Linear codes with complementary duals from regular graphs
invariant under finite groups

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Let $F$ be a finite field of $q$ elements, and $G$ be a transitive group on a finite set $\Omega$. Then there is a $G$-action on $\Omega$, namely a map $\cdot:G \times \Omega \rightarrow \Omega$, $(g, w) \mapsto w^g = g \cdot w$, satisfying $w^{gg'} = (gg')w = g(g'w)$ for all $g, g' \in G$ and all $w \in \Omega$, and that $w^1 = 1w = w$ for all $w \in \Omega$. Let $F\Omega = \{ f : \Omega \rightarrow F \}$, be the vector space over $F$ with basis $\Omega$. Extending the $G$-action on $\Omega$ linearly, $F\Omega$ becomes an $FG$-module called an $FG$-permutation module. We are interested in finding all $G$-invariant $FG$-submodules, i.e., codes in $F\Omega$. The elements $f \in F\Omega$ are written in the form $f = \sum_{w \in \Omega} a_w \chi_w$ where $\chi_w$ is a characteristic function. The natural action of an element $g \in G$ is given by $g(\sum_{w \in \Omega} a_w \chi_w) = \sum_{w \in \Omega} a_w \chi_{g(w)}$. This action of $G$ preserves the natural bilinear form defined by

$$\langle \sum_{w \in \Omega} a_w \chi_w, \sum_{w \in \Omega} b_w \chi_w \rangle = \sum_{w \in \Omega} a_w b_w.$$

By way of illustration we determine all linear codes of length 50 over $F_p$ ($p$ a prime) which admit the projective special unitary group $U_3(5)$ as an automorphism group. By group representation theory means we prove that these can all be realized as submodules of the permutation module $F\Omega$ where $\Omega$ corresponds to the vertex set of the Hoffman-Singleton graph.