EE 5220 - Lecture 27 Fri Mar 15, 2024

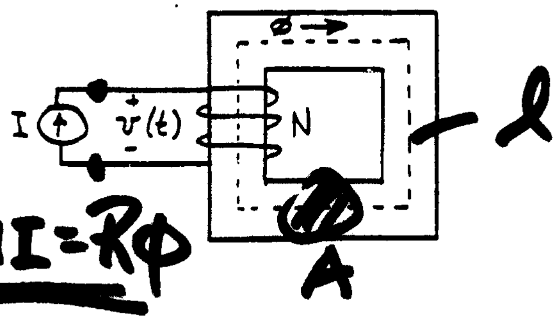
Topics for Today:

- Course Info:
  - Web page: https://pages.mtu.edu/~bamork/ee5220/
  - Book, references, syllabus, more are on web page.
  - Software - Matlab. ATP/EMTP [License - www.emtp.org] ATP tutorials posted on our course web page
  - EE5220-L@mtu.edu (participation = min half letter grade)

- HW#8 - Probs. 9.6, 9.12 due Tues Mar 19th, 9am.
- HW#9 - Probs. 9.2, 9.3, 9.4 due Tues Apr 26th 9am.
- Mid-term: indiv homework set. think of a date that works, maybe early April
- Term Project - due: a) complete reference list and b) fully-detailed table of contents according to format given in Term Project Guidelines.
- Transformer modeling - Section 11.1 of text, plus lecture notes
  - Nonlinear inductor models - Types 93, 98, 96
  - Magnetic materials: B-H characteristics
  - Transformer Inrush - initial conditions
    - Energization inrush
    - Recovery inrush
    - Sympathetic inrush
- Next - take stock of available ATP transformer models

REVIEW OF MAGNETIC CIRCUITS

As a simple example, an ideal single-winding magnetic circuit will be used. The magnetic core in this case is assumed to have no magnetic saturation, even at high levels of flux density.



$$\text{MMF} = \mathcal{F} = \underline{NI = \mathcal{R}}\phi$$

Some of the basic parameters which physically define this circuit are:

A = Cross-sectional area of core

N = Number of turns of the winding

l = Mean (average) path length of core (dashed line)

μ = Magnetic permeability of the core. μ depends on the type of core material. ($\mu = \mu_r \mu_0$)

Some other important magnetic circuit quantities are defined as follows:

Reluctance of magnetic core:

$$R = \frac{l}{\mu A} \ H^{-1} \quad \mathcal{R}$$

Magnetomotive Force:

MMF = N I Amp-Turns

Magnetic Flux:

$$\phi = \frac{\text{MMF}}{R} = \frac{N\ I}{R} \ \text{Webers}$$

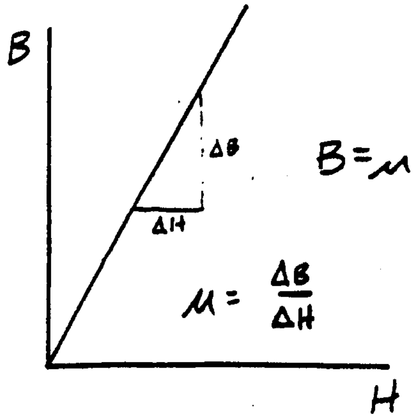
(Direction given by "right-hand rule").

Magnetic Field Intensity:

$$H = \frac{\text{MMF}}{l} \ \frac{\text{Amp-Turns}}{m}$$

Magnetic Flux Density

$$B = \frac{\phi}{A} \ \frac{\text{Webers}}{m^2} \ \text{or} \ \underline{\text{Tesla}}$$



$$B = \mu H$$

$$\mu = \frac{\Delta B}{\Delta H}$$

Flux Linkage

$\lambda = N\underline{\phi} = N A B$ Weber-Turns (or Volt-sec)

Inductance

$$\boxed{L = \frac{\lambda}{I}} = \frac{N^2}{\underline{\mathcal{R}}} = \frac{N^2 \mu A}{l} \quad \frac{\text{Wb-Turns}}{\text{Amp}} \text{ or } \underline{\text{Henries}}$$

Induced Voltage

$$\underline{v(t) = \frac{d\lambda}{dt}} = \boxed{N\frac{d\phi}{dt}} = L\frac{di}{dt} \quad \text{Volts} \qquad \underline{\lambda(t) = \int v(t)\,dt}$$

Note that in this case, the induced voltage v(t) is zero, since the current and flux do not change with time.

USE OF VARIOUS UNITS OF MEASUREMENT

Manufacturer's test reports for various magnetic materials may give parameters in several different units of measurement. The following is a clarification of these different units:

Flux Density (B)

Standard Unit: Tesla = Weber/m²
   Other Unit: Maxwells (lines/inch²) = Tesla x 64500
   Other Unit: Gauss = Tesla x 10^4

Field Intensity (H)

Standard Unit: Ampere-*Turns/m or Amps/m or Amps/cm
   Other Unit: Oerstads = Ampere-Turns/m x $\underline{0.01257}$

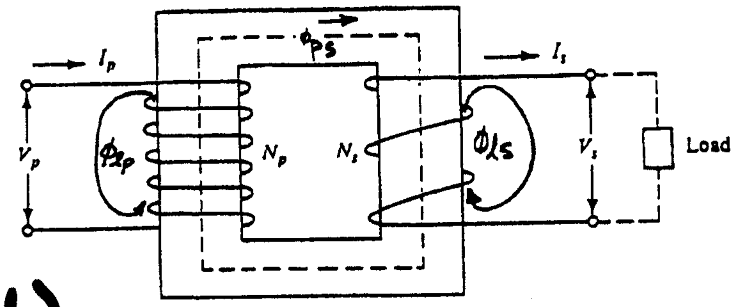
* Note that "Turns" is not really a dimensional unit

Magnetomotive Force (MMF)

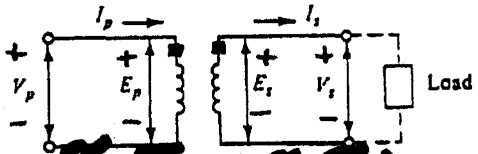
Standard Unit: Ampere-Turns
   Other Unit: Gilberts = Ampere Turns x $\underline{0.4\pi}$

SATURABLE MAGNETIC CIRCUITS

As an example, we consider a 1ø two-winding transformer with a saturable magnetic core.

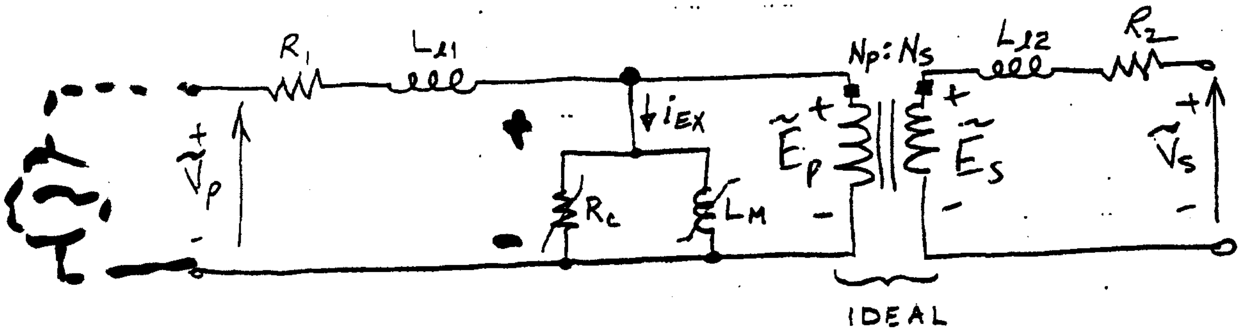


$$e(t) = \frac{d\lambda(t)}{dt}$$



$$\tilde{V}_p I_p^* = \tilde{V}_s I_s^*$$

If flux leakage, winding resistance, and core losses are included, the folowing equivalent circuit can be used:



Note: $\tilde{V}_p$ & $\tilde{V}_s$ are voltages measured at terminals.
$\tilde{E}_p$ & $\tilde{E}_s$ are magnetically induced via core.

R_1 and R_2	AC resistance of windings (linear)
R_c	Resistance representing core losses (nonlinear)
L_m	Magnetizing inductance of the core (nonlinear)
L_{l1} & L_{l2}	Leakage inductance of windings (linear)

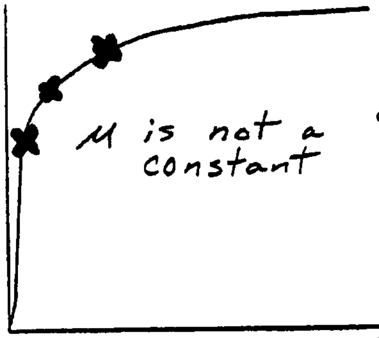
Note that core losses consist of eddy current losses and hysteresis losses, which can be frequency dependent and voltage dependent. Therefore, R_c cannot be represented as a linear resistance.

$\lambda = \Phi N$

Note also that the magnetizing inductance L_M is nonlinear due to magnetic saturation and cannot be represented as a linear inductance.

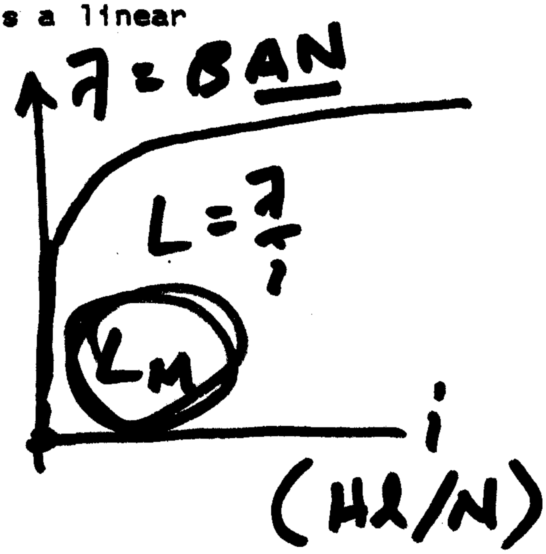
$= \frac{\Phi}{A} = \frac{\lambda}{NA} = B$

k



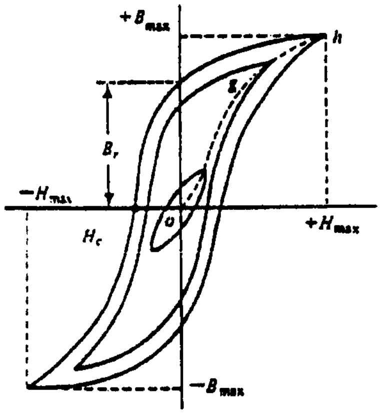
$\frac{Ni}{l} = H$

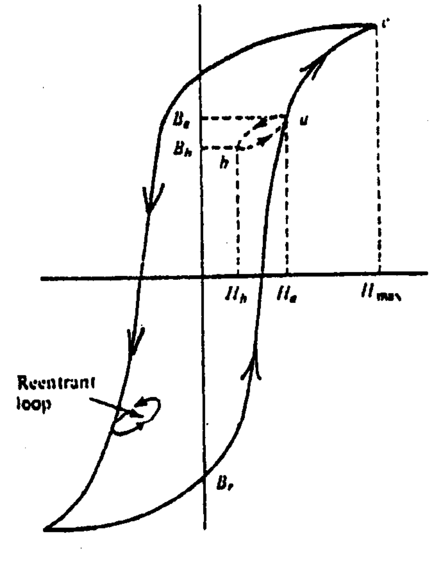
$\Rightarrow$



In other words, μ is not constant, and B & H are not linearly related. Therefore, behavior of the transformer core depends on the B-H relationship at each instant of time. This nonlinearity must be included when doing transient analysis of transformers.

Hysteresis makes the behavior of the core more complicated than the above B-H characteristic indicates. The areas of the hysteresis loops shown below (left) are proportional to the energy required for one steady-state cycle of operation. Each loop corresponds to a different level of voltage. If operation is not steady-state (below, right), a subloop or "re-entrant loop" is followed.





LEAKAGE FLUX

Leakage inductance arises because not all the flux links the windings via the core. Figure 4-1 shows an example in which some flux has leaked from the iron core and completed the magnetic circuit through the air. Such leakage is associated with both the primary and secondary windings. For convenience of illustration a core-type transformer with windings on separate limbs is shown. The principle, however, applies to any transformer (or inductor for that matter). If the windings are placed one on top of the other, as is more usual, there will still be leakage inductance, but probably to a lesser degree.

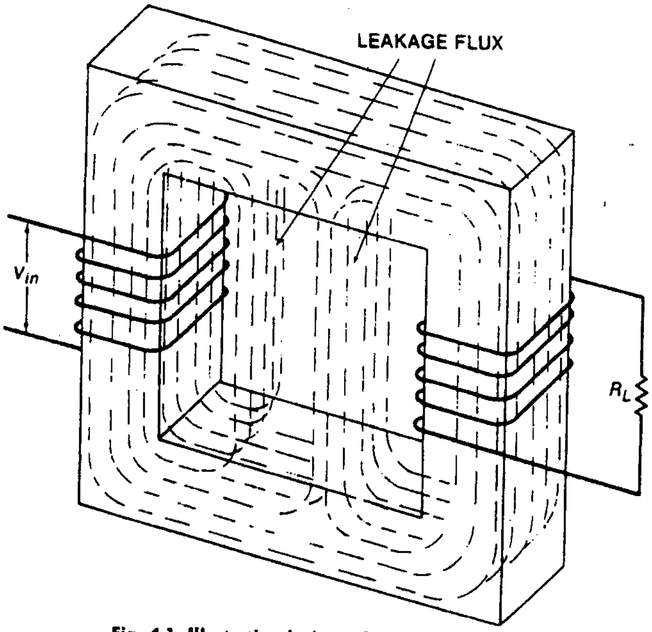


Fig. 4-1. Illustrating leakage flux from the core.

The *effect* of leakage inductance is as though a small part of the total inductance had been detached and placed in series with the winding, as shown schematically in Fig. 4-2, where L_P' and L_S' are the primary winding and secondary winding leakage inductances, respectively. Again, the effect is generally not important except at relatively high frequencies, for then the reactances are high, and being in series, have a marked effect on performance.

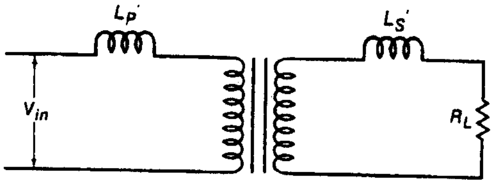
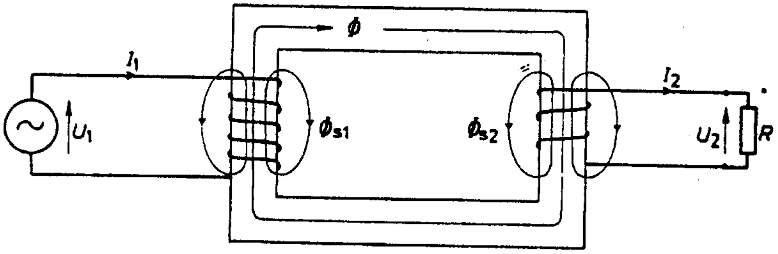
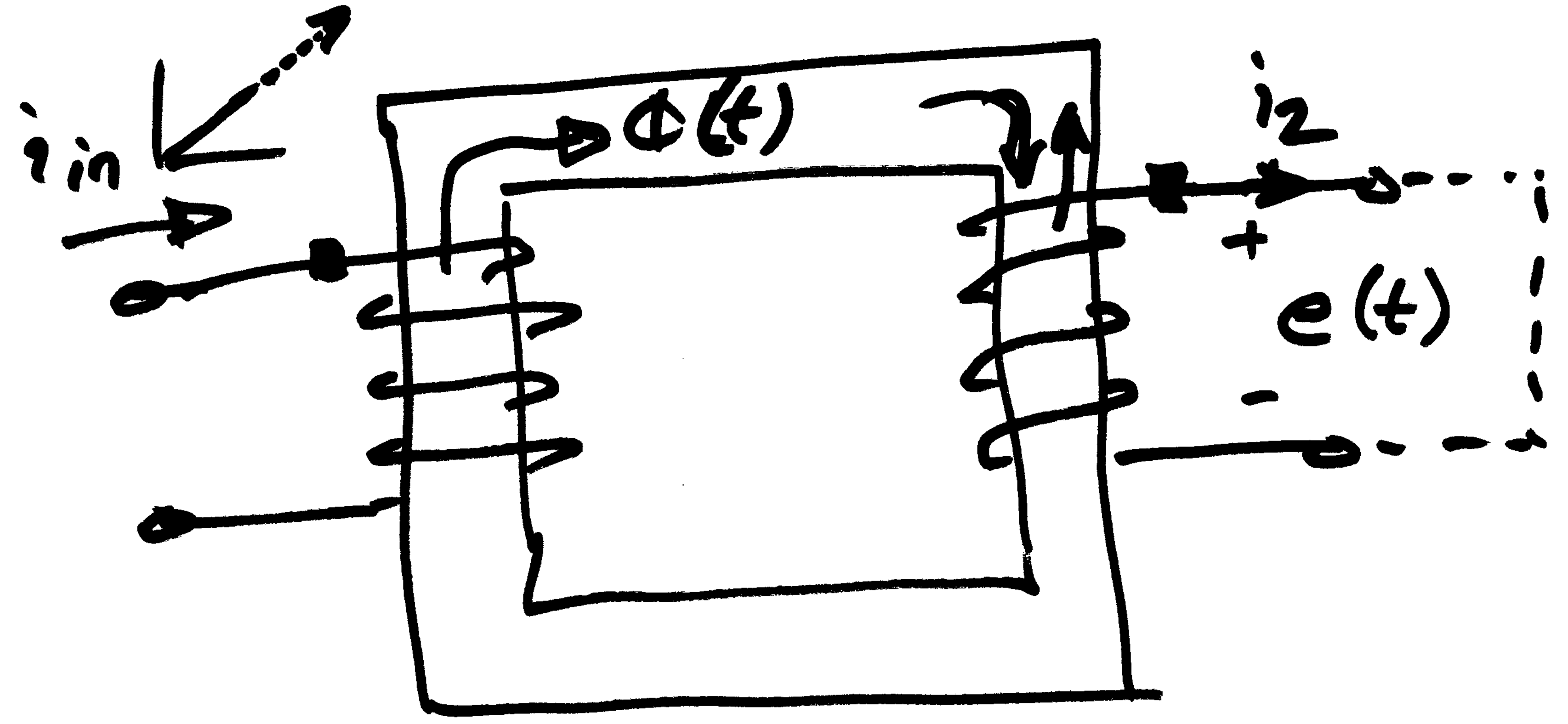


Fig. 4-2. Schematic diagram showing leakage inductances of the circuit in Fig. 4-1.



Lenz' Law

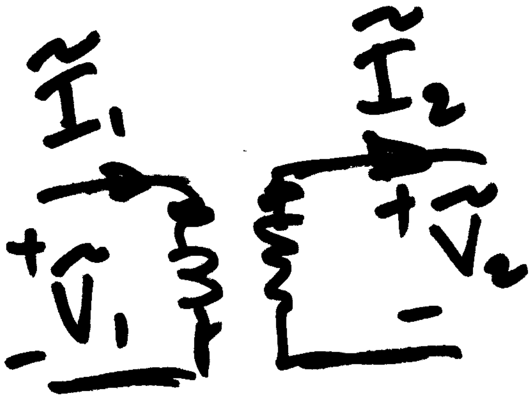
$$e(t) = \frac{d\lambda(t)}{dt} = N\frac{d\phi(t)}{dt}$$



$$\Phi(t) = \frac{Ni}{R}$$

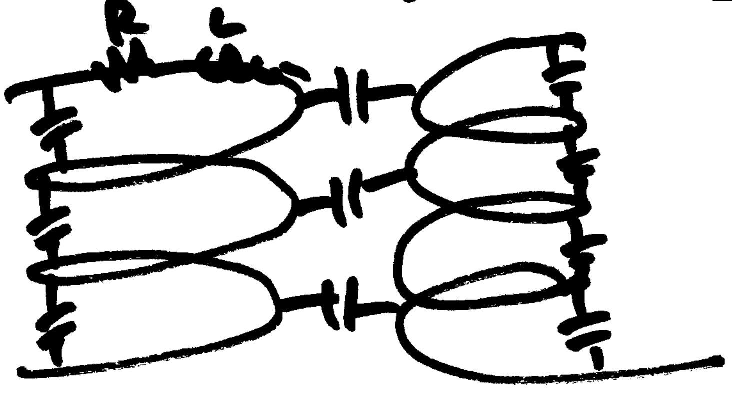
Transformers

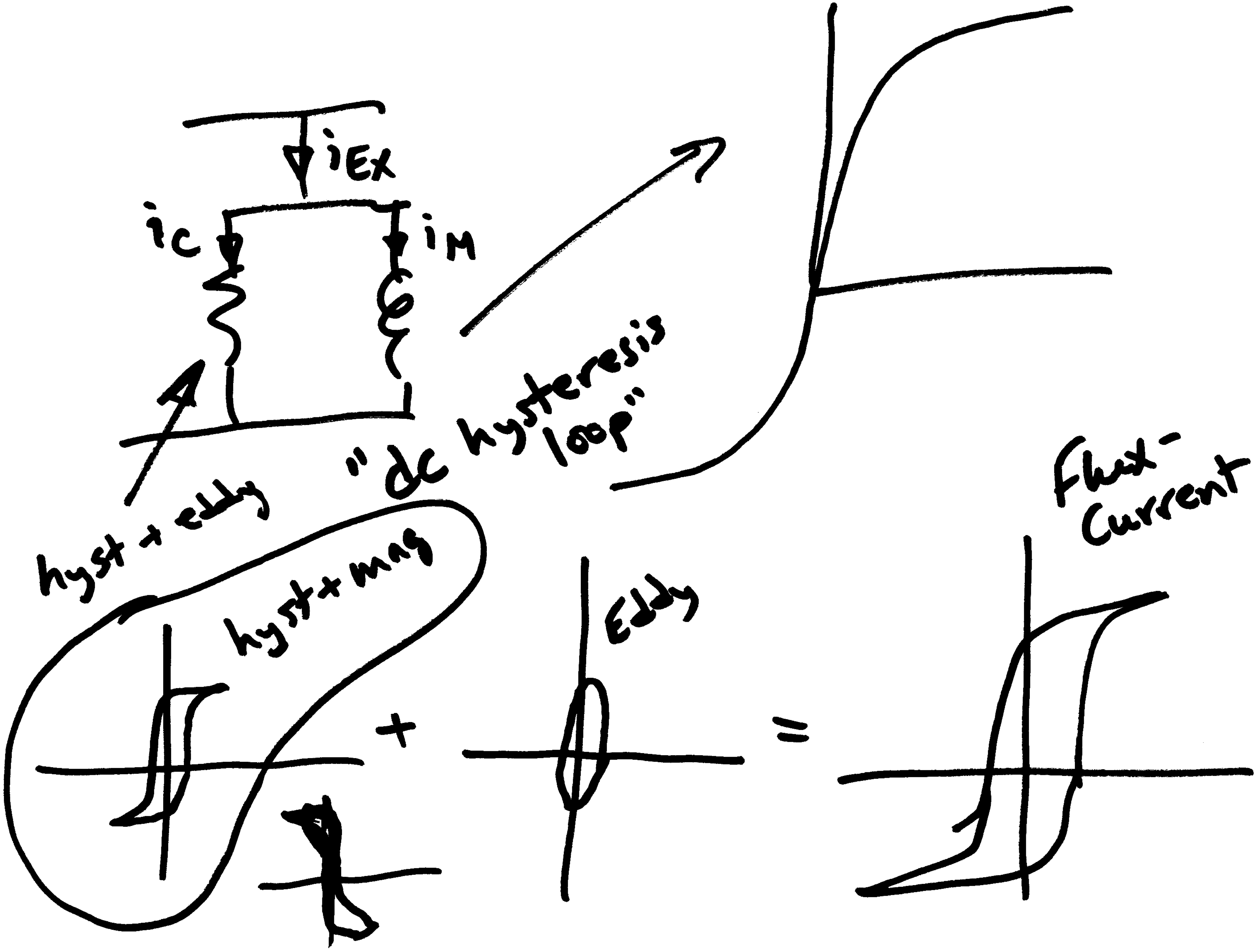
- Basics:

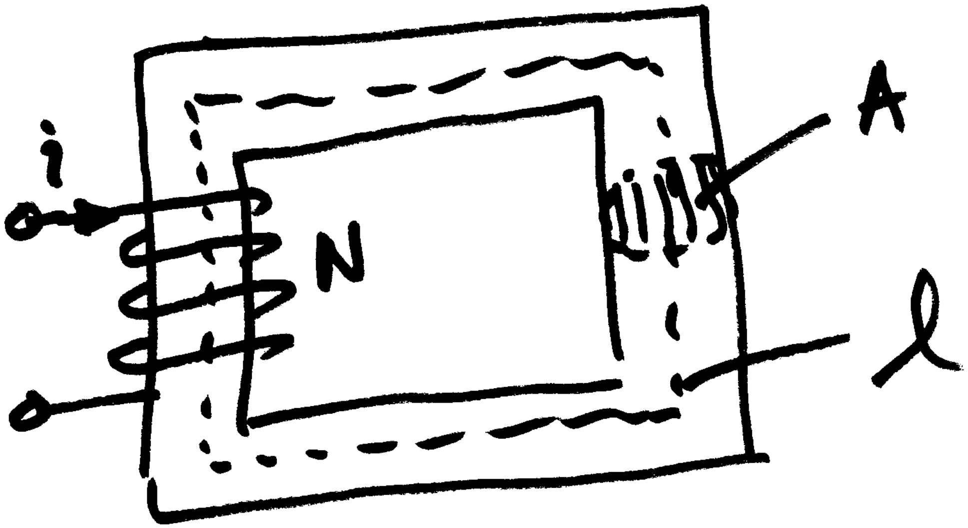


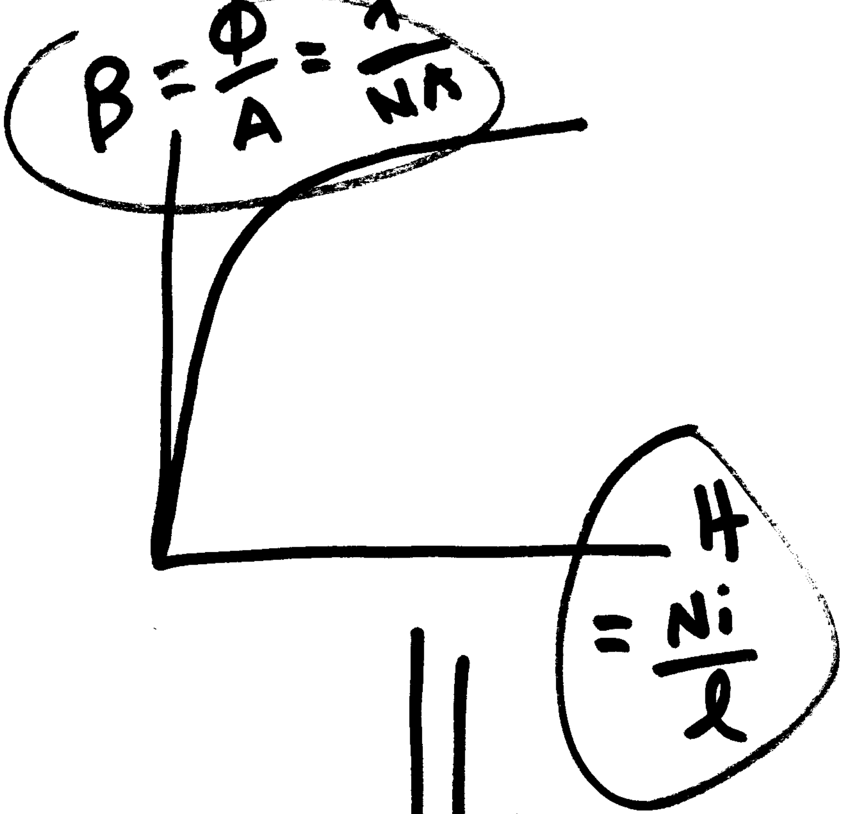
S.S. Phasor Analysis

- How about <u>time domain</u>?
- What about freq > 50 or 60 Hz?
  - Capacitance of Coils
  - Freq-dep of core/laminations
  - " " " Coils




iEx
iC
iM
hyst + eddy
"dc hysteresis loop"
Flux-Current
hyst + mag
Eddy
+
=





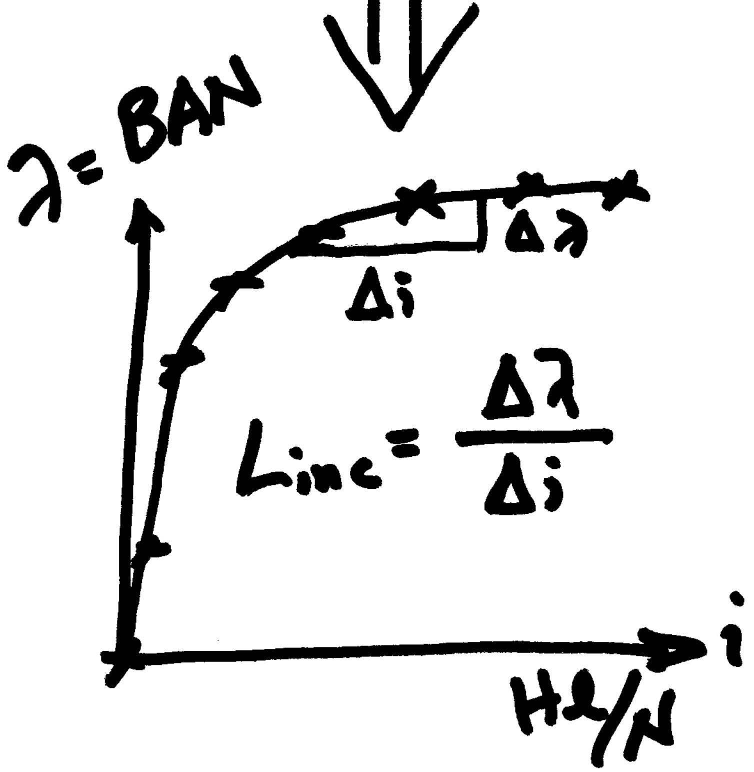
Ampere's Circuital Law

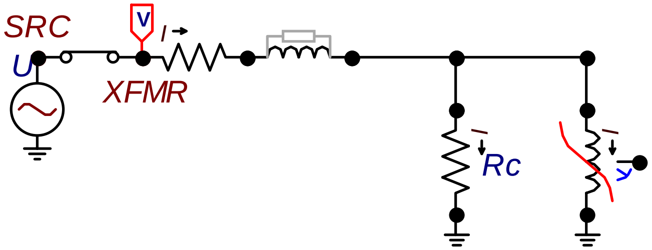
$$Ni = MMF = \Phi R$$

Flux Linkage $\lambda = N\phi$

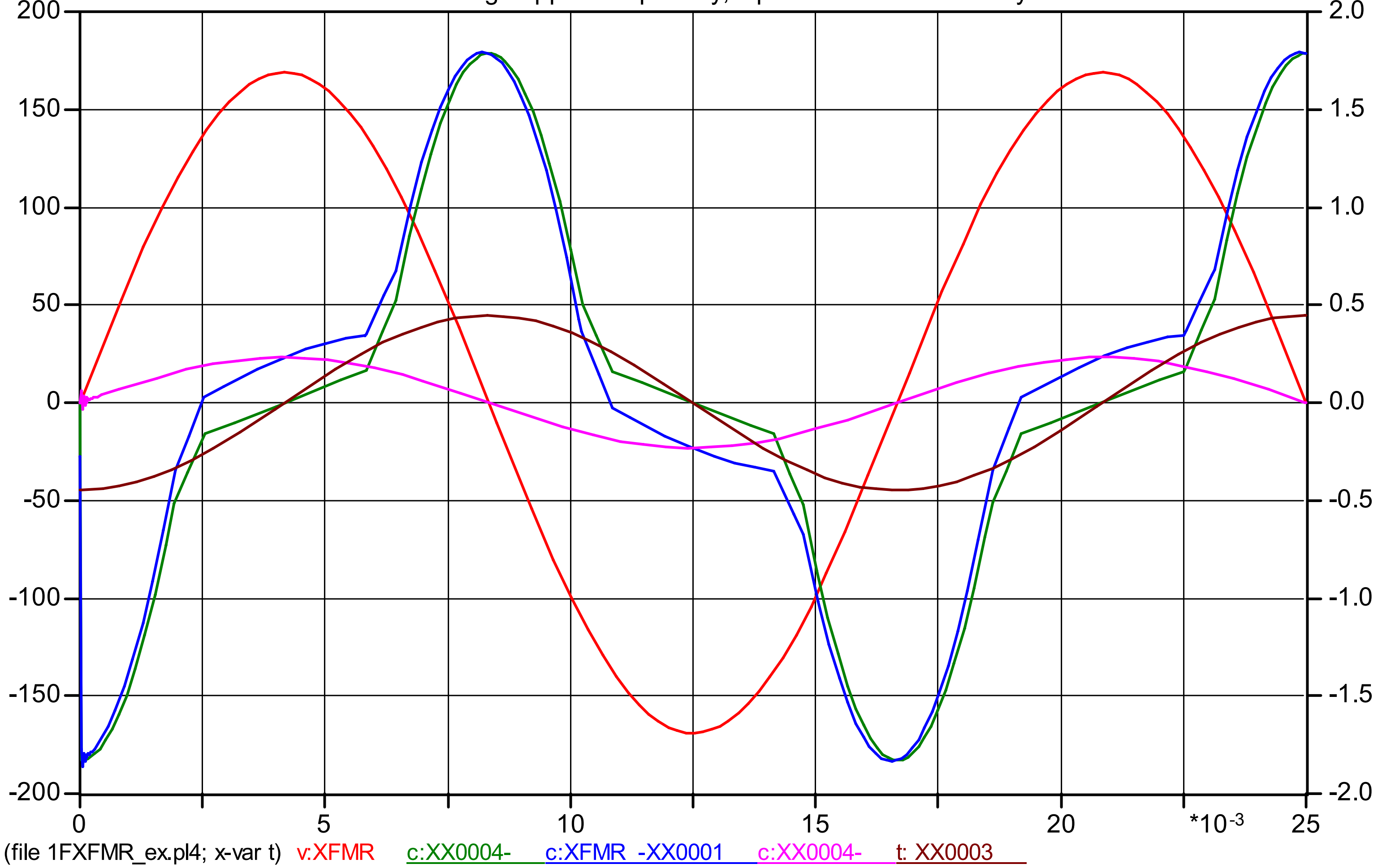
$$e(t) = N\frac{d\phi}{dt} = \frac{d\lambda}{dt}$$

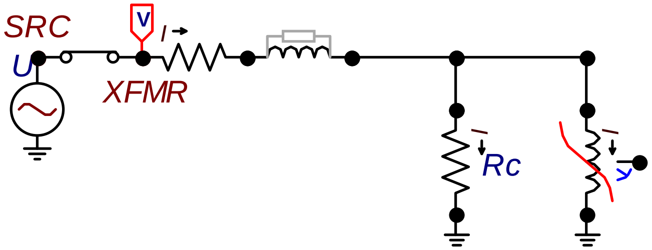
$$L = \frac{\lambda}{i}$$



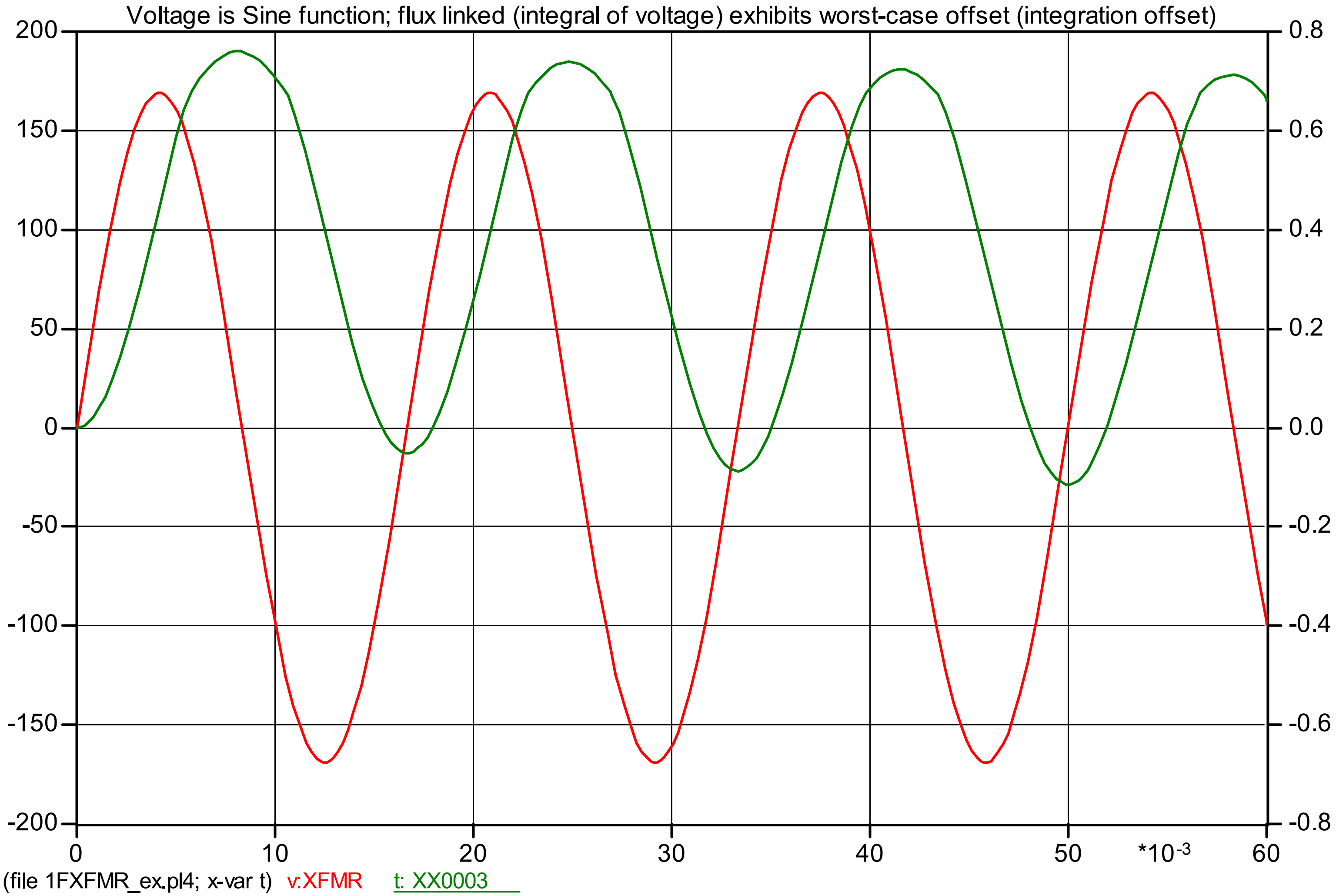


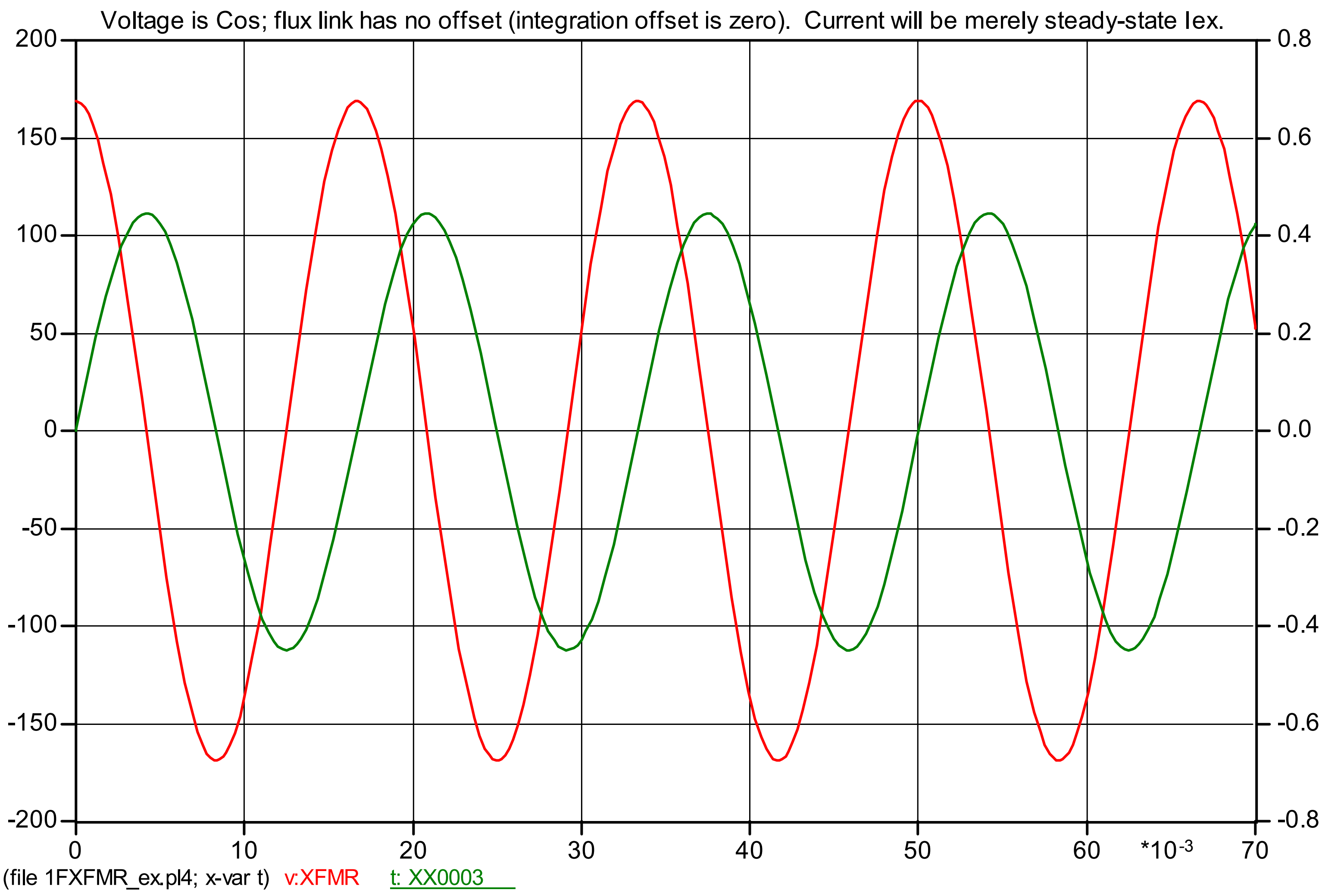
Steady-State Excitation of Single Phase Transformer
Rated voltage applied to primary, Open-circuited secondary



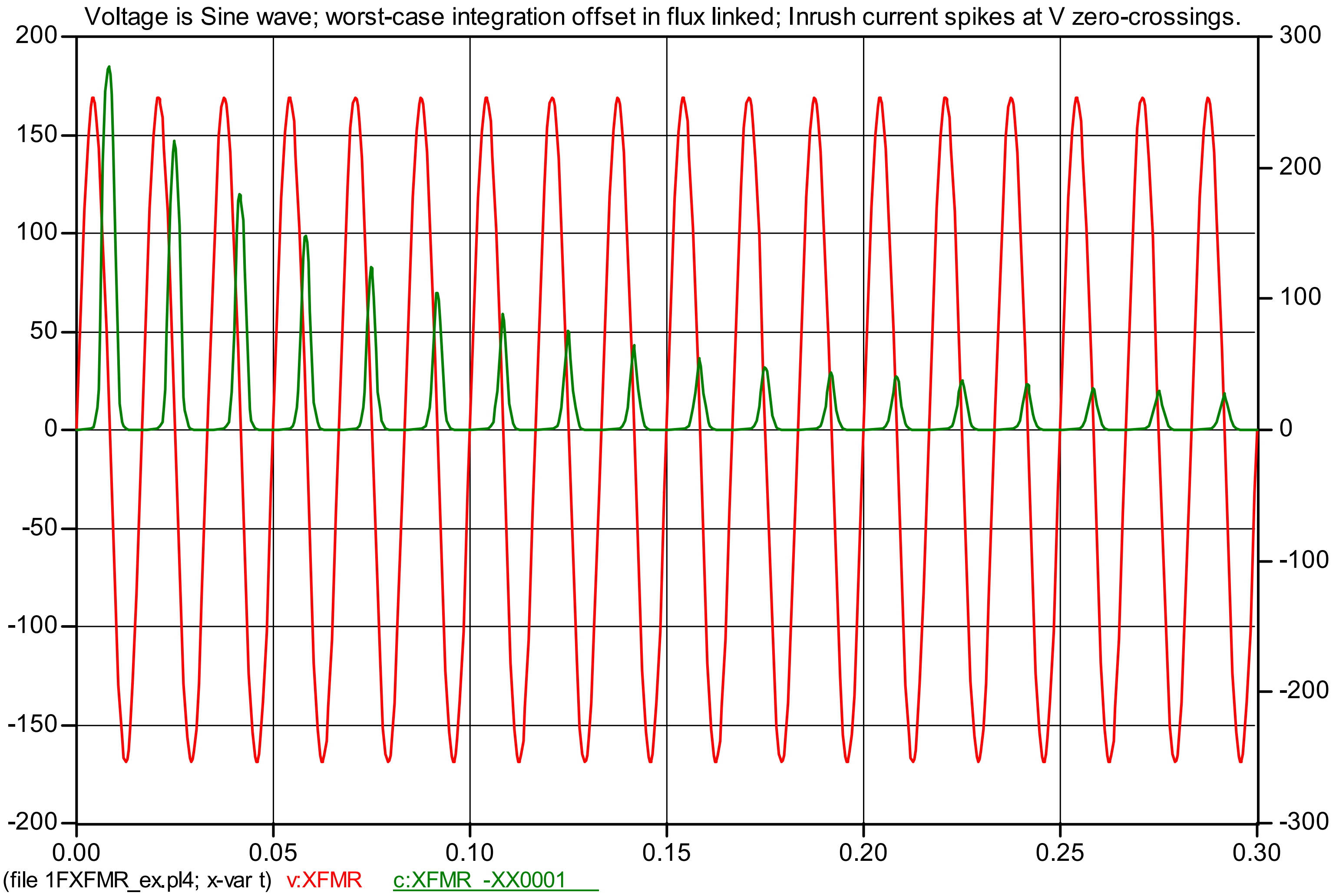


$\lambda(0) = 0$; voltage source (red) is Sine wave which turns on at t=0. Note worst-case integration offset in flux linked (green).





Special case to illustrate how to get rid of integration offset. Energize transformer at plus or minus peak voltage (Cos voltage function) and then the flux linked will have zero offset. (Again, this assumes that residual flux linked λ(0) in transformer core is zero. Unfortunately, λ(0) cannot be known or exactly predicted). Cases below go back to worst-case integration offset to illustrate the characteristics of inrush current. Inrush current spikes lag voltage by 90° as would be expected of an inductancem L_M. Winding resistance R_1 provides damping.



Same case as above, inrush current is overplotted with flux linked. See how flux linked begins with full offset, but the offset decays due to the damping effect of R_1. Rate of decay is not exactly exponential like in a linear R-L circuit, due to nonlinear (saturable) L_M characteristic. Decay is initially quite rapid while L_M is in full saturation, but rate of decay is slower as it progresses (less saturation => smaller current spikes => less damping).