resolvable. It is obvious that $\mathrm{Orb}_{\mathbb{Z}_v}(A)$ is resolvable iff there exists a subset $B \subset \mathbb{Z}_v$ such that $|B| = \frac{v}{k}$ and $A + B \equiv \mathbb{Z}_v$, where $A + B = \{a + b \big| a \in A, b \in B\}$.
In the case when k is a power of a prime p, we will show that the existence of B is equivalent to the existence of an integer s and a subset $B' \subset \mathbb{Z}_{p^s}$ satisfying $A + B' \equiv \mathbb{Z}_{p^s}$ such that $kp\big|p^s\big|v$.
Moreover, by utilizing this result and some number theoretical properties of cyclotomic cosets we will count the number of Singer cycles of lines in $\mathrm{PG}(2n,q)$ which can be decomposed into spreads within each individual cycle when $q+1$ is a prime power.

Keywords: Resolvability of a cyclic orbit, Spead decomposition of a Singer cycle of projective lines

A New Method to Construct Self-dual Codes

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In this paper, we study binary self-dual codes of length $4n$, with a generator matrix of the form

$$G = \begin{pmatrix} I_n & 0 & A & B \\ 0 & I_n & B & A \end{pmatrix}$$

where A, B are binary square matrices of order n. Due to the orthogonal and symmetric matrices in the construction, we call it as OS-construction. Then the connection between the codes over the ring $F_2 + uF_2$ and the self-dual codes obtained from OS-construction will be established.

Keywords: self-dual codes, orthogonal matrix, symmetric matrix, codes over rings

The part-frequency matrix of a partition

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We introduce the combinatorial tool of the title, with potential utility for proofs in partition theory. As applications, we generalize some theorems of Andrews, Dixit and Yee on partitions with restrictions on odd and even parts to all moduli, and construct a new statistic that realizes Ramanujan's congruences mod 5, 7, and 11.

Keywords: partition, Glaisher correspondence

The Gray map for homogeneous weight on R_k and related binary codes

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We describe the homogeneous weight for a family of Frobenius rings namely R_k. Then we find an associated Gray map for these rings by using first order Reed-Muller codes. Quasitwited codes over R_k and their binary images under this homogeneous Gray map are studied. And it is shown that the images of these codes are self-orthogonal and quasicyclic. A substantial number of optimal binary quasicyclic codes of index 8, 16 and 24 are found as new additions to Chen's database of quasicyclic codes.

Keywords: homogeneous weights, cyclic codes, quasi-cyclic codes, codes over rings

Computing Minimum Rainbow and Strong Rainbow Colorings of Block Graphs

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A path in an edge-colored graph G is *rainbow* if no two edges of it are colored the same. The graph G is *rainbow colored* if there is a rainbow path between every pair of vertices. If there is a rainbow shortest path between every pair of vertices, the graph G is *strong rainbow colored.* The minimum number of colors needed to make G rainbow colored is known as the *rainbow connection number*, and is denoted by $rc(G)$. Similarily, the minimum number of colors needed to make G strong rainbow colored is known as the *strong rainbow connection number*, and is denoted by $src(G)$. In this talk, we consider the rainbow and strong rainbow connection numbers of block graphs, which form a subclass of chordal graphs. We give an exact linear time algorithm for strong rainbow coloring block graphs by exploiting a clique tree representation each chordal graph has. We derive a tight upper bound on $rc(G)$, where G is a block graph. We will also discuss some ideas and progress on finding bounds for the rainbow and strong rainbow connection numbers for general graphs.

Keywords: rainbow colored, strong rainbow colored, block graph

On the Hamilton Waterloo Problem for Complete Equipartite Graphs

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Given two 2-factors F_1 and F_2 with n vertices, the Hamilton Waterloo Problem studies the decomposition of the Complete Graph K_n into isomorphic copies of F_1 and F_2. In this work we study a similar problem but decomposing Complete Equipartite Graphs instead of Complete Graphs. To achieve our decompositions we introduce a Graph Product for multipartite graphs. **Keywords:** Graph Decomposition, Hamilton Waterloo Problem, Oberwolfach Problem, Multipartite Graphs, Graph Products

Lattice Basis Reduction techniques based on the LLL algorithm

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In this paper we present results of our experiments based on two different techniques, each requiring a large number of parallel calls of the LLL algorithm, while atempting to solve the famous *Lattice basis reduction* problem. In particular, we obtain best possible results for a number of competition instances in the problem.

Our *Sub-lattices Diffusion Algorithm* is a lattice basis reduction technique based on the LLL algorithm. It relies on performing a large number of LLL reductions on permuted bases of a family of, not necessarily disjoint, sublattices and then fusing the reduced bases of the sublattices. We discuss improvements achieved by using the sub-lattices diffusion algorithm, over a standard (single) LLL reduction. For this, we compare the experimental results of the LLL algorithm, with the sub-lattices diffusion algorithm for random, inflated, orthogonal bases, and ideal lattice bases.

We also present the result of our experiments based on a *hill climbing* algorithm, using a number of parallel LLL executions, to succesfully reduce an initial basis of lattice L. As a test case, we begin with a very short basis H consisting of the rows of a Hadamard matrix, inflate H by multiplying it with a pseudo-random integral unimodular matrix M to obtain a basis B = MH for the same lattice, but of very large weight, and then test our approach on the new basis B. The method produces a basis equivalent to the original one.

Keywords: LLL Lattice Basis Reduction, permutation matrix, Integer unimodular matrix.

Group divisible designs with blocksize 3 and 5 groups.

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A *group divisible design with block size three* (3-GDD) is a triple (X, G, B), where X is a set of *points*, G is a partition of X into subsets called *groups* and B is a set of 3-element subsets of X (called *triples*) such that every pair of points is either in a triple or a group but not both. If there are n_i groups of size g_i, $i = 1, 2, \ldots, r$ we say that the type of the 3-GDD is $g_1^{n_1} g_2^{n_2} g_3^{n_2} \cdots g_r^{n_r}$ and denote such a design by 3-GDD$(g_1^{n_1} g_2^{n_2} g_3^{n_2} \cdots g_r^{n_r})$. They are equivalent to a K_3-decomposition of the complete multipartite graph whose partite sets are the groups.

In this talk we show that a 3-GDD(g^3u^2) exists if and only if g and u have the same parity, $3|u$ and $u \leq 3g$.

We also discuss what is currently known for 3-GDDs with five groups of arbitrary type.

This is joint work with **Charles Colbourn** and **Melissa Keranen**.

Keywords: group divisible design, graph decomposition, 3-GDD

A Normal Quotient Analysis for Some Families of Oriented Four-Valent Graphs

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We analyse the normal quotient structure of several infinite families of finite connected edge-transitive, four-valent oriented graphs. These families were singled out by Marusic and others to illustrate various internal structures for these graphs in therms of their alternating cycles (cycles in which consecutive edges have opposite orientations). We discover new interesting properties of these graph families involving their normal quotients, and in particular, we determine which oriented graphs in each family are 'basic', relative to forming normal quotients, (that is, all their proper normal quotients are degenerate). This new analysis therefore identifies a special subfamily of basic oriented graphs with the property that each graph in the families is a normal cover of a basic one.

Keywords: edge-transitive graphs, oriented graphs, cyclic qutient graph, transitive group.

A modified four circulant construction for self-dual codes

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We propose a modified four circulant construction for self-dual codes and a bordered version of the construction using the properties of λ-circulant and λ-reverse circulant matrices. By using the constructions on $\mathbb{F}_2$ new binary codes of lengths 64 and 68 are obtained. The constructions are applied on R_2. Moreover, new singly-even extremal binary self-dual codes of lengths 66 and 68 are constructed as $\mathbb{F}_2$ and R_1-extensions. More precisely, 3 new codes of length 64, 15 new codes of length 66 and 22 new codes of length 68 are constructed. Codes with these weight enumerators are constructed for the first time in the literature.

Keywords: extremal self-dual codes, Gray maps, circulant constructions, codes over rings.

Non-existence of strongly regular graphs with feasible block graph parameters of quasi-symmetric designs

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A quasi-symmetric design(QSD) is a (v, k, λ) design with two intersection numbers x, y, where $0 \leq x < y < k$. The block graph of QSD is a strongly regular graph(SRG). It is known that there are SRGs which are not block graphs of QSDs. We derive necessary conditions on the parameters of a SRG to be feasible as the block graph of a QSD. We apply these condtions to rule out many infinite families such SRGs.

Keywords: Quasi-symmetric design, Strongly regular graph, Block graph.

Infinite Log-Concavity And r-Factor

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D. Uminsky and K. Yeats [7] studied the properties of the *log-operator* $\mathcal{L}$ on the subset of the finite symmetric sequences and prove the existence of an infinite region $\mathcal{R}$, bounded by parametrically defined hypersurfaces such that any sequence corresponding a point of $\mathcal{R}$ is *infinitely log concave.* Following the similar pattern, we instead study the properties of a new operator $\mathcal{L}_r$ and redefine the parametrically defined hypersurfaces which generalizes the one defined by Uminsky and Yeats [7] and then show that any sequence corresponding a point of the region $\mathcal{R}$, bounded by the new generalized parametrically defined r-factor hypersurfaces, is *Generalized r-factor infinitely log concave.* This in fact redefine and generalizes the log-concavity region $\mathcal{R}$ for Generalized r-factor log-concavity. We also give an improved value of $r_\circ$ found by McNamara and Sagan [4] as the log-concavity criterion using the new log-operator.

References

[1] Boros, G. and Moll, V.,*Irresistible Integrals: Symbolics, Analysis and Experiments in the Evaluation of Integrals.*, Oxford University Press, Cambridge (2004).

[2] Brenti, Francesco: *Log-concave and unimodal sequences in algebra, combinatorics, and geometry*: an update. In Jerusalem combinatorics '93, vol. 178 of Contemp. Math. Amer. Math. Soc., Providence, RI, 1994, pp. 71-89.

[3] Kauers, M., and Paule, P. *A computer proof of Moll's log-concavity conjecture*. Proc. Amer. Math. Soc. 135, 12 (2007), 3847-3856.

[4] P. R. W. McNamara and B. E. Sagan,*Infinite log-concavity: Developments and conjectures*, Advances in Applied Mathematics 44 (2010).

[5] Piero Giacomelli: *Log-concavity of Lucas sequences of first kind*, arXiv:1101.1805v4 [math.NT] 7 Mar 2011.

[6] Stanley, R. P. *Log-concave and unimodal sequences in algebra, combinatorics, and geometry*. In Graph theory and its applications: East and West (Jinan, 1986), vol. 576 of Ann. New York Acad. Sci. New York Acad. Sci.,New York, 1989, pp. 500-535.

[7] Uminsky, D., and Yeats, K. *Unbounded regions of infinitely logconcave sequences*. Electronic Journal of Combinatorics. 14(2007).

On self-dual binary codes invariant under permutation groups

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THIS IS JOINT WORK WITH S. MUKWEMBI AND T. M. MUDZIIRI SHUMBA

We find self-dual codes invariant under some sporadic simple and almost simple groups of various lengths and attempt to characterise them using minimum distances and automorphism groups. We also prove that for $n \geq 4$, there does not exist a self-dual code of length n invariant under S_n. We further show that there is no self-dual binary code of length $n \geq 4$ invariant under A_n. This gives a sharper result on the possible automorphism group of a doubly even self-dual than one given by Günther and Nebe [1].

References

[1] A. Günther and G. Nebe Automorphisms of doubly even self-dual codes, *Bull. London Math. Soc*, 41 (2009), 769-778.

Symmetric functions and quasisymmetric functions

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The ring of quasisymmetric functions is free over the ring of symmetric functions. This result was previously proved by M. Hazewinkel combinatorially through constructing a polynomial basis for quasisymmetric functions. The recent work by A. Savage and O. Yacobi on representation theory provides a new proof to this result. In this talk, I will explain the representation theoretic view and mention some further applications.

Keywords: symmetric functions, quasisymmetric functions, representation theory

New Extremal Self-dual codes from Codes over $\mathcal{R}_{k,m}$

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Self-dual codes take an important place in coding theory due to their relations with designs, lattices and invariant theory.There are different methods to construct self-dual codes. In this study, we have used extension methods to get new self-dual codes over a newly defined family of rings that we denote by $\mathcal{R}_{k,m}$, where $\mathcal{R}_{k,m} = \mathbb{F}_2[u,v]/\left\langle u^k, v^m, uv - vu \right\rangle$. We have defined a duality-preserving Gray map from $\mathcal{R}_{k,m}$ to $\mathbb{F}_2^{km}$, and we find many new binary self-dual codes as the Gray images extensions.

Keywords: *extremal self-dual codes, Gray maps, codes over rings, extension theorems*

Tight Sets and m-Ovoids of Quadrics

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This is a talk about tight sets and m-ovoids of classical polar spaces. Tight sets and m-ovoids are important substructures of classical polar spaces. They are not only interesting in their own right, but also can give rise to other geometric/combinatorial objects, such as translation planes, strongly regular graphs, two-weight codes, etc. In this talk, we will talk about a construction of $\frac{q^2-1}{2}$-tight sets of $Q^+(5,q)$, the Klein quadric, for $q \equiv 5$ or $9 \pmod{12}$, and a recent construction of $\frac{q-1}{2}$-ovoids of $Q(4,q)$, the parabolic quadric of $PG(4,q)$, for $q \equiv 3 \pmod 4$.

Keywords: Generalized quadrangle, m-ovoid, ovoid, polar space, quadric, tight set.

Orthogonal arrays and divisible designs derived from a Baer subplane of a projective plane

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Fuji-Hara and Kamimura (Utilitas Math. 43, 65-70, 1993) gave a method for constructing orthogonal arrays of strength 2 with $q(q-1)$ symbols, for a prime power q, on the complement of a Baer subplane. We describe a construction of orthogonal arrays with the same numbers of constraints and symbols but with smaller sizes and indices. For $q \equiv 2 \pmod 3$ and $q > 2$, our construction gives a new series of orthogonal arrays which cannot be obtained by Bush's direct product construction. This talk is based on a joint work with Nobuko Miyamoto.

Keywords: Orthogonal array, Baer subplane, Projective plane

A novel Approach for Constructing Reversible Codes over Different Alphabets[3]

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In this work we introduce a novel approach to find reversible codes over different alphabets, using so-called coterm polynomials and a module-construction. We obtain many optimal reversible codes with these constructions. We are also able to find reversible and reversible-complement codes that are not necessarily linear cyclic codes. This marks the distinction from the usual construction methods of reversible codes as cyclic codes. We also mention a few applications to DNA codes over different alphabets.

Keywords: reversible codes, coterm polynomials, module construction, reversible complement property

[3]The work presented here is supported by the Scientific and Technological Research Concil of Turkey(TUBITAK), Grant No:113F071

Partitions with distinct parts and unimodality

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In this talk, we discuss the (non)unimodality of the rank-generating function F_λ of the poset of partitions *with distinct parts* whose Ferrers diagrams are contained inside the Ferrers diagram of a given partition λ. This work, in collaboration with Richard Stanley, has in part been motivated by an attempt to place into a more general context the unimodality of $F_\lambda(q) = \prod_{i=1}^{n}(1+q^i)$, namely the rank-generating function associated to the "staircase" partition $\lambda = (n, n-1, \ldots, 1)$, for which determining a combinatorial proof remains an outstanding open problem to this day.
Surprisingly, our type of results present some remarkable similarities to those shown in 1990 by Dennis Stanton, who extended, to an arbitrary partition λ, the study of the unimodality of the q-binomial coefficient — i.e., the rank-generating function of the poset of *arbitrary* partitions whose Ferrers diagrams are contained inside a given rectangular Ferrers diagram. We will be mentioning several conjectures or open problems during the talk, and discuss some of the most recent developments, including a (prize-winning!) paper by Levent Alpoge that has solved my conjecture with Stanley on the unimodality of F_λ when λ is the "truncated staircase" partition $(n, n-1, \ldots, n-(b-1))$, for $n \gg b$.

Keywords: Integer partition; shifted Ferrers diagram; q-binomial coefficient; partition with distinct parts; unimodality; q-analog; generating function; bijective proof.

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