Bounding the Effective Yield Behavior of Mixtures

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Abstract

The yield surface of a mixture of elastic/plastic materials is examined through mathematical techniques. A variational principle is derived from the local yield behavior (which may be anisotropic) and is used to generate upper bounds on the effective yield surface of the mixture. These bounds effectively reduce estimates on the nonlinear problem of yield to estimates on the elastic properties of mixtures of *linearly elastic* "reference materials." Upper bounds obtained from the variational principle are shown to be tighter than previous bounds, provided one uses a linear bound which is tighter than the arithmetic-mean bound. Finally, explicit bounds are derived for the yield surface of anisotropic mixtures of two von Mises type materials when one phase takes the form of ellipsoidal inclusions embedded in a matrix of the second phase, and difficulties arising in applications to polycrystals are discussed.

1 Introduction

The variational principles proposed by Hashin and Shtrikman [1] and developed by Walpole [2] sparked a great deal of research in the behavior of mixtures of materials having linear constitutive laws (see, for example, the reviews in [3] through [6]). It is surprising, then, that similar variational principles were developed only recently for materials with nonlinear behavior (as in [7], [8]). In this paper, we give an example of such a variational principle and the derivation of bounds for the yield behavior of mixtures.

2 Linear Elasticity

For purposes of analogy, we begin with a brief review of some basic results for linearly elastic mixtures: the principle of minimum potential energy and a derivation of the Hashin-Shtrikman/Walpole bounds.

We consider a mixture occupying a region Ω, in which the material at position x has moduli $C(x)$:

$$\sigma(x) = C(x)\,\epsilon(x), \qquad x \in \Omega. \tag{1}$$

$\sigma(x)$ and $\epsilon(x)$ represent the local stress and strain when, for example, boundary displacements $\epsilon^0 x$ are applied. Assuming that the material is at equilibrium and that displacements are continuous (so the materials are "perfectly bonded"), we have

$$\nabla \cdot \sigma = 0 \tag{2}$$