PH2200 Formula Sheet Knight

Electric Charges and Forces

$$F_{1\text{ on }2} = F_{2\text{ on }1} = \frac{K|q_1||q_2|}{r^2}$$

$$q = (N_p - N_e)e$$

$$\vec{E} = \vec{F}_{\text{on q}}/q$$

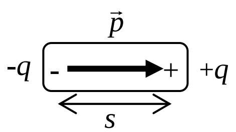
$$\vec{F}_{\text{on B}} = q_B\vec{E}$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r} \quad \text{point charge}$$

The Electric Field

$$\vec{E}_{\text{net}} = \sum_i \vec{E}_i$$

Electric dipole:



$$\vec{p} = (qs,\ \text{from negative to positive})$$

Field on axis $\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{2\vec{p}}{r^3}$

Field in bisecting plane $\vec{E} = -\frac{1}{4\pi\varepsilon_0}\frac{\vec{p}}{r^3}$

linear charge density: $\lambda = \frac{Q}{L}$

surface charge density: $\eta = \frac{Q}{A}$

volume charge density: $\rho = \frac{Q}{V}$

Uniform infinite line of charge:

$$\vec{E} = \left(\frac{1}{4\pi\varepsilon_0}\frac{2\lambda}{r},\ \text{perpendicular to line}\right)$$

Uniform infinite plane of charge:

$$\vec{E} = \left(\frac{\eta}{2\varepsilon_0},\ \text{perpendicular to plane}\right)$$

Uniformly charged sphere:

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r} \quad \text{for } r \ge R$$

Parallel-plate capacitor:

$$\vec{E} = \left(\frac{\eta}{\varepsilon_0},\ \text{from positive to negative}\right)$$

$$(E_{\text{ring}})_z = \frac{1}{4\pi\varepsilon_0}\frac{zQ}{(z^2+R^2)^{3/2}}$$

$$(E_{\text{disk}})_z = \frac{\eta}{2\varepsilon_0}\left[1 - \frac{z}{\sqrt{z^2+R^2}}\right]$$

$$\vec{a} = (q/m)\vec{E}$$

$$\tau = pE\sin\theta$$

Gauss's Law

$$\Phi_e = \vec{E}\cdot\vec{A} = EA\cos\theta \quad \text{constant field}$$

$$\Phi_e = \int_{\text{surface}} \vec{E}\cdot d\vec{A}$$

$$\Phi_e = \oint \vec{E}\cdot d\vec{A} = \frac{Q_{\text{in}}}{\varepsilon_0}$$

$\vec{E}$ at surface of a charged conductor:

$$E_{\text{surface}} = \begin{cases} \frac{\eta}{\varepsilon_0} \\ \text{perpendicular to surface} \end{cases}$$

The Electric Potential

$$U_{\text{elect}} = U_0 + qEs \quad \text{(parallel-plate capacitor)}$$

$$U_{q_1+q_2} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r} \qquad U_{\text{elect}} = \sum_{i<j}\frac{1}{4\pi\varepsilon_0}\frac{q_i q_j}{r_{ij}}$$

$$U_{\text{dipole}} = -\vec{p}\cdot\vec{E} = -pE\cos\theta$$

$$U_{q+\text{sources}} = qV \qquad V = \frac{U_{q+\text{sources}}}{q}$$

$$V = Es \quad \text{(inside a parallel-plate capacitor)}$$

$$E = \frac{\Delta V_C}{d} \quad \text{(parallel plate capacitor)}$$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} \quad \text{point charge}$$

$$V = \sum_i \frac{1}{4\pi\varepsilon_0}\frac{q_i}{r_i}$$

Potential and Field

$$\Delta V = V(s_f) - V(s_i) = -\int_{s_i}^{s_f} E_s\,ds$$

$= \text{the negative of the "area"}$

$$E_s = -\frac{dV}{ds}$$

$$\Delta V_{\text{loop}} = \sum_i (\Delta V)_i = 0$$

$$\Delta V_{\text{bat}} = \frac{W_{\text{chem}}}{q} = \mathcal{E} \quad \text{(ideal battery)}$$

$$C = \frac{Q}{\Delta V_C} \qquad C = \kappa C_0$$

$$C = \frac{\varepsilon_0 A}{d} \quad \text{(parallel-plate capacitor)}$$

$$C_{\text{eq}} = C_1 + C_2 + C_3 + \dots \quad \text{(parallel)}$$

$$C_{\text{eq}} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots\right)^{-1} \quad \text{(series)}$$

$$U_C = \frac{Q^2}{2C} = \frac{1}{2}C(\Delta V_C)^2 \qquad u_E = \frac{\varepsilon_0}{2}E^2$$

Current and Conductivity

Electron current:

$i = \text{rate of electron flow}$

$N_e = i\Delta t$

$i = nAv_d$

$v_d = \frac{e\tau}{m}E$

Conventional current:

$I = \text{rate of charge flow} = ei$

$Q = I\Delta t$

Current density:

$J = I/A$

$J = nev_d = \sigma E$

$\sigma = \frac{ne^2\tau}{m} = \frac{1}{\rho}$

$$\sum I_{\text{in}} = \sum I_{\text{out}}$$

$$E_{wire} = \frac{\Delta V_{\text{wire}}}{L}$$

$$I = \frac{\Delta V_{\text{wire}}}{R} \quad \text{where } R = \frac{\rho L}{A}$$

Fundamentals of Circuits

$$I = \frac{\Delta V}{R}$$

junction law: $\sum I_{\text{in}} = \sum I_{\text{out}}$

loop law: $\Delta V_{loop} = \sum_i (\Delta V)_i = 0$

$$P_{\text{bat}} = I\mathcal{E}$$

$$P_{\text{R}} = I\Delta V_{\text{R}} = I^2 R = \frac{(\Delta V_{\text{R}})^2}{R}$$

$$R_{\text{eq}} = R_1 + R_2 + R_3 + \ldots + R_{\text{N}} \quad \text{(series)}$$

$$R_{\text{eq}} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots + \frac{1}{R_{\text{N}}}\right)^{-1} \quad \text{(parallel)}$$

$$Q = Q_0 e^{-t/\tau} \qquad I = I_0 e^{-t/\tau} \qquad \tau = RC$$

The Magnetic Field

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2} = \left(\frac{\mu_0 |q| v\sin\theta}{4\pi r^2},\ \text{RHR}\right)$$

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{I\Delta\vec{s}\times\hat{r}}{r^2} = \left(\frac{\mu_0 I(\Delta s)\sin\theta}{4\pi r^2},\ \text{RHR}\right)$$

$$B_{\text{long straight wire}} = \frac{\mu_0}{2\pi}\frac{I}{d} \qquad B_{\text{coil center}} = \frac{\mu_0}{2}\frac{NI}{R}$$

$$\vec{\mu} = (AI,\ \text{from south pole to north pole})$$

$$\vec{B}_{\text{dipole}} = \frac{\mu_0}{4\pi}\frac{2\vec{\mu}}{z^3} \quad \text{(on axis of dipole)}$$

$$\oint \vec{B}\cdot d\vec{s} = \mu_o I_{\text{through}}$$

$$B_{\text{solenoid}} = \frac{\mu_o NI}{L}$$

$$\vec{F}_{\text{on q}} = q\vec{v}\times\vec{B} = (|q|vB\sin\theta,\ \text{RHR})$$

$$f_{\text{cyc}} = \frac{qB}{2\pi m} \qquad r_{\text{cyc}} = \frac{mv}{qB}$$

$$\vec{F}_{\text{wire}} = I\vec{L}\times\vec{B} = (ILB\sin\theta,\ \text{RHR})$$

$$F_{\text{parallel wires}} = \frac{\mu_0 L I_1 I_2}{2\pi d}$$

$$\vec{\tau} = \vec{\mu}\times\vec{B} = (\mu B\sin\theta,\ \text{RHR})$$

Electromagnetic Induction

$$\mathcal{E} = vlB$$

$$\Phi_{\text{m}} = \vec{A}\cdot\vec{B} = AB\cos\theta \quad \text{(uniform } \vec{B}\text{-field)}$$

$$\Phi_{\text{m}} = \int_{\text{area of loop}} \vec{B}\cdot d\vec{A}$$

$$\mathcal{E} = N\left|\frac{d\Phi_{\text{per turn}}}{dt}\right|$$

$$\mathcal{E}_{\text{coil}} = \omega ABN\sin\omega t$$

$$V_2 = \frac{N_2}{N_1}V_1$$

Electromagnetic Fields and Waves

$$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{\text{in}}}{\varepsilon_0}$$

$$\oint \vec{B}\cdot d\vec{A} = 0$$

$$\oint \vec{E}\cdot d\vec{s} = -\frac{d\Phi_{\text{m}}}{dt}$$

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{\text{through}} + \varepsilon_0\mu_0\frac{d\Phi_{\text{e}}}{dt}$$

$$\vec{F} = q(\vec{E} + \vec{v}\times\vec{B})$$

$$I_{\text{disp}} = \varepsilon_0\frac{d\Phi_{\text{e}}}{dt}$$

$$v_{\text{em}} = c = 1/\sqrt{\varepsilon_0\mu_0}$$

$$c = \lambda f$$

$$E = cB$$

$$\vec{S} = \frac{1}{\mu_0}(\vec{E}\times\vec{B})$$

$$I = \frac{P}{A} = S_{\text{avg}} = \frac{1}{2c\mu_0}E_0^2 = \frac{c\varepsilon_0}{2}E_0^2$$

$$p_{\text{rad}} = \frac{F}{A} = \frac{I}{c} \quad \text{(perfect absorber)}$$

$$I = I_0\cos^2\theta$$

$$I_{\text{transmitted}} = \frac{1}{2}I_0 \quad \text{(incident light unpolarized)}$$

Physical Constants

$K = 8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2$

$\varepsilon_0 = 8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2$

$e = 1.60\times10^{-19}\ \text{C}$

$m_{\text{e}} = 9.11\times10^{-31}\ \text{kg}$

$m_{\text{p}} = 1.67\times10^{-27}\ \text{kg}$

$\mu_0 = 1.26\times10^{-6}\ \text{T}\cdot\text{m/A}$

$c = 3.00\times10^{8}\ \text{m/s}$

Useful Geometry

Circle

Area = πr^2

Circumference = $2\pi r$

Sphere

Surface area = $4\pi r^2$

Volume = $\frac{4}{3}\pi r^3$

Cylinder

Lateral surface area = $2\pi rL$

Volume = $\pi r^2 L$

PH2100 in Brief

$$\vec{F}_{\text{net}} = \sum_i \vec{F}_i = m\vec{a}$$

$$\vec{F}_{\text{A on B}} = -\vec{F}_{\text{B on A}}$$

$$F_{\text{spring}} = -k\Delta s$$

Constant Acceleration:

$$x_{\text{f}} = x_{\text{i}} + v_{\text{i}x}\Delta t + \tfrac{1}{2}a_x(\Delta t)^2$$

$$y_{\text{f}} = y_{\text{i}} + v_{\text{i}y}\Delta t + \tfrac{1}{2}a_y(\Delta t)^2$$

$$v_{\text{f}x} = v_{\text{i}x} + a_x\Delta t$$

$$v_{\text{fy}} = v_{\text{iy}} + a_y\Delta t$$

$$v_{\text{f}x}^2 = v_{\text{i}x}^2 + 2a_x(x_{\text{f}} - x_{\text{i}})$$

$$v_{\text{fy}}^2 = v_{\text{iy}}^2 + 2a_y(y_{\text{f}} - y_{\text{i}})$$

Uniform Circular Motion:

$$v = \frac{2\pi r}{T} \qquad \omega = \frac{2\pi\ \text{rad}}{T}$$

$$\theta_{\text{f}} = \theta_{\text{i}} + \omega\,\Delta t$$

$$a_r = \frac{v^2}{r} = \omega^2 r$$

Energy Conservation

$$K = \tfrac{1}{2}mv^2$$

$$E_{\text{mech}} = K + U$$

$$K_{\text{f}} + U_{\text{f}} = K_{\text{i}} + U_{\text{i}}$$

$$P = \vec{F}\cdot\vec{v} = Fv\cos\theta$$