EE 4222 - Problem Set 2
Due: February 04, 2005 Name________________________________ No.___

1. $V_{AN} = 150\angle 0$, $V_{BN} = 120\angle -120$, and $V_{CN} = 125\angle 90$. Find:

   A. V_{AN0}, V_{BN0}, V_{CN0}, V_{AN1}, V_{BN1}, V_{CN1}, V_{AN2}, V_{BN2}, and V_{CN2}.
   B. V_{AB0}, V_{BC0}, V_{CA0}, V_{AB1}, V_{BC1}, V_{CA1}, V_{AB2}, V_{BC2}, and V_{CA2}.

2. Find V_{AB}, V_{BC}, and V_{CA} for problem 1 by:

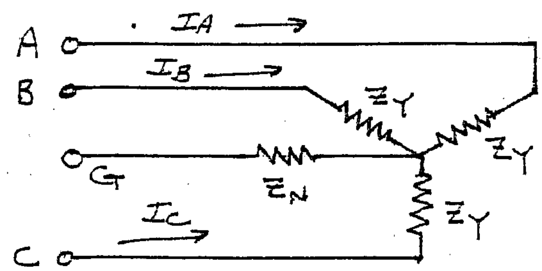
   A. Combining the sequence voltages of problem 1, part B
   B. Combining the a-b-c voltages (i.e., $V_{AB} = V_{AN} - V_{BN}$)

3. A three-phase, wye connected, induction motor is supplied by a three-wire source. The line-to-line voltage magnitudes (as measured with a voltmeter) are found to be:

   $V_{AB} = 240$ V. $V_{BC} = 210$ V. $V_{BC} = 225$ V.

   Percent unbalance in the supply is defined as the ratio of the magnitude of the negative sequence to the magnitude of the positive sequence, expressed in percent. What is the percent unbalance in the source supplying the motor?

4. In the circuit on the right, $V_{AG} = 100\angle 0$, $V_{BG} = 50\angle -120$, and $V_{CG} = 100\angle 90$. $Z_Y = 12.0\angle 30\ \Omega$ and $Z_N = 2.0\angle 90\ \Omega$. What are the phase currents I_A, I_B, and I_C?

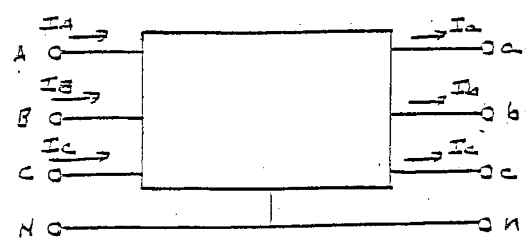


5. In the circuit on the right, the capital letters indicate the input side and the small letters correspond to the output side. The a-b-c values for the input and the 0-1-2 values for the output are:

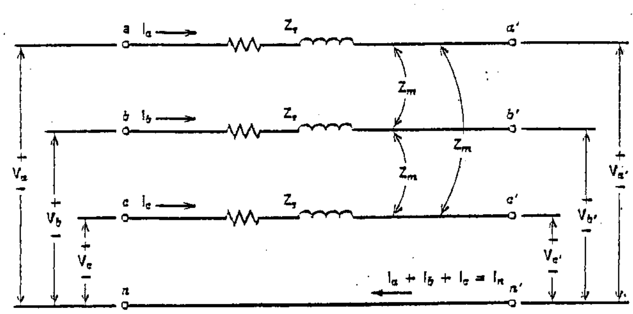
   In: $V_{AN} = 1.0\angle 0$, $V_{BN} = 1.0\angle -90$, $V_{CN} = 1.0\angle +90$,
   $I_A = 1.0\angle 60$, $I_B = 1.0\angle -60$, $I_C = 0$

   Out: $V_{a0} = 0.4\angle 0$, $V_{a1} = 0.8\angle 0$, $V_{a2} = 0.1\angle 180$,
   $I_{a0} = 0.6\angle 0$, $I_{a1} = 0.5\angle 0$, $I_{a2} = 0.5\angle 0$

   What is the complex power absorbed by the circuit?



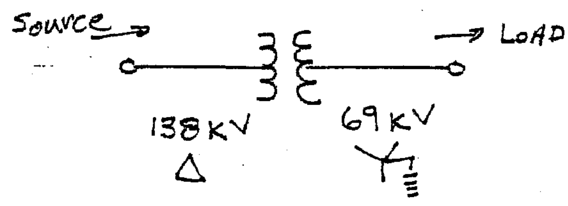
6. In the circuit on the right, find $[Z_{012}]$ and V_a, V_b, and V_c. Given:
   $X_s = 8$ and $X_m = 2$;
   $V'_a = 100\angle 0$, $V'_b = 100\angle -120$ and $V'_c = 80\angle 90$;
   $I_{A0} = 1.0\angle 90$, $I_{A1} = 2.0\angle -30$, and $I_{A2} = 6.0\angle 30$.



7. A 500 MVA, 138 kV -Δ : 69 kV - grounded Y connected transformer is supplying current to an unbalanced load connected to the Y side of the transformer from a source connected to the Δ side of the transformer as shown below. The per unit line currents on the transformer's base to the load are:

$$I_A = 1.25\angle 0,\quad I_B = 1.25\angle\text{-}120^0,\ \text{and}\ I_C = 0.00$$

A. What are the per unit line currents flowing in the lines to the source?
B. What are the magnitudes of these currents in amperes?



8. If the load side line-to-neutral p.u. voltages in problem 7 are:

$$V_{AN} = 1.31696\angle\text{-}4.715,\ V_{BN} = 1.31696\angle\text{-}115.285,\ \text{and}\ V_{CN} = .125\angle\text{-}60$$

and the transformer has a series pre unit reactance of X = .10,

What are the actual phasor values of the line-to-line voltages on the high voltage side?

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1. A) $V_{a0} = 30.81 \angle 13.2$ $\quad V_{a1} = 127.8 \angle -9.4$ $\quad V_{a2} = 15.1 \angle 113.8$

$V_{b0} = 30.81 \angle 13.2$ $\quad V_{b1} = 127.8 \angle -129.4$ $\quad V_{b2} = 15.1 \angle -126.2$

$V_{c0} = 30.81 \angle 13.2$ $\quad V_{c1} = 127.8 \angle 110.6$ $\quad V_{c2} = 15.1 \angle -6.2$

B) $V_{AB0} = 0$ $\quad V_{AB1} = 221.3 \angle 20.6^\circ$ $\quad V_{AB2} = 26.1 \angle 83.8$

$V_{BC0} = 0$ $\quad V_{BC1} = 221.3 \angle -99.4^\circ$ $\quad V_{BC2} = 26.1 \angle -156.2$

$V_{CA0} = 0$ $\quad V_{CA1} = 221.3 \angle 140.6^\circ$ $\quad V_{CA2} = 26.1 \angle -36.2$

2. A) $V_{AB} = 221.3 \angle 20.6 + 26.1 \angle 83.8 = 234.2 \angle 26.3$

$V_{BC} = 221.3 \angle -99.4 + 26.1 \angle -156.2 = 236.6 \angle -104.7^\circ$

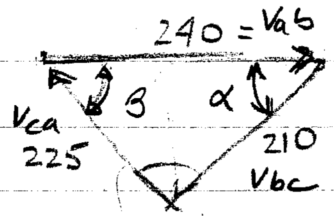
$V_{CA} = 221.3 \angle 140.6 + 26.1 \angle -36.2 = 195.2 \angle 140.2$

B) $V_{AB} = V_{AN} - V_{BN} = 150 \angle 0 - 120 \angle -120 = 234.3 \angle 26.3$

$V_{BC} = V_{BN} - V_{CN} = 120 \angle -120 - 125 \angle 90 = 236.7 \angle -104.7$

$V_{CA} = V_{CN} - V_{AN} = 125 \angle 90 - 150 \angle 0 = 195.3 \angle 140.2$

3.



$a^2 = b^2 + c^2 - 2bc \cos\alpha$

$\cos\alpha = \dfrac{b^2 + c^2 - a^2}{2bc}$

$$\alpha = \cos^{-1}\left(\frac{240^2 + 210^2 - 225^2}{2(240)(210)}\right) = \cos^{-1}(.5067) = 59.56^\circ$$

$$\beta = \cos^{-1}\left(\frac{240^2 + 225^2 - 210^2}{2(240)(225)}\right) = \cos^{-1}(.59375) = 53.58^\circ$$

$$\begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} A^{-1} \end{bmatrix} \begin{bmatrix} 240 \angle 0 \\ 210 \angle -120.44 \\ 225 \angle 126.42 \end{bmatrix} = \begin{bmatrix} 0 \\ 224.7 \angle 2 \\ 17.3 \angle -20 \end{bmatrix} \Rightarrow \underline{\underline{7.7\%}}$$

$$\left(\frac{17.3}{224.7}\right)$$

4. $$\begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} A^{-1} \end{bmatrix} \begin{bmatrix} 100\angle 0 \\ 50\angle -120 \\ 100\angle 90 \end{bmatrix} = \begin{bmatrix} 31.34\angle 37.1 \\ 80.61\angle -11.9 \\ 4.47\angle -150 \end{bmatrix}$$

$$I_0 = \frac{31.34\angle 37.1}{12\angle 30 + 6\angle 90} = 1.97\angle -12.0$$

$$I_1 = \frac{80.61\angle -11.9}{12\angle 30} = 6.72\angle -41.9 \Rightarrow \begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} = \begin{matrix} 8.18\angle -36.8 \\ 5.11\angle -146.6 \\ 7.37\angle 61.6 \end{matrix}$$

$$I_2 = \frac{4.47\angle -150}{12\angle 30} = .372\angle 180$$

5. $$S_{IN} = (1.0\angle 0)(1\angle -60) + (1\angle -90)(1\angle 60) + 0 = 1.932\angle -45^\circ$$
$$S_{OUT} = 3\left[(.4\angle 0)(.6\angle 0) + (.8)(.5) + (.1\angle 180)(.5)\right] = 1.7700\angle 0$$

$$S_{ABS} = S_{IN} - S_{out} = 1.42\angle -106.5 = -.404 - j\,1.366$$

6. $$\begin{bmatrix} Z_{0,12} \end{bmatrix} = j\underline{\begin{bmatrix} 12 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}} \quad j\begin{bmatrix} V_0' \\ V_1' \\ V_2' \end{bmatrix} = \begin{bmatrix} A^{-1} \end{bmatrix} \begin{bmatrix} 100\angle 0 \\ 100\angle -120 \\ 80\angle 90 \end{bmatrix} = \begin{matrix} 16.8\angle -7.5 \\ 90.7\angle -8.4 \\ 16.8\angle 112.5 \end{matrix}$$

$$V_0 = 16.8\angle -7.5 + (12\angle 90)(1\angle 90) = 5.16\angle -25.2$$
$$V_1 = 90.7\angle -8.4 + (6\angle 90)(2\angle -30) = 95.8\angle -1.76^\circ \Rightarrow \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{matrix} 86.6\angle 28.7 \\ 148.0\angle -120 \\ 80.9\angle 81.5 \end{matrix}$$
$$V_2 = 16.8\angle 112.5 + (6\angle 90)(6\angle 30) = 52.7\angle 117.6^\circ$$

7. A) $$\begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} A^{-1} \end{bmatrix} \begin{bmatrix} 1.25\angle 0 \\ 1.25\angle -120 \\ 0 \end{bmatrix} = \begin{bmatrix} .4167\angle -60 \\ .8333\angle 0 \\ .4167\angle 60 \end{bmatrix} \xrightarrow[\text{side}]{\text{TO }\Delta} \begin{bmatrix} 0 \\ .8333\angle 30 \\ .4167\angle 30 \end{bmatrix}$$

$$\begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} = A \begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix} = \begin{matrix} 1.25\angle 30 \\ .722\angle -120 \\ .722\angle 180 \end{matrix}$$

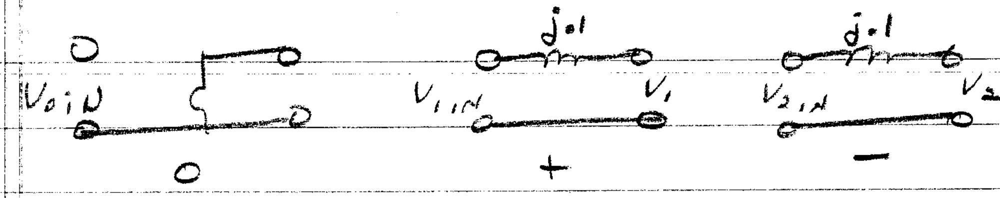
7 B)

$$I_{Base} = \frac{500 \times 10^6}{\sqrt{3} \times 138 \times 10^3} = 2{,}092$$

$|I_A| = 2615 \angle 30 \quad |I_B| = 1510 \angle{-120} \quad |I_c| = 1510 \angle 180$

8)

$$\begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} = A^{-1} \begin{bmatrix} 1.31696 \angle{-4.715} \\ 1.31696 \angle{-115.285} \\ .125 \angle{-60} \end{bmatrix} = \begin{matrix} .54167 \angle{-60} \\ .83334 \angle 0 \\ .41667 \angle 60 \end{matrix}$$



$V_{0\,iN} = 0$

$V_{1\,iN} = .83334 \angle 0 + .1 \angle 90 (.83334 \angle 0) = .8375 \angle 5.711^\circ$

$V_{2\,iN} = .41667 \angle 60 + (.1 \angle 90)(.41667 \angle 60) = .41857 \angle 65.71^\circ$

with phase shift:

$V_{1\,iN} = .83750 \angle 35.711^\circ$

$V_{2\,iN} = .45834 \angle 35.711^\circ$

L-L)

$V_{1\,L-L} = .83750 \angle 65.711^\circ$ (change base mag, no $\sqrt{3}\times$)

$V_{2\,L-L} = .45834 \angle 5.711^\circ$

$$\begin{bmatrix} V_{ab} \\ V_{bc} \\ V_{ca} \end{bmatrix} = \begin{bmatrix} A \end{bmatrix} \begin{bmatrix} 0 \\ .83750 \angle 65.711 \\ .45834 \angle 5.711^\circ \end{bmatrix} = \begin{matrix} 1.13813 \angle 45.299^\circ \\ .37916 \angle{-54.289} \\ 1.13813 \angle{-153.88} \end{matrix}$$

$V_{ab} = 157.1\text{ kV} \angle 45.3$, $V_{bc} = 52.3\text{ kV} \angle{-54.3}^\circ$

$V_{ca} = 157.1\text{ kV} \angle{-153.9}$