Improved Bounds on the Effective Yield Surface of Inhomogeneous Rigid/Plastic Materials

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Abstract

The yield surface of a mixture of rigid/perfectly-plastic materials is examined through mathematical techniques. A new variational principle is derived from the local yield behavior (which may be anisotropic) and is used to generate upper bounds on the effective yield surface of the mixture. These bounds effectively reduce estimates on the nonlinear problem of yield to estimates on the elastic properties of mixtures of *linearly elastic* "reference materials." Upper bounds obtained from the variational principle are shown to be tighter than previous bounds, provided one uses a linear bound which is tighter than the arithmetic-mean bound. Finally, explicit bounds on the yield surface of mixtures of two von Mises type materials are calculated in order to demonstrate the power of the new variational principle.

1 Introduction

Many materials deform elastically under small applied stresses, but exhibit plasticity under sufficiently large stress. The set of stresses at which the deformation changes from elastic to plastic is characteristic of the given material and is called the "yield surface." Naturally this is an important characteristic in the processing of materials, where the goal is to produce permanent deformations.

The simplest types of yield surfaces are those of isotropic materials, for which there are two widely–used models. Tresca (1864) and Saint Venant (1870) proposed a model in which plastic deformation begins when the shear stress exceeds a certain critical value. In this model the yield surface is bounded by a set of planes in stress space. von Mises' (1913) model exhibits plastic deformation when the second invariant of stress becomes too large, and can be thought of as a smooth approximation to the Tresca/Saint Venant model (cf. Kachanov (1971)).

Although the Tresca/Saint Venant and von Mises models are in reasonable agreement with empirical data for certain materials, they can not describe materials which exhibit plastic anisotropy. Single crystals which deform by slip on certain planes ("slip planes") were studied by Schmid and Siebel (1931) in the beginning of the century and were shown to have anisotropic plasticity. Schmid proposed that these crystals deformed when the shear stress *resolved on to one of these slip planes*

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reached a critical value. Thus yielding depended on the orientation of the crystal lattice relative to the applied shear stress.

Because polycrystalline metals are made up of single crystallites with varying orientations, it was natural to ask what yield behavior such a mixture would have, based on the single–crystal behavior. G.I. Taylor (1938) developed a model in the 1930's which calculated the yield surface for an aggregate of cubic crystallites based on the assumption that each crystallite undergoes the same deformation as the aggregate. On the other hand, Sachs (1928) used a model in which all of the grains experienced the same stress. These estimates provide upper and lower bounds, respectively, on the yield surface.

There has been little work done on bounding yield surfaces for heterogeneous materials since the work of Sachs and of Taylor (and related work of Bishop and Hill (1951)). However, there has been an enormous growth in the literature dealing with the effective linear elastic properties of heterogeneous materials. Most notable are probably the work of Hashin and Shtrikman (1962a), who introduced new variational principles, and others who incorporated 2-point and n-point statistics in bounds (e.g., Beran and Molyneux (1966) and Kröner (1977)). Hashin and Shtrikman's primary innovation was the introduction of a homogeneous comparison medium, a method which has recently been extended to bounds on nonlinear inhomogeneous media (Ponte-Castañeda (1991), Willis (1910)). Also quite recently, a new technique dubbed the "translation method" has been very useful in generating bounds for linear properties (cf. Milton (1990)). Some reviews of results for linear heterogeneous materials may be found in Willis (1981), McCoy (1981), and Hashin (1983).

In the following, we will show that these results for linear media can be used to generate improved bounds on the yield surface of mixtures of rigid/perfectly-plastic materials. Concurrent work by Ponte-Castañeda and deBotton (1992) and Michel and Suquet (1992) has generated variational principles and bounds similar to those described here. Their results apply to mixtures of materials obeying a von Mises yield criterion and are derived in the context of homogenization. It should be noted that the bounds derived herein are applicable also to mixtures of *an*isotropic materials, allowing these bounds to be calculated for polycrystals deforming by slip (Sam, Olson, and Adams (1993)). Thus these bounds can make use of recent advances in automated experimental determination of two-point statistics of lattice orientation in polycrystals (see, for example, Adams, Wright, and Kunze (1993)).

The first two sections of the paper summarize and reformulate the information already known about yield in order to establish a basic variational principle. In section (4), the energy of a fictitious linear heterogeneous reference medium is introduced into the variational principle in order to derived the new upper-bound variational principle. This principle is comprised of a first part which may be bounded by any of the applicable linear theories, and a second (nonlinear) part involving the local yield surface. After a brief description of the effects of incompressibility, we show that the new upper-bound variational principle reproduces the Taylor bound when the arithmetic mean bound is used on the linear part. This means that using a better linear bound always gives us a yield bound tighter than Taylor's. In the final section, we work two illustrative examples for mixtures of von Mises materials, one under the assumption of macroscopic isotropy and the other without this restriction.

2 Preliminaries

We shall be concerned primarily with materials that experience little elastic deformation before reaching yield, so that they may be modeled by a rigid perfectly-plastic material. We begin by