

## FILTERING REQUIREMENTS FOR DISTANCE RELAYS

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### 1. INTRODUCTION

Protective relays must filter their inputs to reject unwanted quantities and retain signal quantities of interest.

Distance relays have especially critical filtering requirements, because they must make precise measurements quickly, even with corruption from dc offsets, ccvt transients, travelling-wave reflections, and other interference.

In this discussion, we first identify filtering requirements or criteria for different relays. We then limit the discussion to relays needing precise measurements of the system-frequency component of the signals, such as distance relays.

The next step is to review and conceive many different filtering methods which may meet the filtering requirements.

- Are some methods better than others?
- Are some "good" ideas ill-founded in theory?
- Can we find common ground between seemingly-disparate methods?
- Are there guidelines to help us decide what works?
- Does faster sampling guarantee faster protection?

We attempt to answer filtering questions through requirement-assessment, analysis, simulation and examples.

Several digital filters were evaluated in one of our previous papers. Because of the space limitation here, we consider and compare CAL, cosine, and Fourier filters only. We also examine the differences between finite and infinite impulse response filters.

### 2. FILTERING REQUIREMENTS FOR PROTECTIVE RELAYS

Filtering requirements depend on the protection principle and the application.

In almost all relays, the system frequency components are the information, and everything else interferes. Among the exceptions are relays using harmonic-restraint, and peak-sensitive voltage relays, which may need to detect off-frequency events.

Because distance relays measure impedance, and because impedance is defined at a given frequency, distance relay filters must save only the fundamental frequency.

### 3. SIGNAL PROPERTIES IN FAULTED POWER SYSTEMS

When the resistance-inductance behavior of the power system dominates, the voltages and currents are, as usual, sinusoids with exponentially-decaying dc offsets. The

offsets can severely affect the currents, but seldom importantly affect the voltages.

Reflections on longer lines produce relatively high-frequency oscillations. A wavelength at 60 Hz is about 3100 miles; a quarter-wave is 775 miles. Therefore, lines have to be relatively long before the frequency of the reflections encroach on the power-system frequency. This is fortunate, because the frequency difference makes filtering easy.

Nonlinear loads, power transformers, and instrument transformers can produce harmonics.

Capacitive series compensation introduces subsystem frequency transients. A rough calculation for the subsystem frequency is the square root of the fraction of system compensation. So, for 50% compensation (i.e.,  $X_c = 1/2 X_L$  in the faulted loop), the subsystem oscillation is around 70%. This is very close to the system frequency, and presents a significant filtering problem.

Capacitive-coupled voltage transformers also produce low-frequency transients. The overdamped nature of the transients makes them resemble dc offset.

Given these signal and "noise" properties, we propose filtering requirements and philosophy for distance relays, and other relays which require accurate representation of the system-frequency components.

### 4. FILTER DESIGN CHARACTERISTICS

The filter must have certain characteristics, no matter how we build it: analog, digital, electromechanical, or some combination.

What are the characteristics?

1. Bandpass response, about the system frequency, because all other components are of no interest.
2. Dc and ramp rejection to guarantee decaying-exponentials are filtered out.
3. Harmonic attenuation or rejection to limit effects of nonlinearities.
4. Reasonable bandwidth for fast response.
5. Good transient behavior.
6. Simple to design, build, and manufacture.

Precisely choosing filtering characteristics, based on the relay requirements, is our best guarantee that our filter design will be successful in the laboratory and in the field.

It would be a serious mistake to simply select a filtering concept and "prove" it in EMTP and model power systems tests. If we do not carefully study the requirements and the characteristics, then there is much greater likelihood that some day, some system will present the relay with unforeseen conditions, not evaluated and addressed in systems tests.

**How Should We Synthesize and Implement the Filtering?**

Ultimately, we wish to build the filter using analog and/or digital electronic techniques. Relay requirements of

polarizing memory, and system requirements of fault locating and event recording essentially insist on a digital sampled data-system design.

A digital design gives us a choice between finite and infinite impulse response filtering, whereas analog filters practically limit us to infinite impulse responses.

The outputs of finite impulse response (FIR) filters depend on a finite-time-history of the input; whereas outputs of infinite impulse response (IIR) filters depend on all prior history of the input.

FIR filters subjectively make good sense for protection for two reasons.

1. FIR filters quickly forget the pre-fault condition, and work on analyzing the faulted system. Once the filters fill up with fault data, their phasor estimates of the faulted voltage or current are no longer corrupted with pre-fault data.
2. FIR filters naturally have zeros in their frequency responses. It is relatively easy to put them where we want them, e.g., at dc and harmonics.

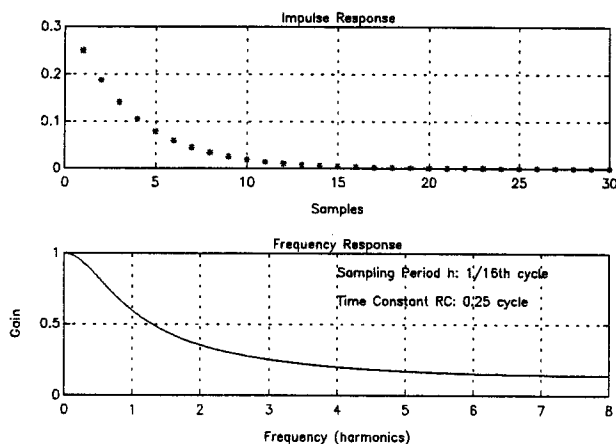


Figure 4.1 IIR Lowpass Filter  $y_k = (1/4)x_k + [1-(1/4)]y_{k-1}$

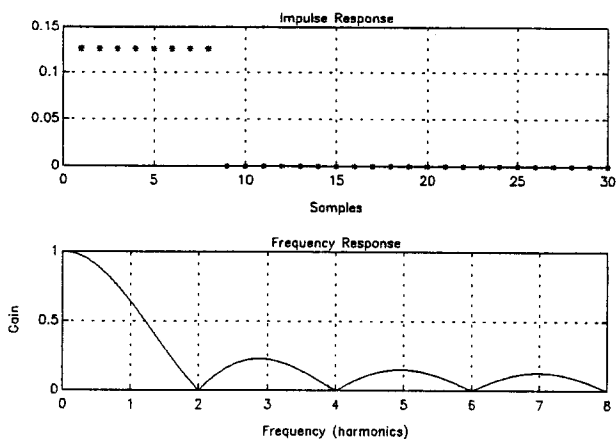


Figure 4.2 FIR Lowpass Filter  $y_k = [x_k + \dots + x_{k-7}]/8$

Figures 4.1 and 4.2 compare an IIR filter to an FIR filter. Both are, for simplicity, lowpass filters. The impulse response of the IIR filter is samples of a decaying exponen-

tial, and therefore lasts forever. That is why the present output depends on all prior history of the input.

We choose an FIR filter such that its low frequency response looks similar to the IIR filter. However, the FIR impulse response clearly includes only a finite time history of the input. The output depends only on the most recent eight samples.

The complete frequency domain characteristics are different. The IIR filter is sharper in the low frequency region (not always an advantage); and the FIR filter has zeros. In practice, we can put those zeros to work to notch out harmonics.

Impulse Response Effects on Frequency Response

The shorter we make the impulse response, the faster the relay becomes. What happens to other performance features? Figure 4.3 shows the frequency responses for three cosine filters: half-cycle, one-cycle and two-cycle. We choose a 1/2, 1, 2 sequence to show clearly the difference.

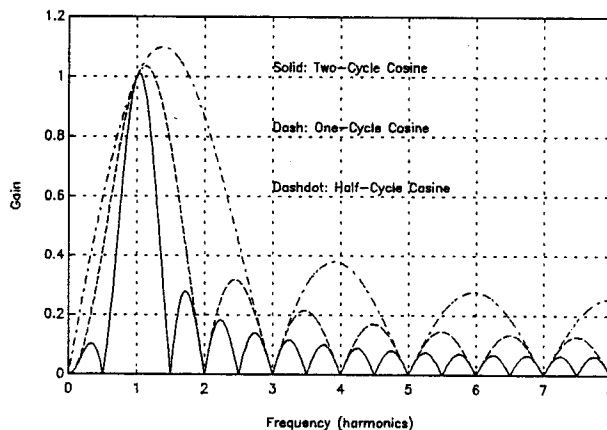


Figure 4.3 Half-, One- and Two-Cycle Cosine Filters

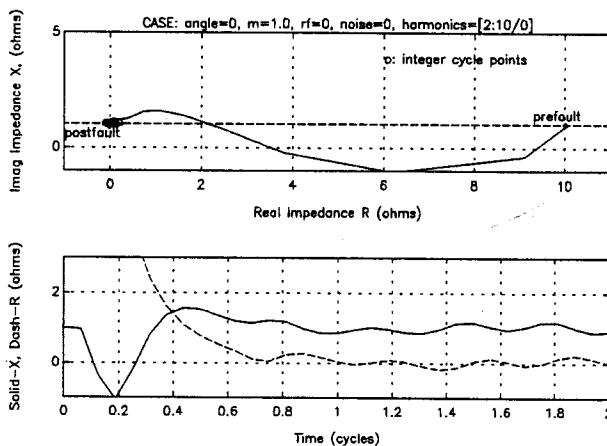


Figure 4.4 Impedance Plot of Half-Cycle Cosine Filter

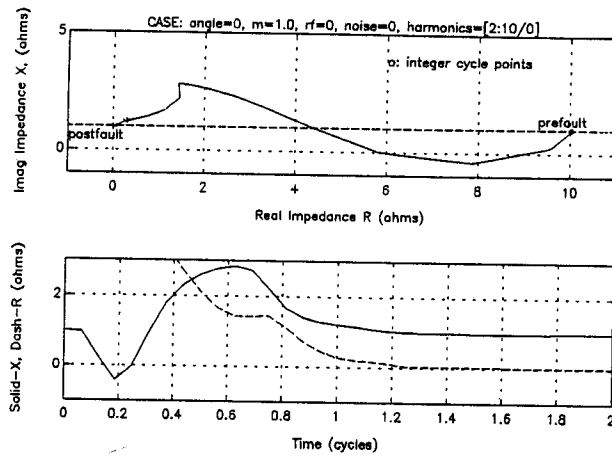


Figure 4.5 Impedance Plot of One-Cycle Cosine Filter

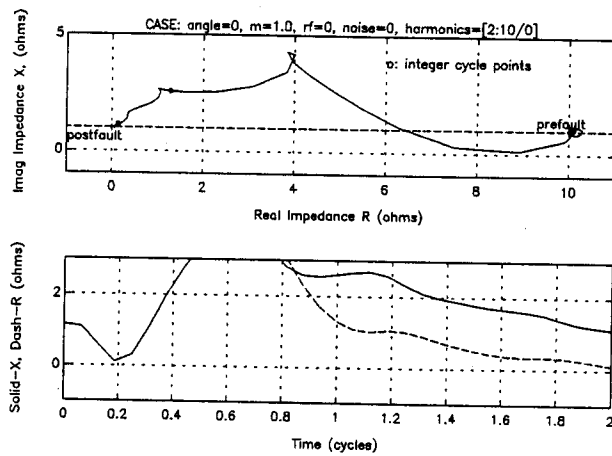


Figure 4.6 Impedance Plot of Two-Cycle Cosine Filter

The longer impulse responses have narrower frequency responses. The one-cycle cosine filter has zeros at dc and at the harmonics of 60 Hz. We lose rejection of the even harmonics when we reduce the filter to half-cycle. The time-response graph of the half-cycle cosine filter in Figure 4.4 shows the penalty for increasing speed: poor transient response. The half-cycle filter is not a double-differentiator, and has poor ability to reject exponentials. The impedance-plane trajectory spirals, indicating severe over-reaching.

The two-cycle window is unnecessarily slower (Figure 4.6) compared with the one-cycle window (Figure 4.5), and its transient performance is insignificantly better.

These impedance time-responses come from filter simulations which we shall discuss in the next section.

### How Does Sampling Rate Affect Relay Operating Time?

Sampling faster means shorter operating times, but the improvement is tempered by filter delay. Figure 4.7 plots operating times for a certain fault condition, as a function of the sampling rate. For each value of the sampling rate, we have optimized the digital and analog filter pair. Increasing the rate from four to eight samples/cycle decreases the operating time by about 1/8 cycle, at the cost of double computations. Doubling the sampling rate again yields only a reduction of about 1/16 cycle, again with

double the computations. Doubling from 16 to 32 samples/cycle speeds up the operation by only 1/32 of a cycle.

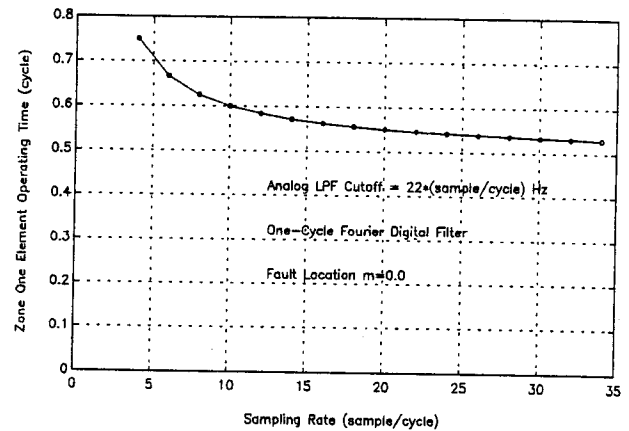


Figure 4.7 Operating Time vs Sampling Rate

For remote faults, the operating times are all longer; but the speedup times remain about the same.

Why is the speedup so minor? The reason is that the digital filters are all based on a one-cycle window. The speedup comes mainly from reduced analog lowpass filter delay and reduced processing latency.

## 5. FILTER EVALUATION

### The Power System Model

We shall evaluate filters from two aspects: their steady-state and transient performance. When the filtering window of a filter covers partially pre-fault and partially post-fault data, the filter is in a transient period. After its filtering window includes all post-fault data, the filter is then in a post-fault steady state.

The frequency response, or Bode magnitude plot, of a filter is an excellent tool to study the filter's steady-state performance. We can visualize the filter's frequency characteristics: what signal gets passed? what is blocked? However, the frequency response represents the steady-state behavior of filters. Also, only time-invariant filters, whose filter coefficients do not change with time, have frequency response plots. The Kalman filter, for example, does not have frequency response plots.

To investigate the filter transient performance, like over-reaching and settling time, and to study time-variant filters, we need time-domain filter simulations. Filter simulations confirm the filter steady-state properties as well.

We want filter simulations to be as simple and basic as possible, so we can get useful and clear results efficiently. We also want the simulation environment to be controllable, so that different desired filter properties can be unveiled clearly and separated.

Fault data generation and collection are one of the key elements of simulations. For this purpose, we set up a one-phase power system model, as shown in Figure 5.1, to generate fault voltages and currents. White noise and harmonics are options one can choose to contaminate generated fault data. We use the white noise to emulate the

high frequency noises caused by unmodeled distributed capacitance and other sources. Harmonics could result from nonlinear devices in a power network.

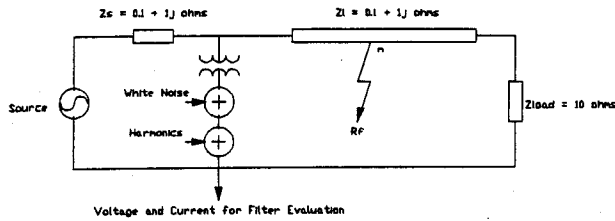


Figure 5.1 Power System Model

The simple power system model helps us probe the filter's ability to reject exponentially-decaying dc offsets, high frequency noise and harmonics. It obviously does not cover all possibilities arising from a real, complicated power network. The simulations are later complemented by EMTP testing of complete schemes.

One set of voltage and current waveforms generated from the power system model is shown in Figure 5.2. The fault is at the end of the line with no fault resistance. An inception angle of zero gives full dc offset. The postfault data are corrupted by adding white noise with a variance of 0.1, plus 20% second, 15% third and 10% fifth harmonics. The variance of the white noise and the magnitudes of harmonics are in terms of percentage of the postfault voltage and current magnitudes.

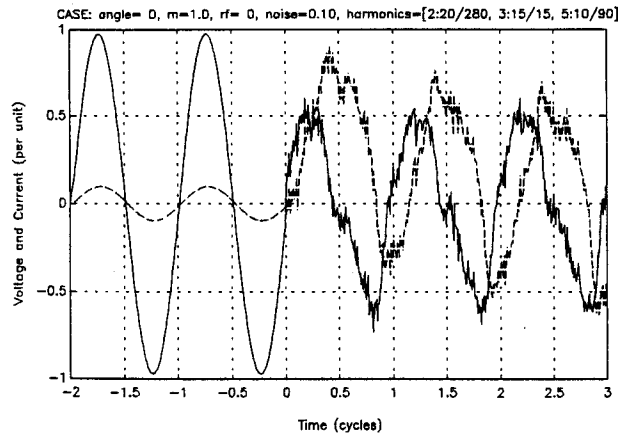


Figure 5.2 Generated Voltage and Current

### Filter Evaluation System

The model system used to evaluate filters is shown in Figure 5.3. It includes an analog lowpass filter, analog to digital conversion (A/D), a digital filter and impedance calculations.

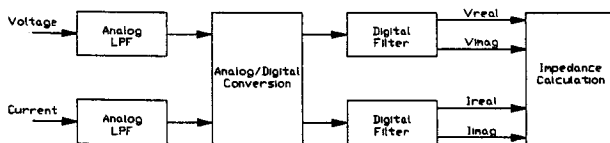


Figure 5.3 Evaluation of Analog and Digital Filters

The impedance is a complex value. Its calculation requires the phasors, or real and imaginary parts, of voltages and currents. Phasors can be obtained by two different methods. One is through an orthogonal filter pair, such as the sine and cosine Fourier filter. When filtering a signal, the filter pair simultaneously gives two filtered outputs with a 90-degree phase shift, which thus constitute the real and imaginary parts of a phasor. Alternatively, the present and the quarter-cycle earlier outputs of one filter are 90 degrees apart. One filter plus a quarter-cycle delay is thus another way to get phasors.

We might expect that the orthogonal filter pair method should be a quarter-cycle faster than the filter plus delay method. However, as we shall see in the next section, this is not necessarily true. The method filters a quantity twice to get a phasor. It can cost twice as many calculations.

## 6. COMPARISON OF DIGITAL FILTERS

We want a digital filter which rejects both dc and ramps (these two are the main ingredients of exponentially-decaying dc offsets), rejects all harmonics, has a bandpass filter characteristic, and has fast, well-behaved transient responses.

FIR filters with less than a one-cycle window cannot reject all harmonics. We have seen some effects on the frequency response when shortening the cosine filter in Section Four. Even worse, the lower harmonics (second and third) are usually the first ones to be sacrificed when shortening the window. For this reason, we limit our discussions only to one-cycle-window FIR filters. We shall use a sampling rate of 16 samples per cycle in the following. The analog lowpass filter is a second order Butterworth with a cutoff frequency of 360 Hz.

We evaluate and compare filters in the order: CAL, cosine, and Fourier.

### 1. CAL, Cosine Filters

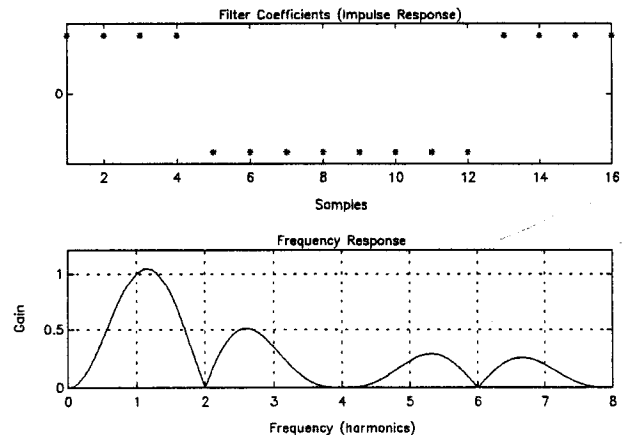


Figure 6.1 CAL Filter

The CAL filter is the simplest filter we evaluated. Its coefficients are  $\pm 1$ . The filtering process uses only addition and subtraction. This eliminates time-consuming multiplications. It is therefore the most computationally-efficient filter. The CAL filter is a double differentiator. It can nicely reject dc and ramp components of inputs and

therefore the exponentially-decaying dc offset. From the filter frequency response, shown in Figure 6.1, we see that the filter does not reject odd harmonics. The analog lowpass filter should be designed to help the CAL filter reject harmonics.

The cosine filter has its coefficients evenly sampled from a cycle of a cosine waveform. It is similar to the CAL filter in terms of the double differentiator property which is so essential to effectively reject exponentially-decaying dc offsets. From the cosine filter's frequency response shown in Figure 6.2, we see that the filter rejects exactly all harmonics and has a bandpass filtering property.

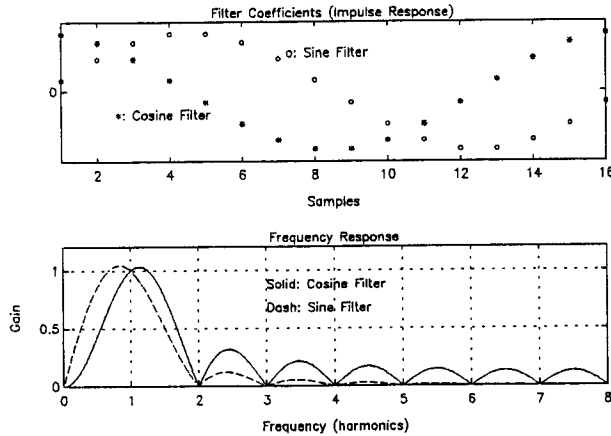


Figure 6.2 Cosine and Sine Filters

The dc, fundamental and odd-harmonic performances of the CAL and cosine filters are essentially the same: excellent.

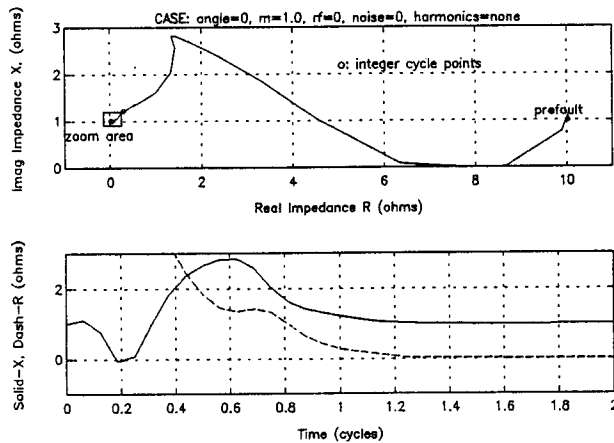


Figure 6.3 Impedance Plot of Cosine Filter

One impedance plot of the cosine filter is given in Figure 6.3.

The imaginary part of voltage or current phasors comes from a quarter-cycle delayed filter output. From the start of a fault, it takes a cycle for the fault to fill the filter, and another quarter-cycle delay to complete the quadrature component. The worst-case filter speed is thus one and one-quarter cycles.

## 2. Fourier Filter

The cosine filter is so promising that we investigated ways to improve it. One natural thought is to eliminate the quarter-cycle delay needed to get the quadrature component. This directs us to a filter orthogonal to the cosine filter, which is the sine filter. The frequency response of the sine filter is shown together with that of the cosine filter in Figure 6.2. The response looks like the cosine filter pushed toward low frequencies. The sine filter has better high frequency attenuation and the same total harmonic rejection. However, we pay for this better high frequency attenuation by sacrificing ramp rejection (double differentiation) capability. Because it lacks ramp rejection, the Fourier filter pair has poor transient response.

Is the Fourier filter a quarter-cycle faster than the cosine filter? Let us look at what happens when there are dc offsets. Figure 6.4 shows the impedance response of the Fourier filter with full dc offset. The imaginary part of the postfault impedance is one ohm. The zoomed version of the impedance plot (Figure 6.5) shows that the postfault impedance circles around the postfault point, and takes a long time to settle. After 1.75 cycles, the Fourier filter still has ten percent overreaching and underreaching. The cosine filter, however, gives less than two percent impedance variation after one and one-quarter cycles. Therefore, the cosine filter is faster and more accurate than the Fourier filter, whenever dc offsets accompany fault currents.

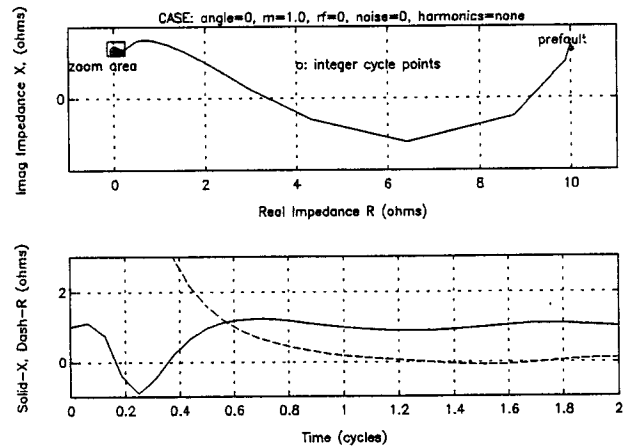


Figure 6.4 Impedance Plot of Fourier Filter

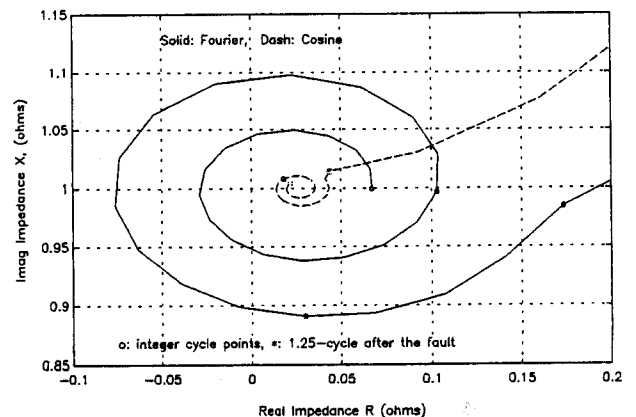


Figure 6.5 Comparison of Cosine and Fourier Filters

Among all possible fault incident angles from 0 to 360 degrees, there are exactly two points where a fault does not cause any dc offset. From our simulations, we have seen worse Fourier filter transient overreaching and underreaching if the fault incident angle is more than 10 degrees from those two points. That is to say, assuming random, uniform fault angle incidence, the cosine filter performs better than the Fourier filter 8 out of 9 times!

## 7. CONCLUSION

1. Fault currents and voltages used in protective relays are contaminated with exponentially-decaying dc offsets, harmonics and other interference. For protective relays which rely on precise fundamental quantities, we need to extract postfault voltages and currents as quickly and as accurately as possible. An ideal filter for such relays is a narrow bandpass filter.

2. FIR filters have advantages over IIR filters. FIR filters have zeros naturally in their frequency response. We can arrange these zeros to reject harmonics exactly. An FIR filter uses finite samples of an input for its output. Once the fault inception point propagates through the filtering window, its output is no longer corrupted with pre-fault data. The outputs of IIR filters, however, rely on the entire history of an input. This is contrary to the basic requirement of protective relays.

3. An FIR filter with a less than one-cycle window cannot reject all harmonics. The filter is usually more prone to low harmonics.

4. The one-cycle cosine filter is the best filter we evaluated. It rejects exponentially-decaying dc offsets, rejects all harmonics, comes close to the desired bandpass filtering, and has good transient response. The cosine filter outperforms the Fourier filter, when dc offsets are present. This is clearly shown in Figure 6.5.

5. The advantage of higher sampling rates on the relay speed diminishes, when a filtering window is fixed. The improvement in speed comes from decreasing the analog lowpass filter delay and computational latency.

## 8. REFERENCES

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## 9. BIOGRAPHICAL SKETCHES

**Edmund O. Schweitzer** received BSEE and MSEE degrees from Purdue University in 1968 and 1971, respectively. He earned a PhD at Washington State University (WSU) in 1977. His professional experience includes electrical engineering work at Probe Systems in California and the National Security Agency in Maryland. He served as an assistant professor at Ohio University and as an assistant and an associate professor at WSU. Since 1983, he has directed the activities of Schweitzer Engineering Laboratories, Inc. (SEL), the company he founded in Pullman, Washington. SEL designs, manufactures, and markets digital protective relays for power system protection.

Schweitzer started investigating digital relays during PhD studies at WSU in 1976, which produced both his doctoral dissertation and SEL. His university research was supported by Bonneville Power Administration, Electric Power Research Institute, and various utilities. Although the company has grown significantly, Schweitzer is still involved in the development of new relays and auxiliary equipment.

He is a Fellow of the Institute of Electrical and Electronic Engineers (IEEE), is a member of Eta Kappa Nu and Tau Beta Pi, and has authored or co-authored 30 technical papers.

**Daqing Hou** received BS and MS degrees in Electrical Engineering from the Northeast University of Technology, China, in 1981 and 1984, respectively. He received a PhD in Electrical and Computer Engineering from Washington State University in 1991. Since 1990, he has been with Schweitzer Engineering Laboratories, Inc., Pullman, Washington, where he is currently a development engineer. His work includes system modeling, simulation and signal processing for power system digital protective relays. His research interests include multivariable linear systems, system identification and signal processing.

Hou is a member of the IEEE and has authored or co-authored several technical papers.