EE 5220 - Lecture 2 Wed Jan 10, 2024

Topics for Today:

- Startup
    - Web page: http://www.ee.mtu.edu/faculty/bamork/ee5220/
    - Book, references, syllabus, more are on web page.
    - Software - ATP/EMTP, Matlab
    - EE5220-L@mtu.edu (participation = min half letter grade)
    - Lectures - new videostreams, some archived videos also
    - Daily lecture notes scanned and .pdf file archived
    - Exercises posted as pdf on Canvas.
    - Grading: grad students must achieve BC (75%) or higher.
    - Prereqs: - Circuit Analysis RLC Responses, EE5200
    - Do all exercises in Ch.1 (solutions are posted) note typo.
- Chapters 1 and 2, probs 1.2, 1.3, 2.2, 2.3, 2.7 due next Wed.
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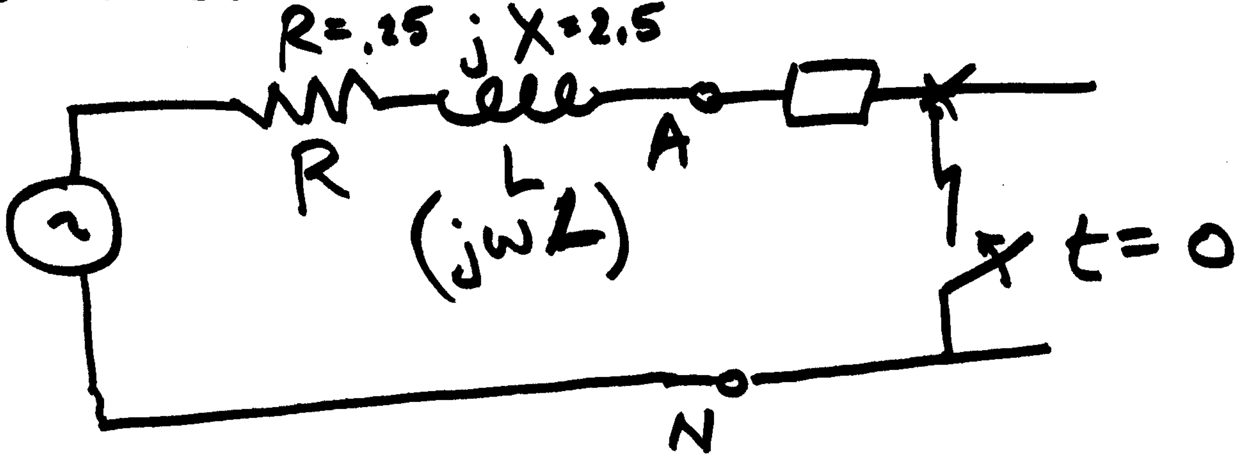
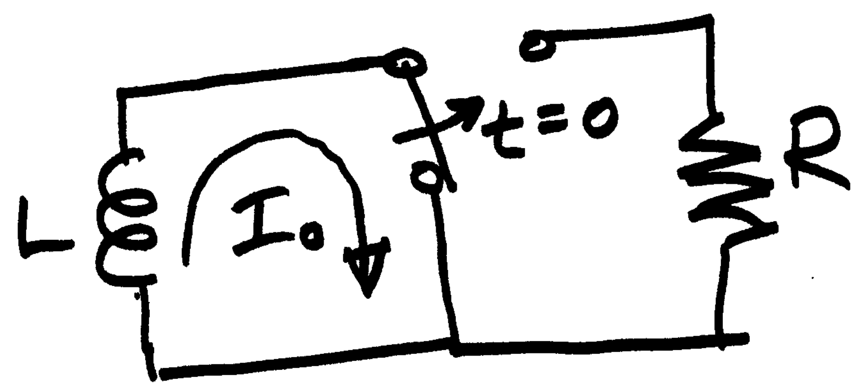
- Get your textbook! Dig out your old Circuits book!
- Review of Laplace, RLC circuits
- Basic use of ATP (prereq/tutorials from EE5200), see http://www.ee.mtu.edu/faculty/bamork/ee5200/

Overview – RL Circuits



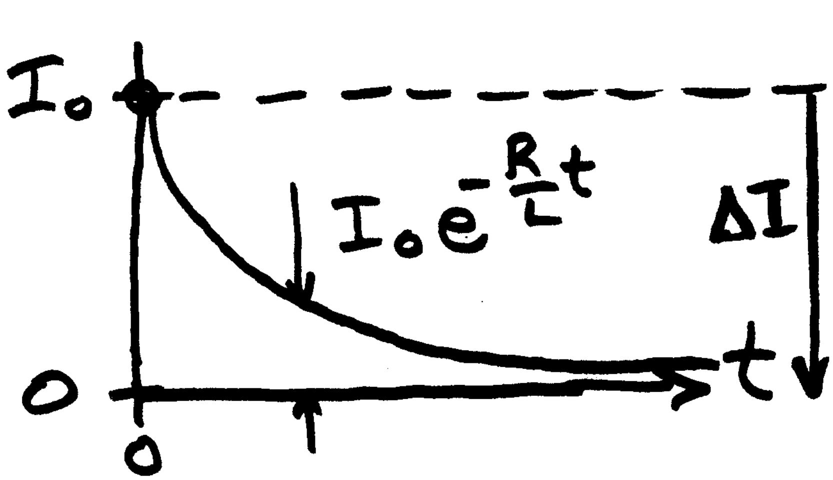
BASIC:



1) Identify Initial & Final State

$$i_L(0) = I_o = i_L^- = i_L^+$$

$$i_L(\infty) = 0$$



$$i_L(t) = I_\infty - \underline{\underline{\Delta I}}\, e^{-\frac{R}{L}t}$$

$$= I_\infty - \underbrace{(I_\infty - I_o)}\, e^{-\frac{R}{L}t}$$

$$= 0 - (0 - I_o)\, e^{-\frac{R}{L}t}$$

$$= +I_o e^{-\frac{R}{L}t}$$

RL time constant

$$\tau = \frac{L}{R} \text{ s}$$

Consider $\quad e^{-\frac{t}{\tau}} = e^{-\frac{R}{L}t}$

Increase $R \rightarrow$ Faster rate of change, less time to final state.

Increase $L \rightarrow$ Slower rate of change, longer to final state.

Stored energy:

$= \frac{1}{2} L i^2$ Joules

$= \frac{1}{2} C v^2$

$= \frac{1}{2} K X^2$ (spring)

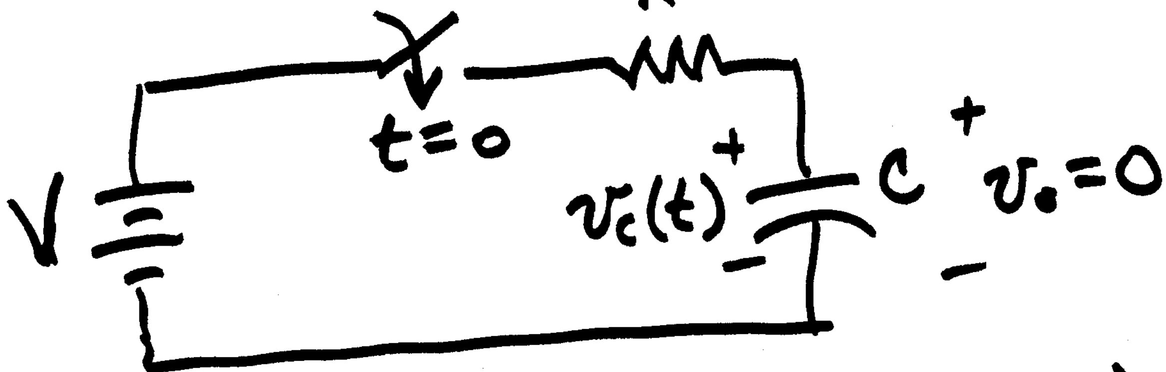
$= \frac{1}{2} I_o \omega^2$ (fly-wheel)

$\uparrow$

"H"

RC Circuits -

$\tau = RC$



Initially: $v_c(0) = v_c(0^+) = v_c(0^-) = 0$

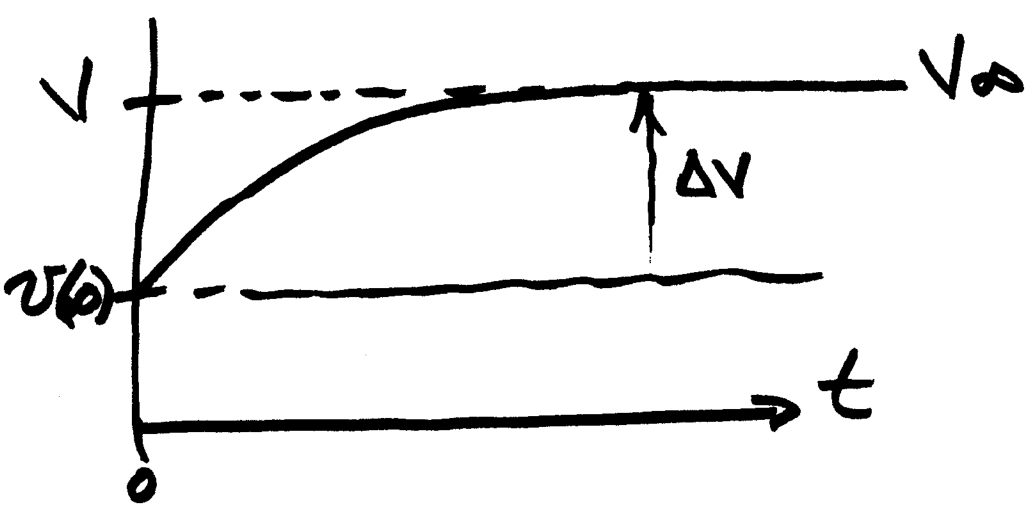
Final State: $v_c(\infty) = V$

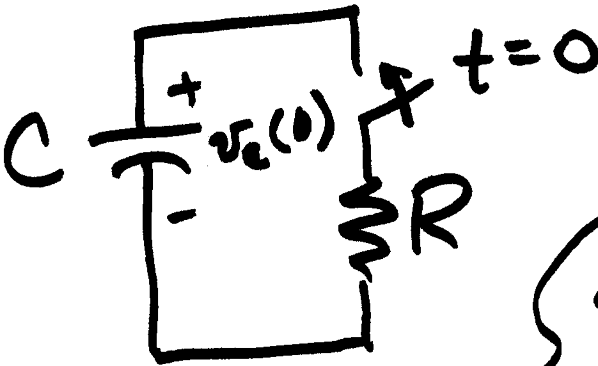
$$\underline{\Delta V_c} = V_\infty - V_0 = V - V_0$$

Note: ΔV_c drives response!

$$v_c(t) = V - \Delta V e^{-\frac{t}{RC}}$$

$$= \cancel{V} \boxed{\underline{v_c(\infty)} - \Delta V e^{-\frac{t}{RC}}}$$

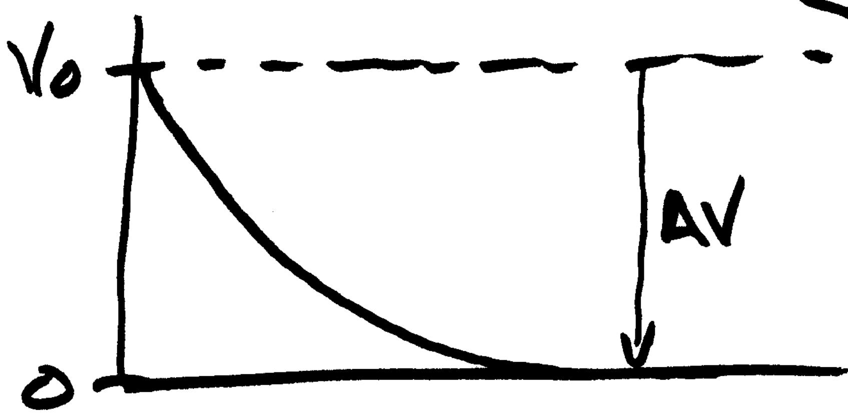




$v_c(0) = V_0$

$v_c(\infty) = 0$

$\Delta V = \underbrace{V_\infty - V_0}$



$$v_c(0) = \cancelto{0}{v_c(\infty)} - \underline{\Delta V} e^{-\frac{t}{RC}}$$

$$= V_c(0)\, e^{-\frac{t}{RC}}$$

time constants

Compare τ to 60-Hz period

$\frac{2.5}{.25} = \frac{X}{R} \cong 10$ for Transmission System

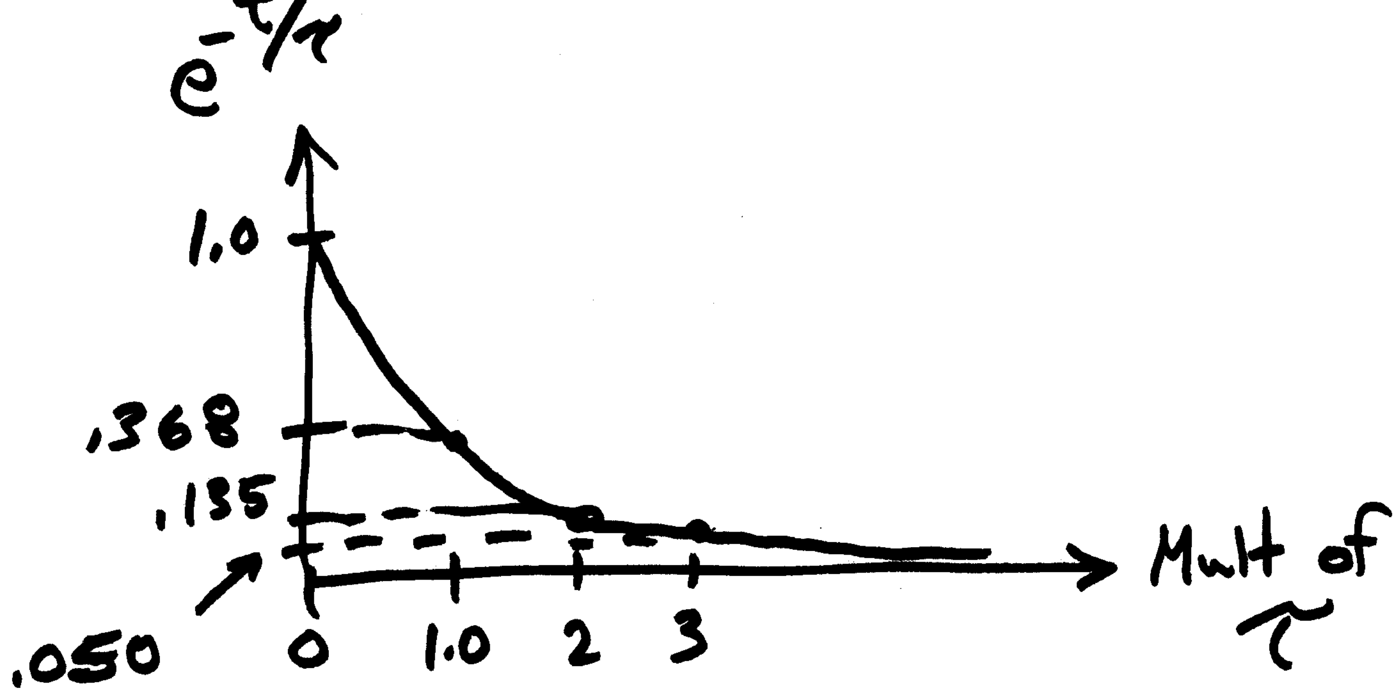
$$\tau = \frac{L}{R} = \frac{X}{\omega R} = \frac{2.5}{377 \times .25} \cong 0.028\,s$$

~~0~~ ms $\Longleftrightarrow$ $1/60\,s = \underline{16.67\,ms}$
28 ms

$$e^{-\frac{t}{\tau}}$$

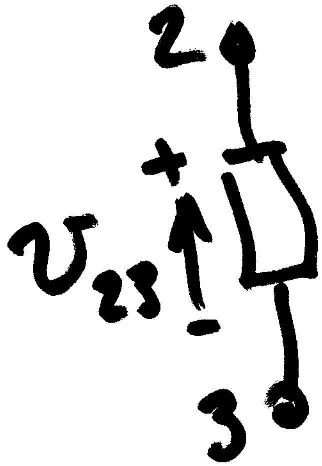
At one time constant

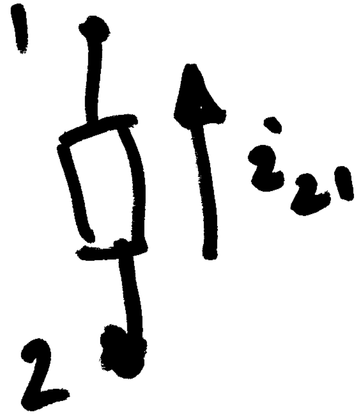
$$e^{-1} = 0.368$$

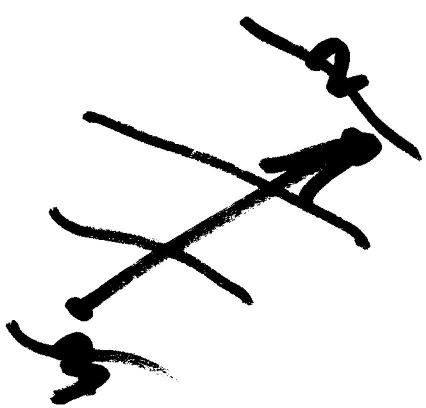


v_{23} i_{21} P

v_1 from to Q







EEE350 **RLC CIRCUITS TRANSIENT SOLUTION GUIDE**

B.A. Mork

The following general solutions may be used to calculate the transient response of an RLC circuit. The values of α and ω_o must first be found and used to determine whether response is overdamped, underdamped, or critically damped.

RLC NATURAL RESPONSE - Parallel or series: $v_C(t)$, $i_C(t)$, $v_L(t)$, $i_L(t)$

$f(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$	Overdamped
$f(t) = e^{-\alpha t}(B_1 \cos \omega_d t + B_2 \sin \omega_d t)$	Underdamped
$f(t) = D_1 t e^{-\alpha t} + D_2 e^{-\alpha t}$	Critically Damped

RLC STEP RESPONSE - Parallel or series: $v_C(t)$, $i_C(t)$, $v_L(t)$, $i_L(t)$

$f(t) = f(\infty) + A_1 e^{s_1 t} + A_2 e^{s_2 t}$	Overdamped
$f(t) = f(\infty) + e^{-\alpha t}(B_1 \cos \omega_d t + B_2 \sin \omega_d t)$	Underdamped
$f(t) = f(\infty) + D_1 t e^{-\alpha t} + D_2 e^{-\alpha t}$	Critically Damped

Coefficients for a particular case are determined by evaluating initial conditions. The expressions for each general solution and its derivative evaluated at $t=0^+$ are given below.

OVERDAMPED:

$$f(0^+) = A_1 + A_2 \quad [+ f(\infty) \text{ if step response}]$$

$$\frac{d\,f(0^+)}{dt} = s_1 A_1 + s_2 A_2$$

UNDERDAMPED:

$$f(0^+) = B_1 \quad [+ f(\infty) \text{ if step response}]$$

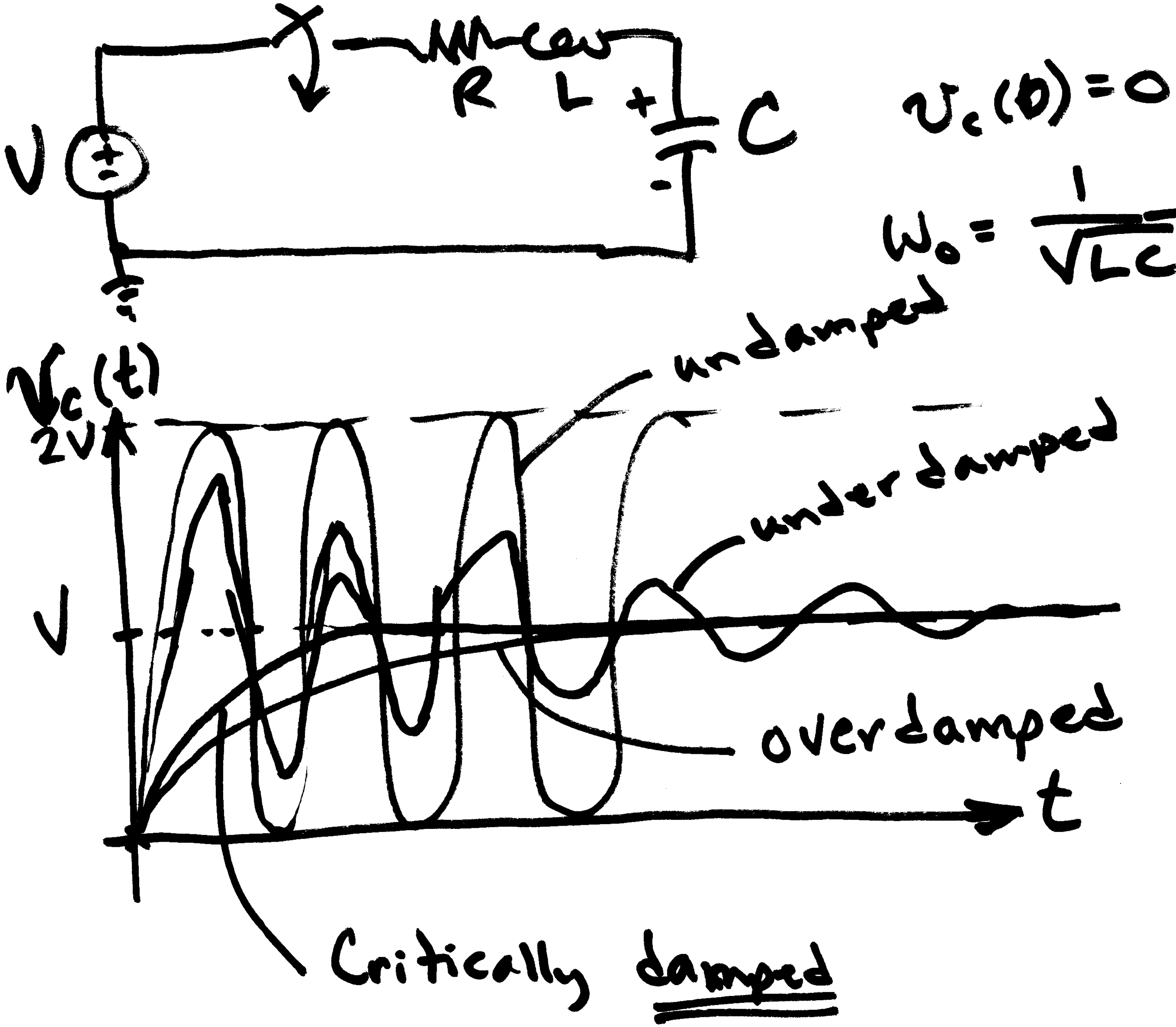
$$\frac{d\,f(0^+)}{dt} = -\alpha B_1 + \omega_d B_2$$

CRITICALLY DAMPED:

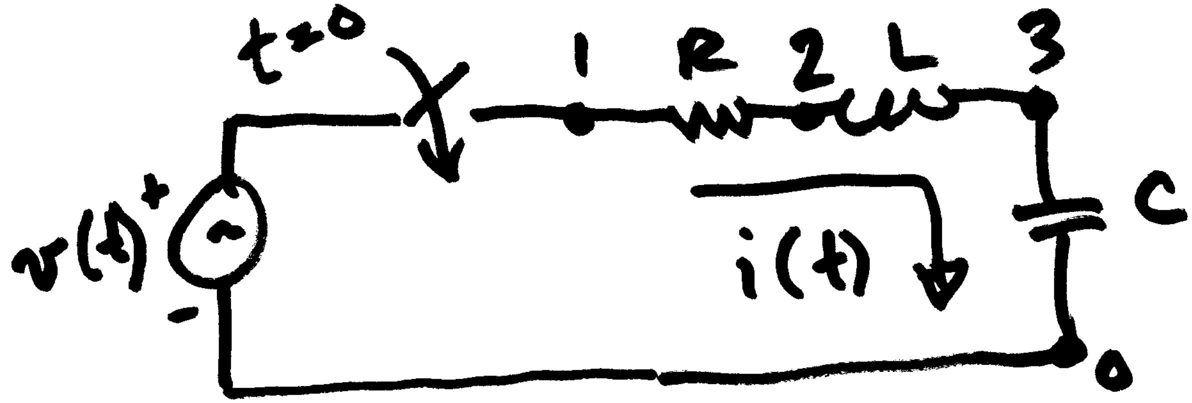
$$f(0^+) = D_2 \quad [+ f(\infty) \text{ if step response}]$$

$$\frac{d\,f(0^+)}{dt} = D_1 - \alpha D_2$$

The numeric initial values of the function and its derivative at $t=0^+$ are obtained from your knowledge of the particular circuit: parameter values, initial conditions, type of switching occurring, and differential relationships of current and voltage in the inductor and capacitor at $t=0^+$.



V
R
L
C
$v_c(0) = 0$
$\omega_0 = \frac{1}{\sqrt{LC}}$
$v_c(t)$
2V
V
undamped
underdamped
overdamped
t
Critically damped



$+$

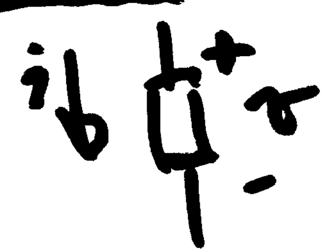
$v_{30} = v_3$

$-$

- Diff. Eqns — <u>Handout</u>
- Laplace
- Intuitive methods.

Review of Ckts! — EE5200

- Subscript notations
- Active vs. <u>passive</u> sign conv.
- P, Q flow
- Phase angles



$p(t) = v(t)\, i(t)$

$= p_{in}$

$\vec{v}_{23}$

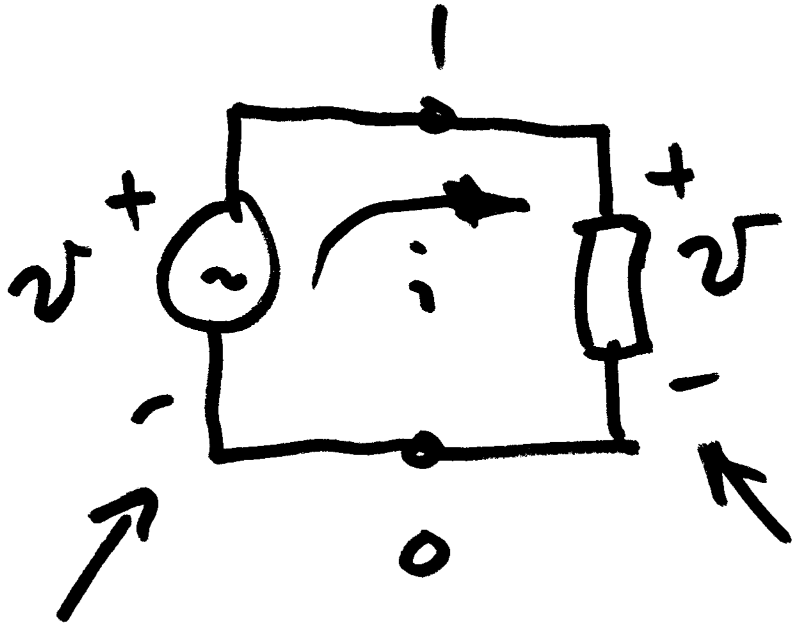
v_1

$\vec{i}_{21}$

from to

$\vec{P}$

$\vec{Q}$



$$p = vi = p_{out}$$

$$p = vi = p_{in}$$

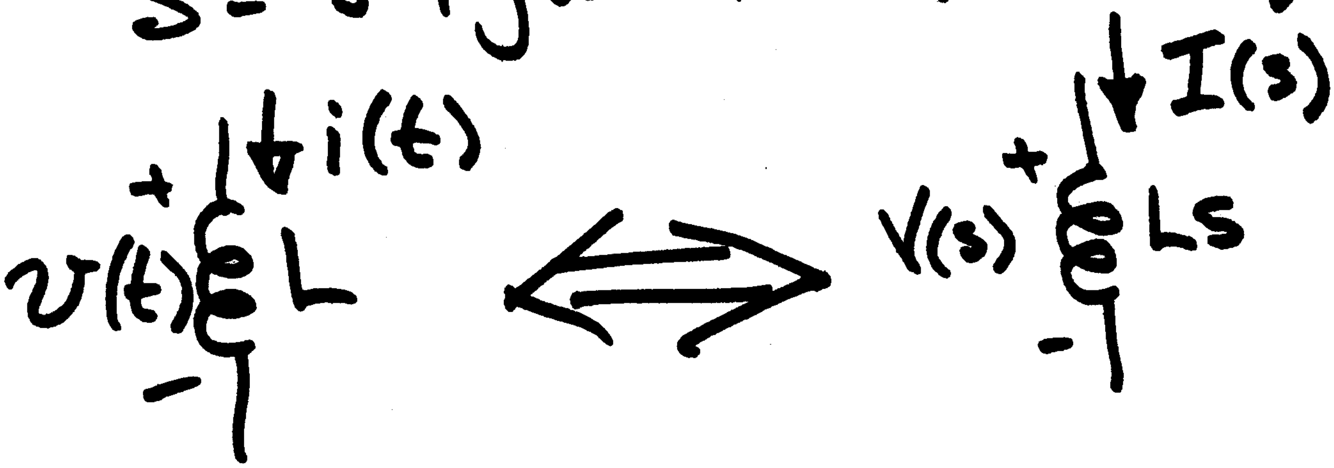
APPENDIX A
LAPLACE TRANSFORM TABLE

Laplace transform, $F(s)$	Time function, $f(t)$, $t \geqslant 0$
1	$\delta(t_0)$, unit impulse at $t = t_0$
$\dfrac{1}{s}$	$U(t)$, unit step function
$\dfrac{1}{s^2}$	t
$\dfrac{2}{s^3}$	t^2
$\dfrac{n!}{s^{n+1}}$	t^n
$\dfrac{1}{s+a}$	e^{-at}
$\dfrac{1}{(s+a)(s+b)}$	$\dfrac{e^{-at} - e^{-bt}}{b-a}$
$\dfrac{1}{(s+a)^n}$	$\dfrac{1}{(n-1)!}\, t^{n-1}e^{-at}$
$\dfrac{s+\alpha}{(s+a)(s+b)}$	$\dfrac{1}{(b-a)}\left[(\alpha - a)e^{-at} - (\alpha - b)e^{-bt}\right]$
$\dfrac{1}{(s+a)(s+b)(s+c)}$	$\dfrac{e^{-at}}{(b-a)(c-a)} + \dfrac{e^{-bt}}{(c-b)(a-b)} + \dfrac{e^{-ct}}{(a-c)(b-c)}$
$\dfrac{\omega}{s^2+\omega^2}$	$\sin \omega t$
$\dfrac{s}{s^2+\omega^2}$	$\cos \omega t$
$\dfrac{\omega}{(s+\omega)^2}$	$\omega t e^{-\omega t}$
$\dfrac{1}{(1+Ts)^n}$	$\dfrac{t^{n-1}e^{-t/T}}{T^n(n-1)!}$

Laplace transform, $F(s)$	Time function, $f(t)$, $t \geqslant 0$
$\dfrac{1}{s(1 + Ts)}$	$1 - e^{-t/T}$
$\dfrac{1}{s(1 + Ts)^2}$	$1 - \dfrac{t + T}{T} e^{-t/T}$
$\dfrac{\omega}{(s + a)^2 + \omega^2}$	$e^{-at} \sin \omega t$
$\dfrac{(s + a)}{(s + a)^2 + \omega^2}$	$e^{-at} \cos \omega t$
$\dfrac{\omega_n^2}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)}$	$1 + \dfrac{e^{-\zeta\omega_n t}}{\sqrt{1 - \zeta^2}} \sin (\omega_n \sqrt{1 - \zeta^2}\, t - \alpha)$ where $\cos \alpha = -\zeta$
$\dfrac{\omega_n^2}{s^2(s^2 + 2\zeta\omega_n s + \omega_n^2)}$	$t - \dfrac{2\zeta}{\omega_n} + \dfrac{1}{\omega_n \sqrt{1 - \zeta^2}} e^{-\zeta\omega_n t} \sin (\omega_n \sqrt{1 - \zeta^2}\, t - \theta)$ where $\theta = 2 \tan^{-1} \dfrac{\sqrt{1 - \zeta^2}}{-\zeta}$
$\dfrac{s}{(1 + Ts)(s^2 + \omega_n^2)}$	$\dfrac{-1}{(1 + T^2\omega_n^2)} e^{-t/T} + \dfrac{1}{\sqrt{1 + T^2\omega_n^2}} \cos (\omega_n t - \theta)$ where $\theta = \tan^{-1} \omega_n T$
$\dfrac{s}{(s^2 + \omega_n^2)^2}$	$\dfrac{1}{2\omega_n} t \sin \omega_n t$
$\dfrac{1}{(s + b)[(s + a)^2 + \omega^2]}$	$\dfrac{e^{-bt}}{(b - a)^2 + \omega^2} + \dfrac{e^{-at} \sin (\omega t - \theta)}{\omega[(b - a)^2 + \omega^2]^{1/2}}$ where $\theta = \tan^{-1} \dfrac{\omega}{b - a}$
$\dfrac{2abs}{[s^2 + (a + b)^2][s^2 + (a - b)^2]}$	$\sin at \sin bt$
$\dfrac{1 + as + bs^2}{s^2(1 + T_1 s)(1 + T_2 s)}$	$t + (a - T_1 - T_2) + \dfrac{b - aT_1 + T_1^2}{T_1 - T_2} e^{-t/T_1} - \dfrac{b - aT_2 + T_2^2}{T_1 - T_2} e^{-t/T_2}$

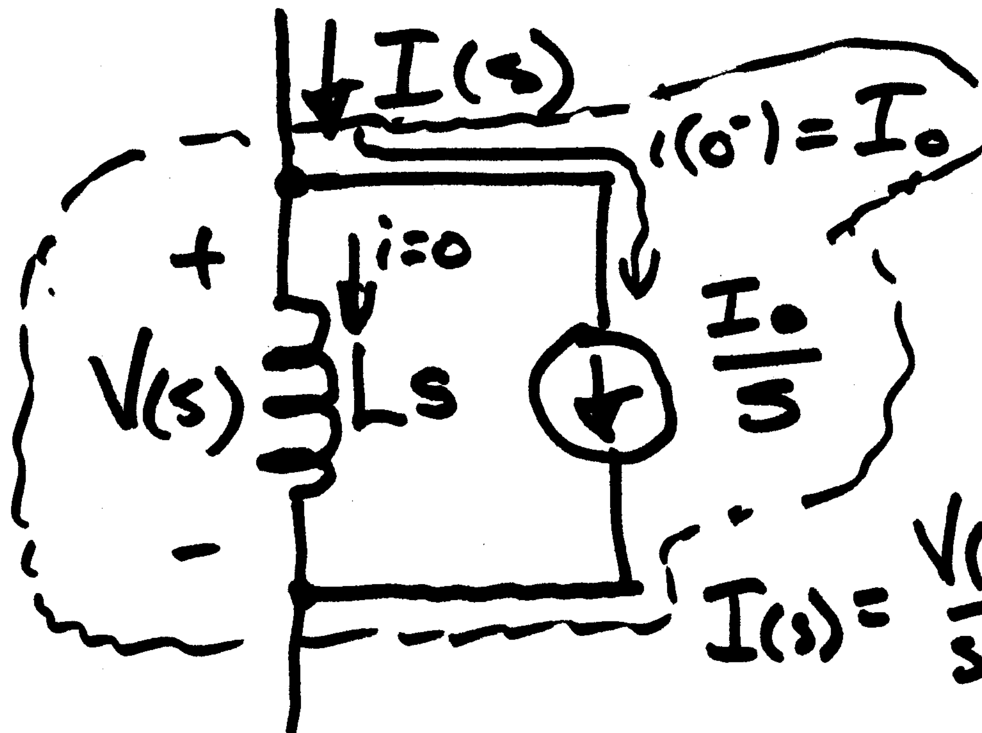
Laplace Review

$s = \sigma + j\omega$ (complex freq)



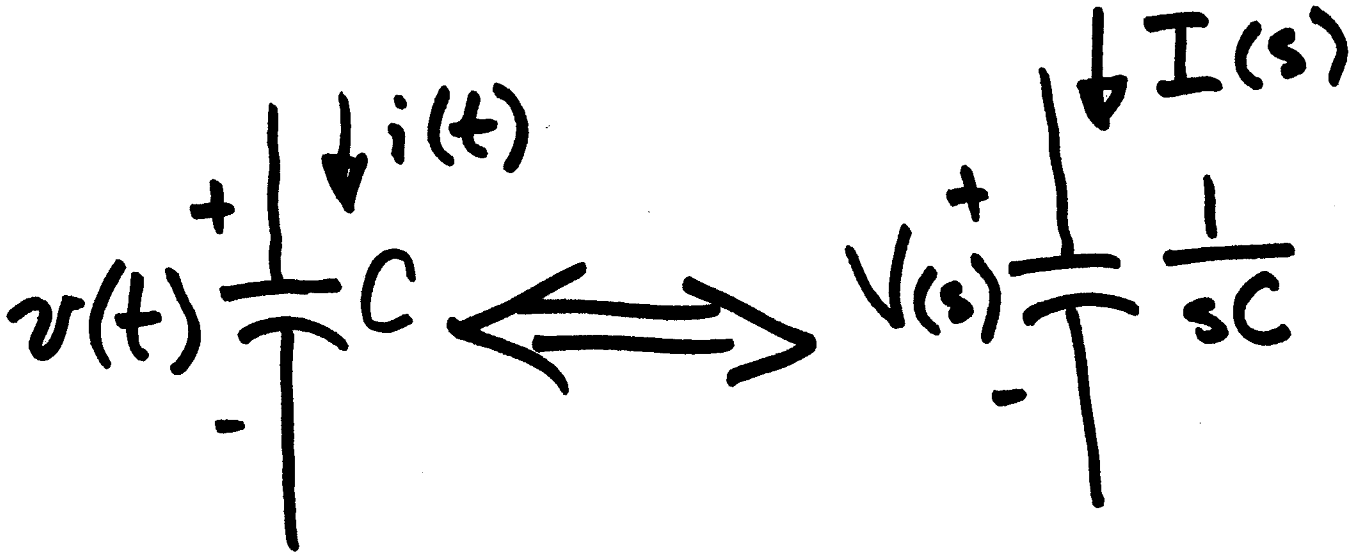
Initial Conditions:

if $i(0^-) = I_0$ then

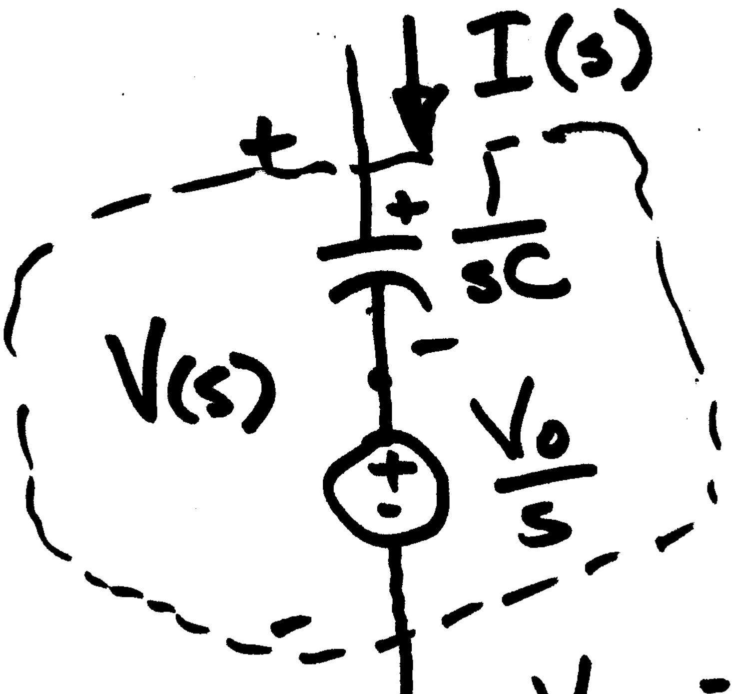


$$I(s) = \frac{V(s)}{sL} + \frac{I_0}{s}$$

$$V(s) = sL\left(I(s) - \frac{I_0}{s}\right) = \underline{sLI(s) - LI_0}$$



Initial Conditions:
if $v(0^-) = V_0$ then



$$V(s) = \frac{I(s)}{sC} + \frac{V_0}{s}$$

$$I(s) = sCV(s) - CV_0$$