A Spectral Graph Sparsification Approach to Scalable Vectorless Power Grid Integrity Verification

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(I) Motivation

- Smaller feature sizes and reduced supply voltages make performance of modern ICs more vulnerable to supply voltage variations.
- It is indispensable to verify the robustness of PDNs, ensuring that supply voltage fluctuations do not exceed certain thresholds.
- Power grid voltage integrity verification is critical to reliable PDNs design.

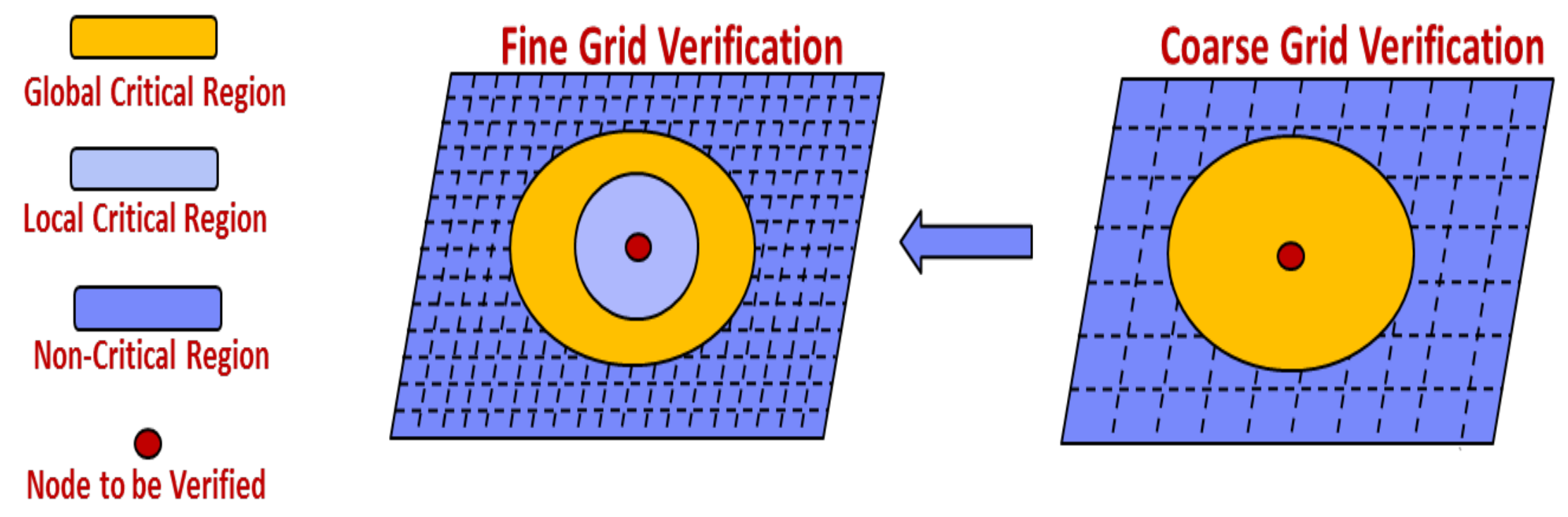
(II) Background

- **Linear programming (LP) formula for maximizing voltage drop on a node i:**

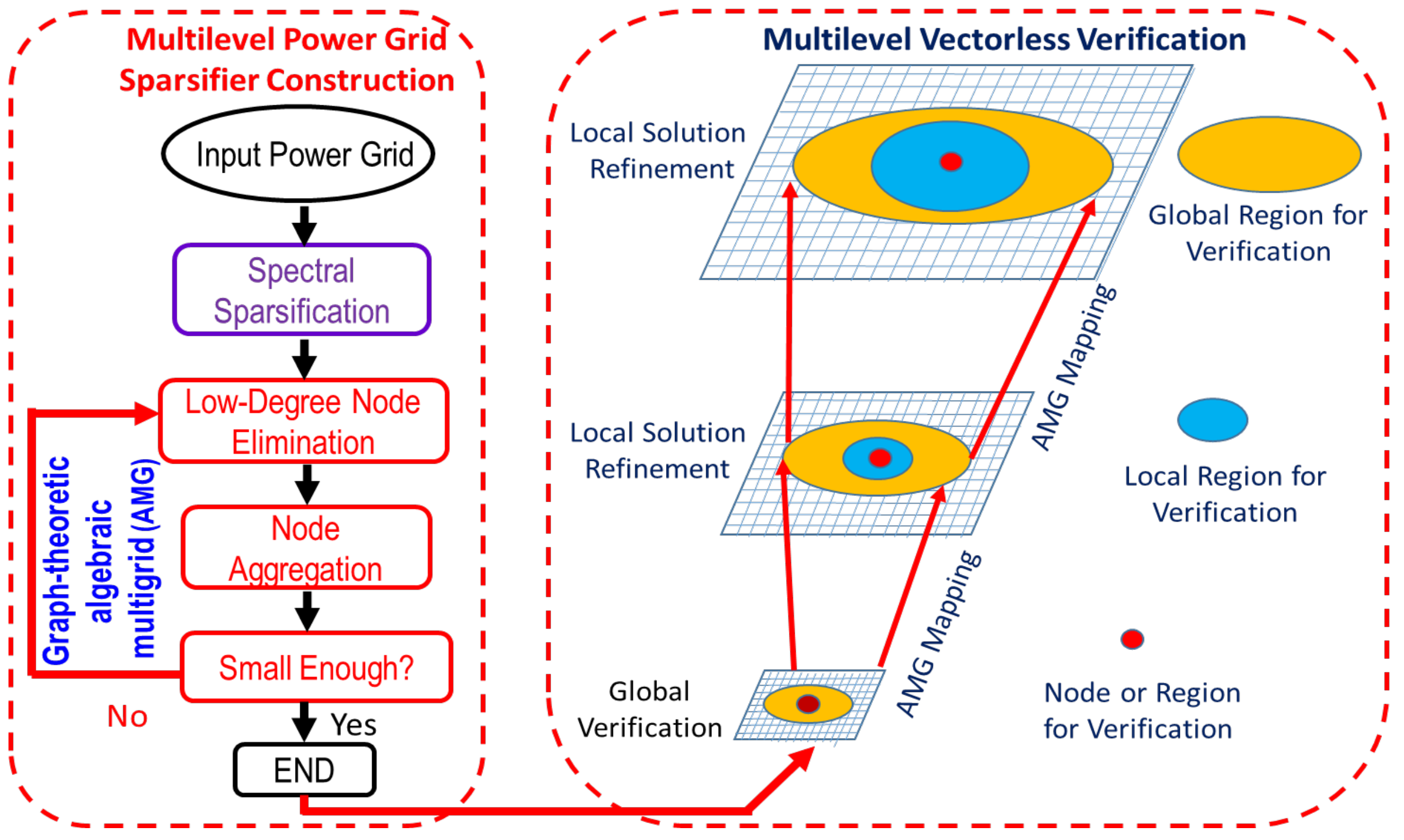
  Maximize: $v_i = e_i^T E^{-1} b$ **for** $i = 1, \cdots, n$ **,where** $E = G^{-1}$**,** $G \cdot v = b$ **and** $e_i^T = [0, \cdots, 1, \cdots, 0]$

  S. t. the local and global current constraints: $b^L \le b \le b^U, 0 \le Qb \le b_g$

- **Multilevel voltage integrity verification framework (Feng, DAC'13):**
  - Create a hierarchy of voltage verification problems ordered from finest to coarsest levels
  - Successively/incrementally tackle the coarsest to the finest level verification problems



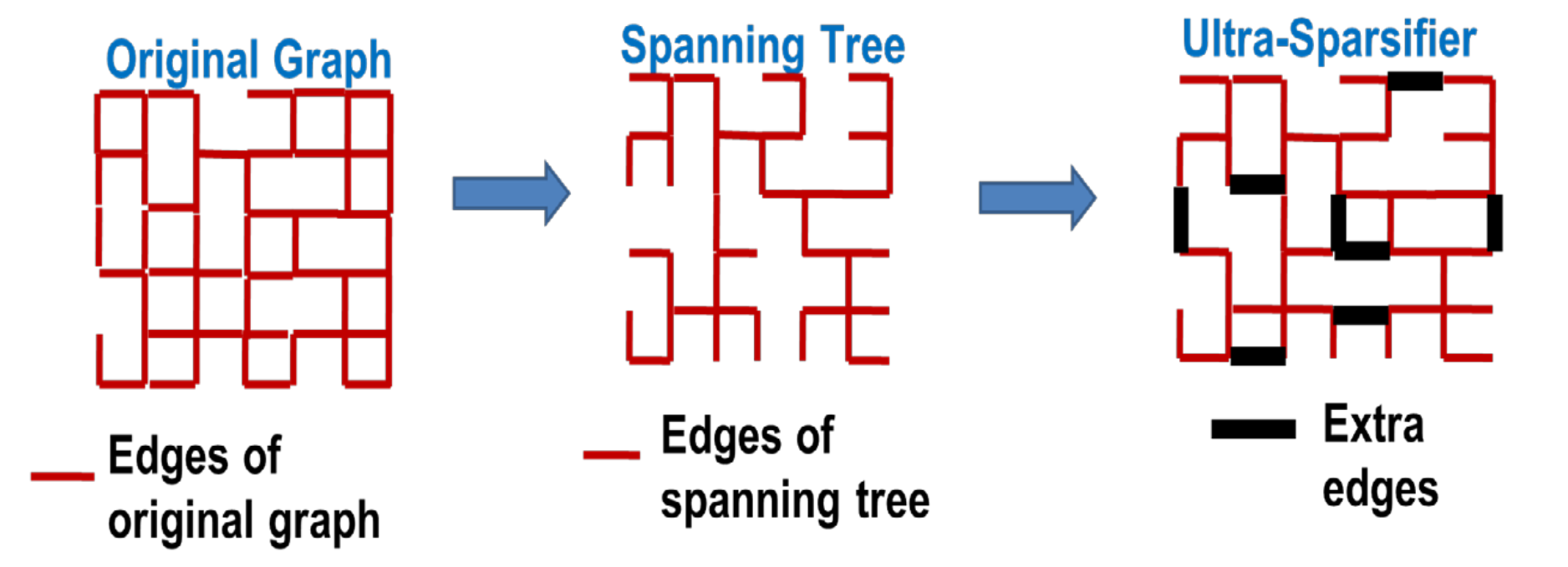
(III) Overview of Proposed Method



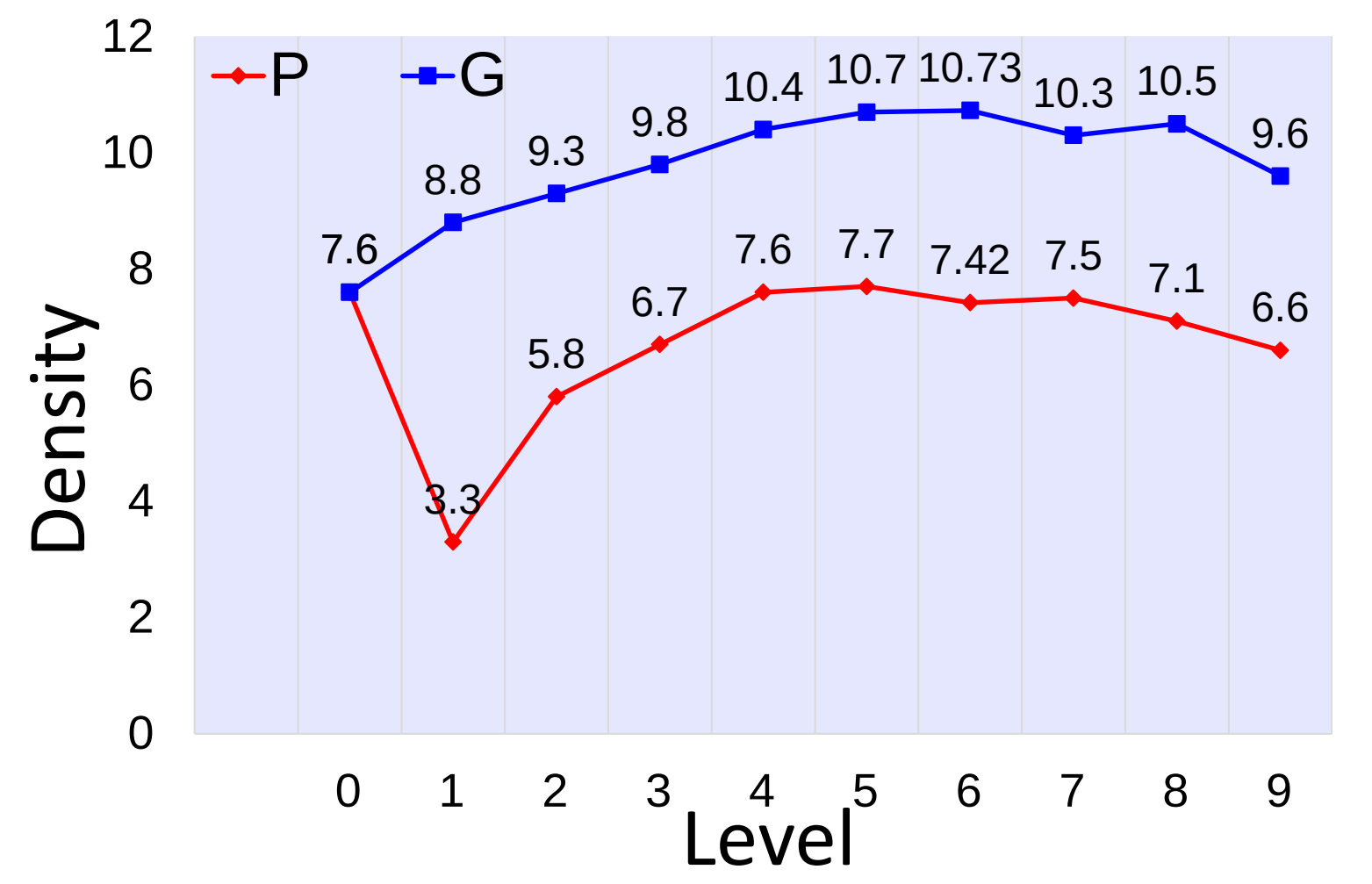
- **Key components in the proposed method**
  - **1. Multilevel spectral power grid sparsifier construction**
    - **Spectral graph sparsification techniques are utilized to produce sparsifiers;**
    - **Hierarchical grids are constructed based on graph-theoretic algebraic multigrid;**
  - **2. Multilevel vectorless integrity verification**
    - **Adjoint sensitivities calculation at each level;**
    - **Current constraints mapping from fine grid to coarse grid;**
    - **Verification solution vector mapping from coarse grid to fine grid;**
    - **LP solver will be adopted for verification of each level problem;**
    - **Solution refinement is critical for improving the verification accuracy.**

(IV) Multilevel Power Grid Sparsifier Construction

- **Spectral graph sparsification (Spielman et al., ACM Comm.'13; Feng, DAC'16)**



1. Construct an initial low-stretch spanning tree from original graph G
2. Spectrally embed off-tree edges via power iteration
3. Rank off-tree edges according to spectral critical levels
4. Add a small amount of off-tree edges to the spanning tree
5. Return the final spectral graph sparsifier P



- Sparsified power grid can well retain effective resistance of original power grid, which is equivalent to preserve the adjoint sensitivity.
- Ultra-sparse power grids will result in nearly linear time of adjoint sensitivity calculation for each LP solve.

- **Multilevel sparsified power grid construction**

➢ Graph-theoretic algebraic multigrid: Lean Algebraic Multigrid (O. Livne et al., SIAM'12)

1. Node Elimination: disconnected nodes & low degree nodes.
2. Node aggregation: aggregate similar(nearby) nodes into clusters to form next coarser level.

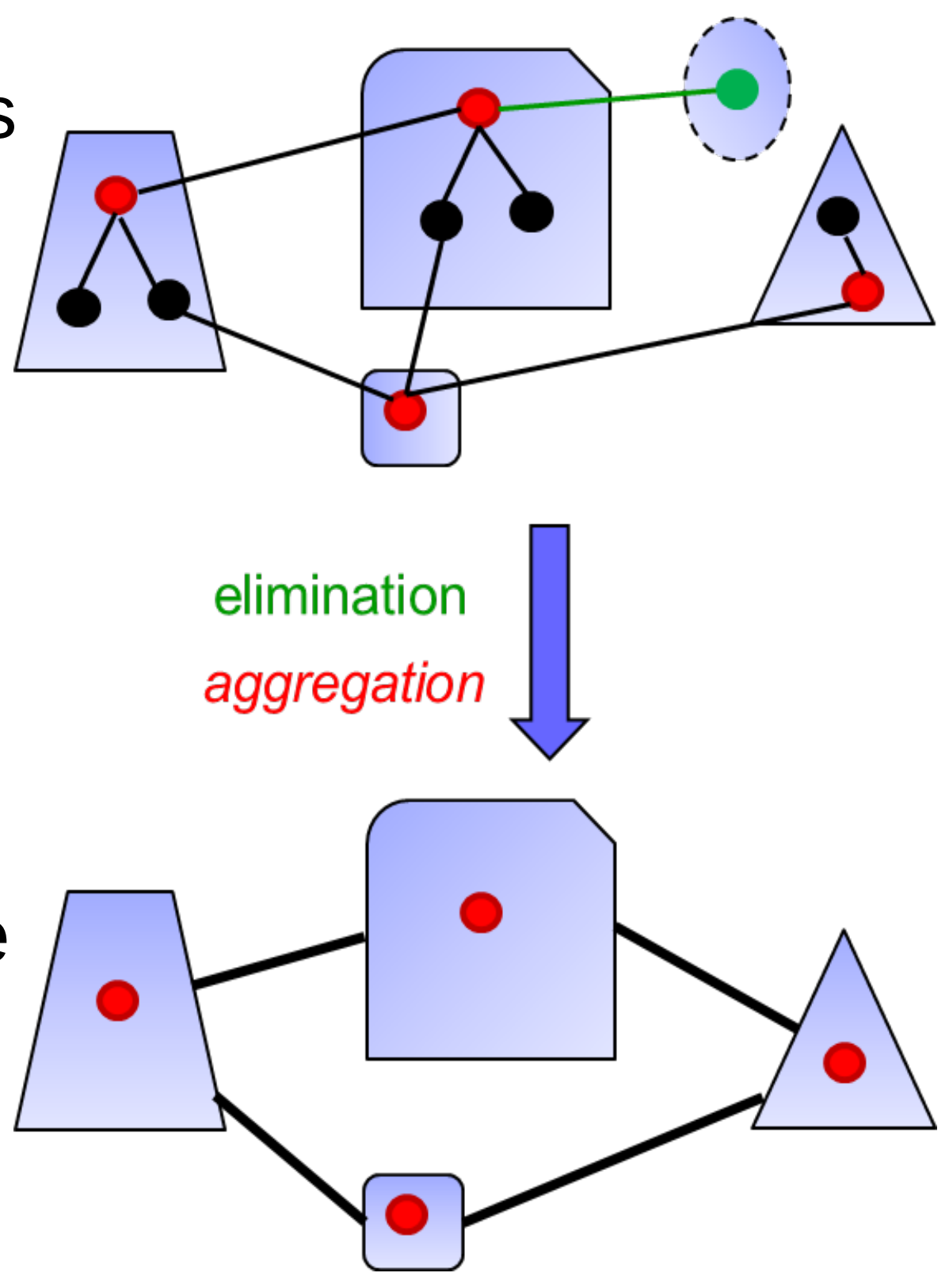
- K Test vectors $X^{(1)}, X^{(2)}, \cdots, X^{(K)}$, formed by Gaussian Seidel relaxation sweep to $Ax = 0$.
- Affinity c_{uv} between node u and node v:

$$c_{uv} = \frac{|(X_u, X_v)|^2}{(X_u, X_u)^2 (X_v, X_v)^2}, \quad (X_u, X_v) = \sum_{k=1}^{K} X^{(k)} Y^k$$

- The larger c_{uv}, the closer node u and node v.
- c_{uv} is a global measurement for the node distance, which can be viewed as a stronger notation than local measurement.

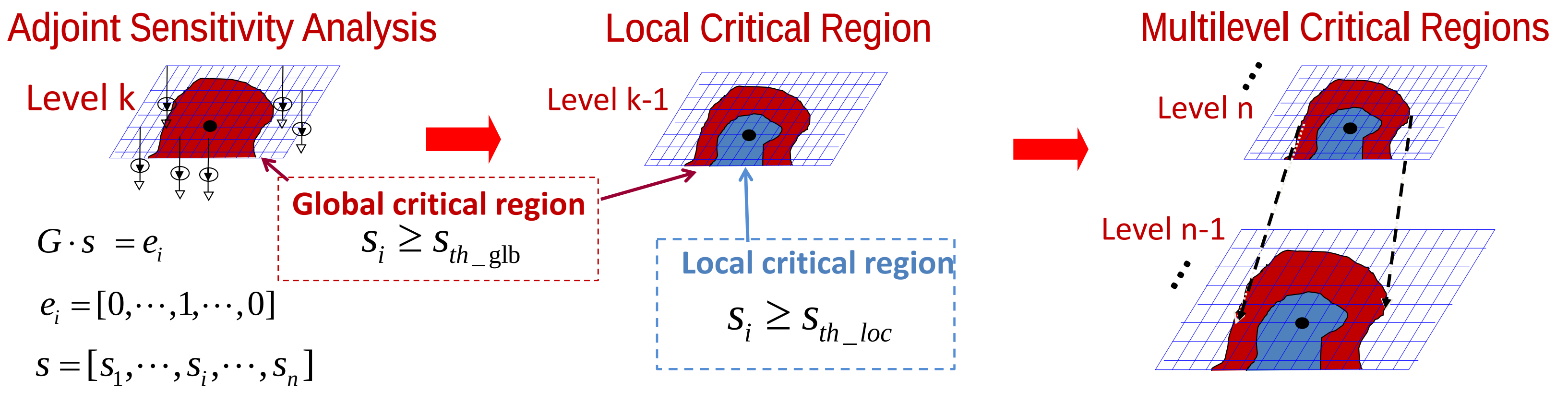
➢ Number of variables in LPs can be dramatically reduced.

➢ Adjoint sensitivity calculations are much faster on reduced grids.



(V) Multilevel Vectorless Verification

- **Adjoint sensitivity analysis is used to find critical regions**
  - If the adjoint sensitivity of a specific current source is smaller than the threshold s_{th}, it will fall into the *global or local critical regions*
    - Global critical regions are defined for the coarsest level grid
    - Local critical regions are defined for the coarser to finest level grids



$$G \cdot s = e_i$$

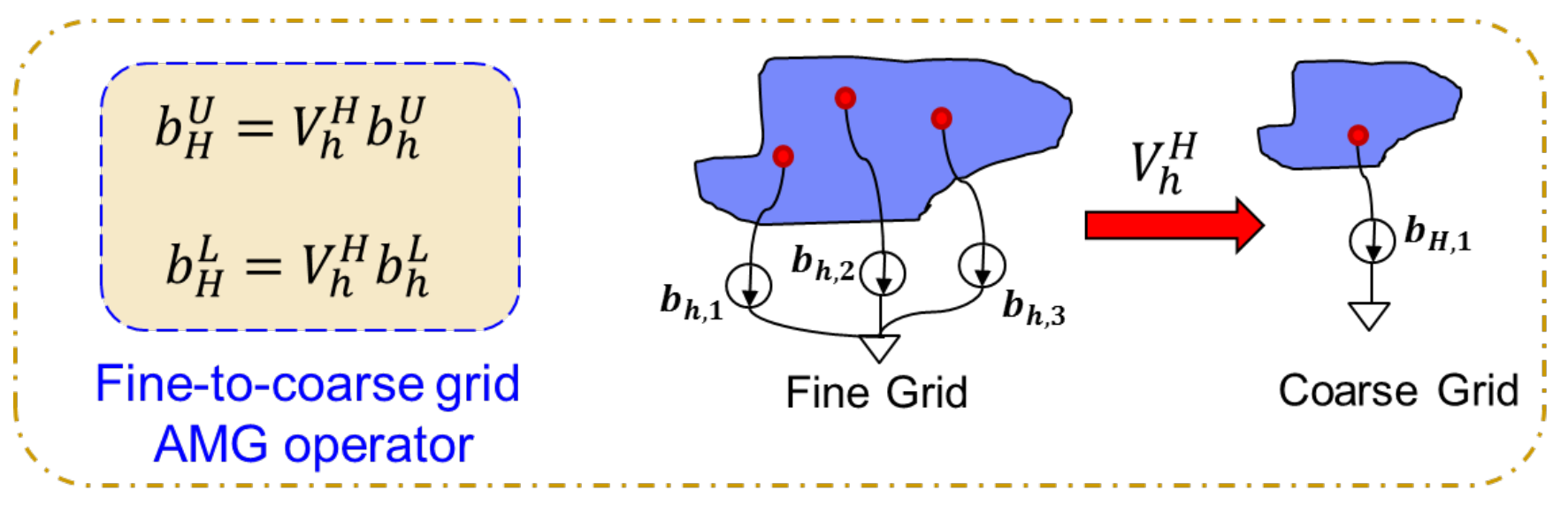
$$e_i = [0, \cdots, 1, \cdots, 0]$$

$$s = [s_1, \cdots, s_i, \cdots, s_n]$$

$$s_i \ge s_{th\_glb}$$

$$s_i \ge s_{th\_loc}$$

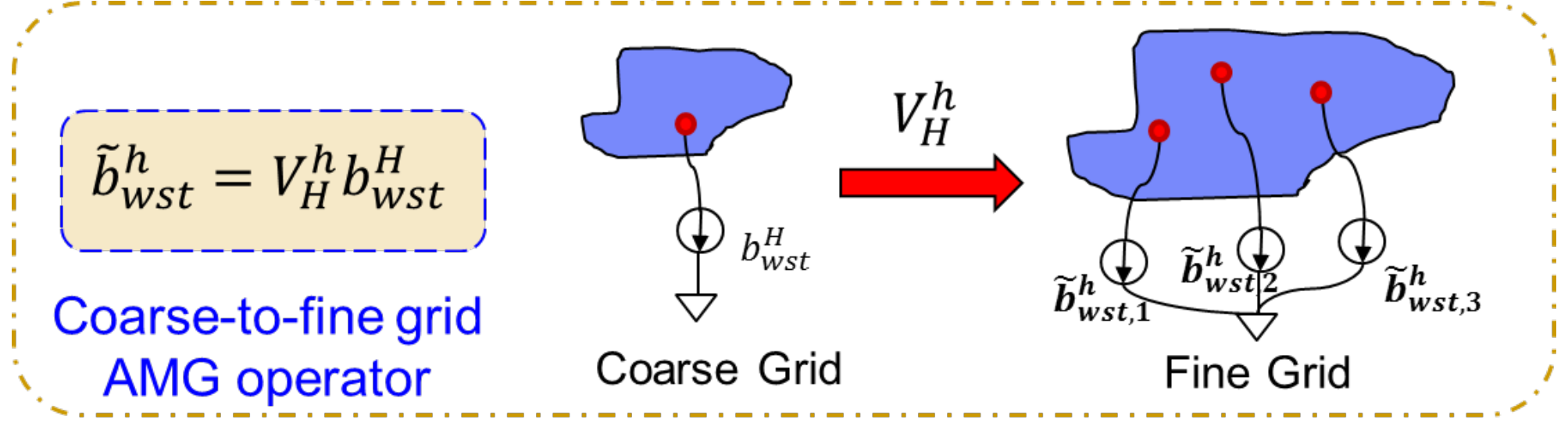
- **Current constraints/solution mapping**
  - Current constraints and solution vector are mapped using AMG operators.



$$b_H^U = V_h^H b_h^U$$

$$b_H^L = V_h^H b_h^L$$

Fine-to-coarse grid AMG operator



$$\tilde{b}_{wst}^h = V_H^h b_{wst}^H$$

Coarse-to-fine grid AMG operator

- **Extra voltage difference contributed by the currents outside the critical region:**

$$v_e = \max_{\forall b_i \notin C_{\text{glb}}} \left(\sum_{i=1}^{n} s_i b_i \right) \le \max_{\forall b_i \notin C_{\text{glb}}} \left(\sum_{i=1}^{n} s_{th} b_i \right) = s_{th} \left| b - b_p \right|_1,$$ where b_p is a current vector including all the currents within C_{glb}

- **Upper/lower bounds for worst case voltage difference:** $\overline{v}_{wst} \le v_{wst}^* \le v_{wst}$

$$\overline{v}_{wst} \le v_{wst}^* \le \overline{v}_{wst} + v_e \le \overline{v}_{wst} + s_{th} \left| b - b_p \right|_1 = v_{wst}$$

(VI) Experimental Results

Comprehensive Results

➢ Test cases are from IBM and Tsinghua University power grid benchmarks.

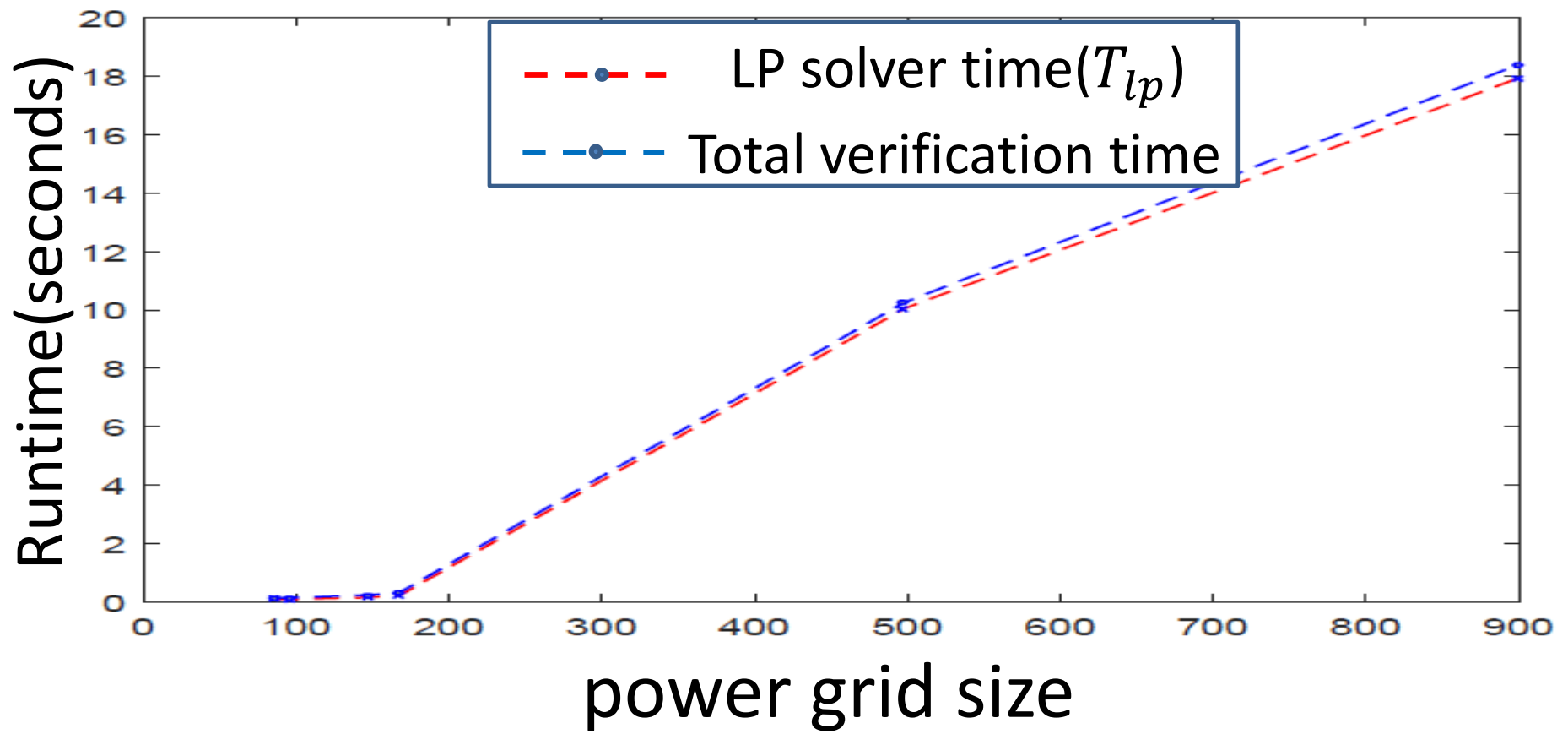
	Power Grid Specifications			Single Level			Multilevel w/o Sparsification				Multilevel w/ Sparsification			
CKT	N.#	C.#	L.#	T_{chol}	T_{solve}	T_{lp}	T_{chol}	T_{solve}	T_{lp}	Err(%)	T_{chol}	T_{solve}	T_{lp}	Err(%)
ibmpg3	0.85M	90K	2	17.4	1.39	1.79	37.12	0.87	0.21	2.34%	1.40	0.029	0.11	2.58%
ibmpg4	1.0M	100K	2	22.4	1.57	2.10	48.04	1.16	0.35	3.42%	3.72	0.06	0.08	2.26%
ibmpg6	1.7M	170K	2	16.3	2.84	3.19	30.30	0.67	0.45	1.23%	5.41	0.10	0.20	2.27%
ibmpg7	1.5M	150K	2	30.3	2.44	3.15	66.50	1.57	0.32	3.98%	4.86	0.08	0.15	1.0%
thupg1	5.0M	500K	3	92.2	9.20	11.43	433.5	5.06	27.15	1.0%	11.09	0.21	10.03	1.0%
thupg2	9.0M	900K	3	963.2	43.16	282.3	2527.4	398.05	45.10	1.0%	47.23	0.46	17.93	1.0%

Performance Speedup

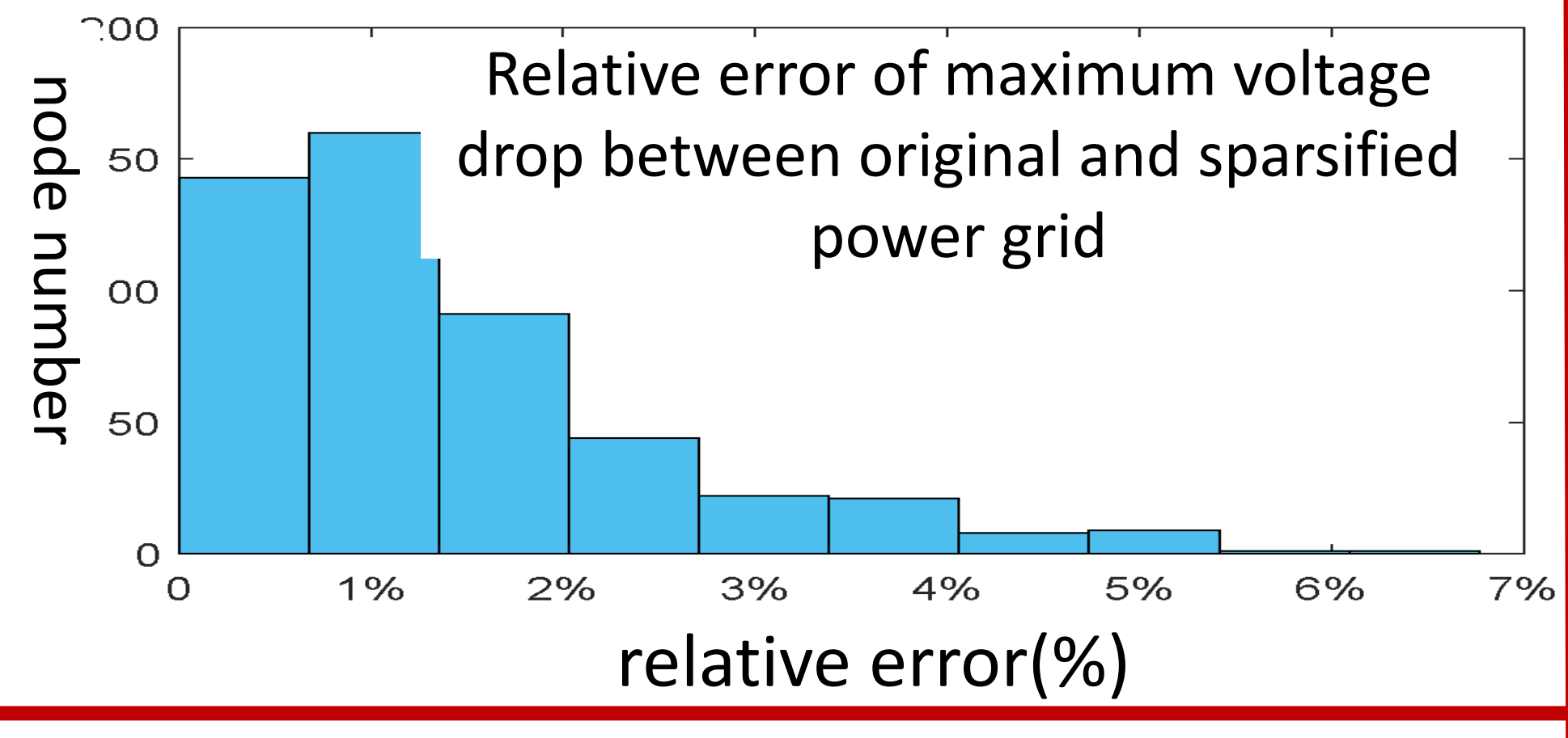
	Single Level			Multilevel w/o Sparsification		
CKT	T_{chol}	T_{solve}	T_{lp}	T_{chol}	T_{solve}	T_{lp}
ibmpg3	12.4X	47.8X	16.3X	26.5X	30.0X	2.0X
ibmpg4	10.4X	26.2X	26.3X	12.9X	19.3X	4.4X
ibmpg6	3.0X	28.4X	16.0X	5.6X	6.7X	2.3X
ibmpg7	6.2X	30.5X	21.0X	13.7X	19.6X	2.1X
thupg1	8.3X	43.8X	1.1X	40.0X	24.1X	2.7X
thupg2	20.4	93.8X	15.7X	53.7X	865X	2.5X

Scalability

➢ Good scalability for large-scale power grid



Accuracy of Spectral Sparsification



Accuracy & Efficiency

➢ Tradeoff between solution quality and verification cost.