

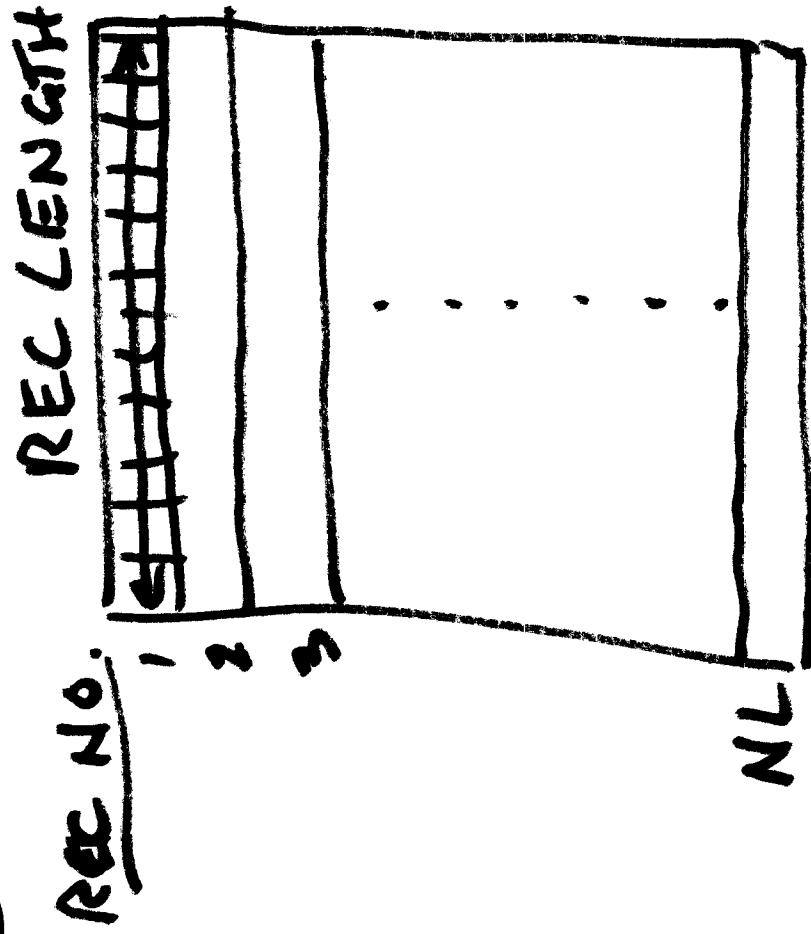
Topics for Today:

- Questions?
- Questions/Comments on Homework #4 ?
- Building [Y] for 14-bus IEEE system.
- Useful Matlab functions: fgetl, sscanf, format '%*23c%f', spalloc, sparse, spy
- Newton Iteration, example for one-variable case
- Newton Iteration, example for two-variable case
- Loadflow Formulation: “NR Details” handout (Week 4)
- NR Algorithm implementation.

Coming up:

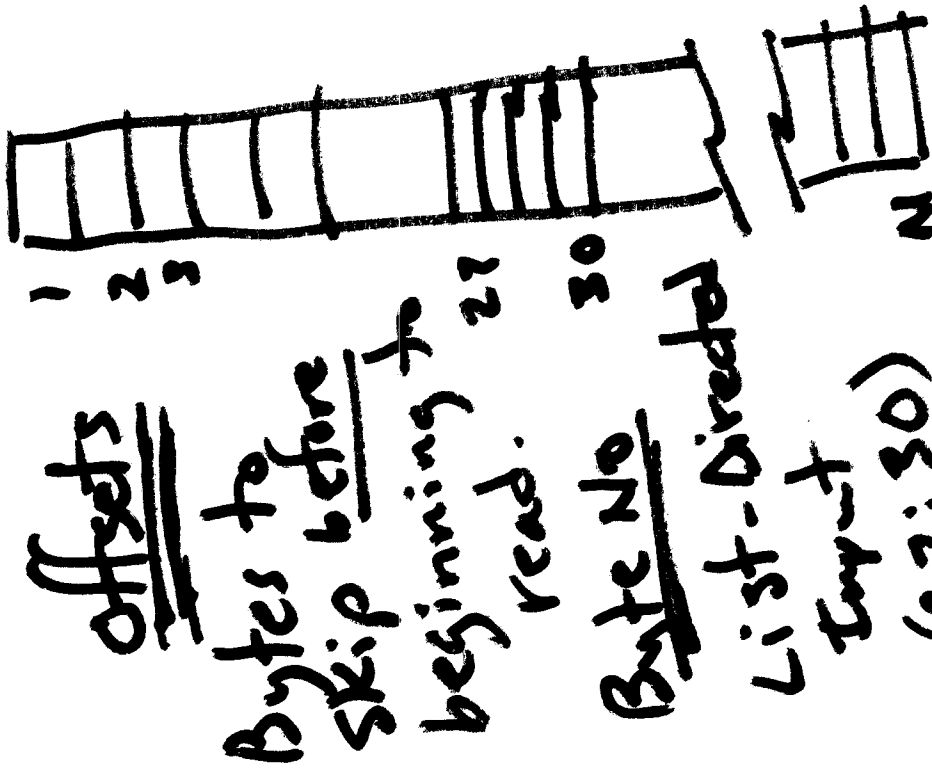
- More MatLab - build Jacobian, solve for $\Delta\delta$ and ΔV , iterate.
- Data structures, LU factorization, reordering to avoid zero divides and/or speed up solution.

② Direct Access



FIXED LENGTH
DIRECT ACCESS

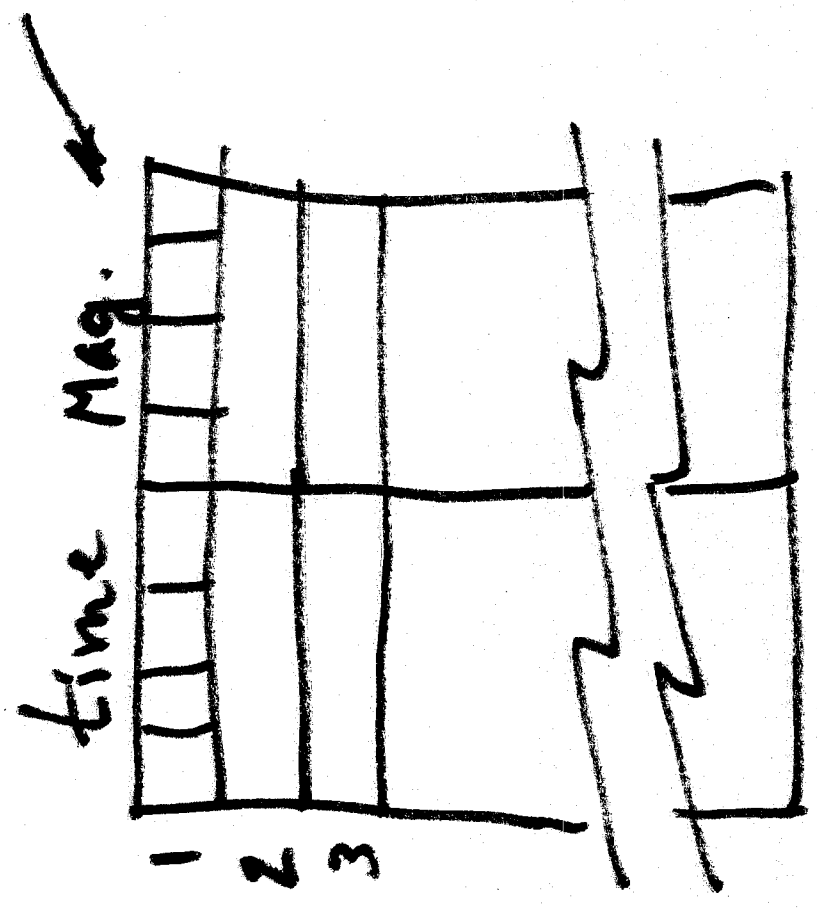
Storage



N-BYTES

EX:

Binary File

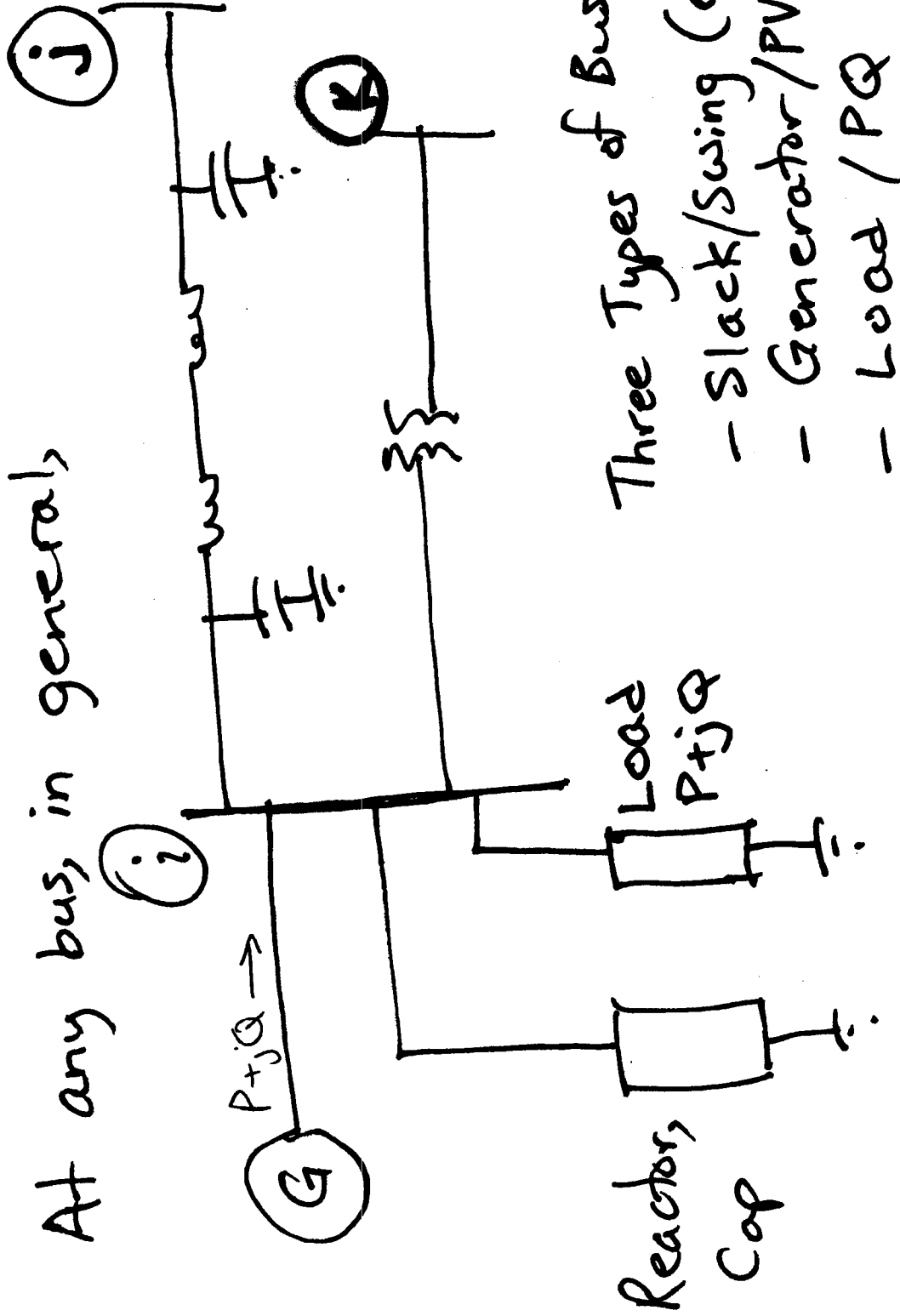


REC LENGTH = 8 Bytes

- Time: Single Rec Real
- Mag: " " "

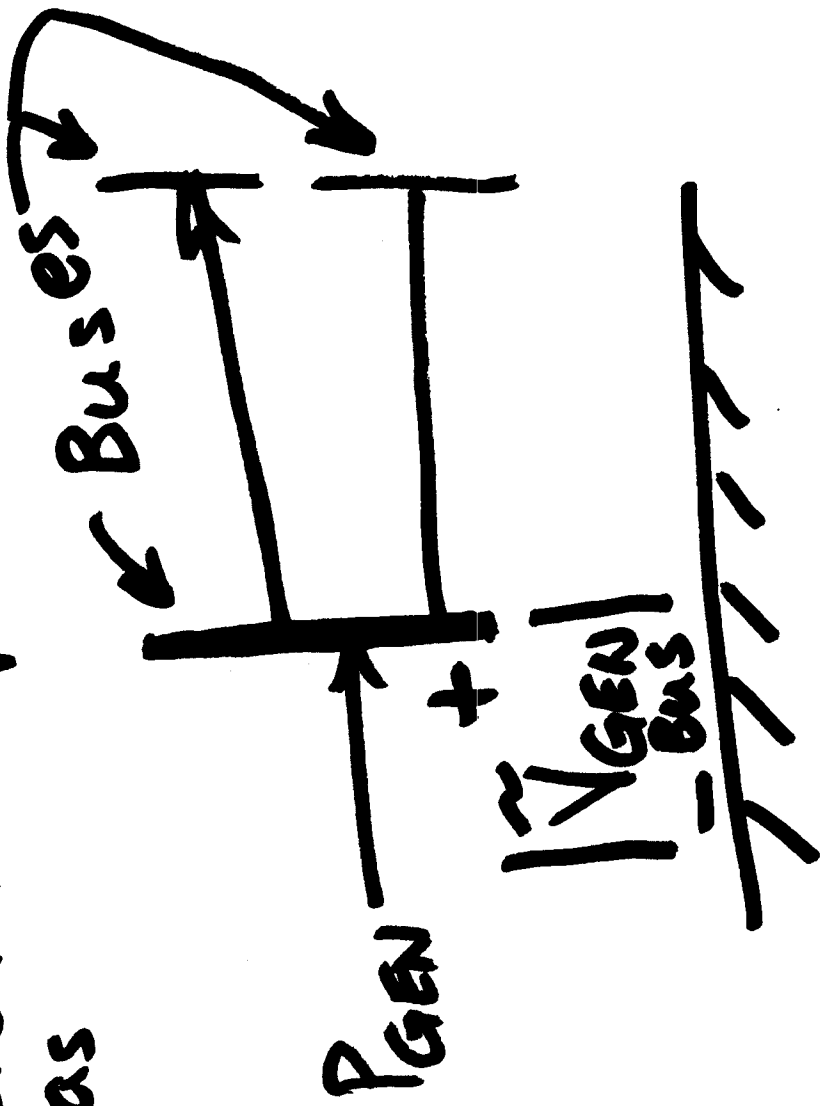
Load Flows Setup

At any bus, in general,



Load Flow

Gen is represented
as



Governor
- Control P_{GEN}

Exciter:
- Control V_{Bus}

KEY
 ✓ = known
 ? = unknown

Bus Type	Variables			
	$ V $	δ	P	Q
Slack	✓	✓	?	?
Gen	✓	?	✓	?
Load	?	?	✓	✓

$V_{bus} = |V| \angle \delta$ Net P & Q into bus
 due to gen and load

Does not include
 Lines, XFMR, reactor.

- Next - Combine
- $[Y_{bus}]$
 - Knowns & unknowns, produce $[J]$, $[f^m]$
 - Apply NR iteration to solve.

Load Flow Formulation

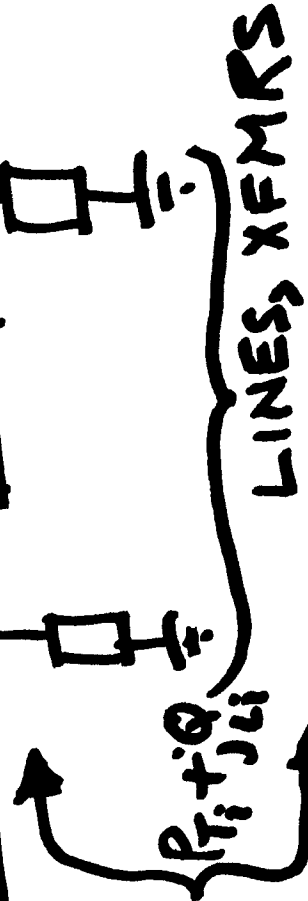
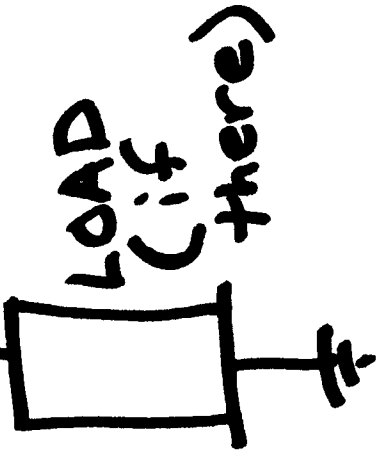
2

At each bus i

P_{Gi} →

GEN (if there)

$P_i + jQ_i$

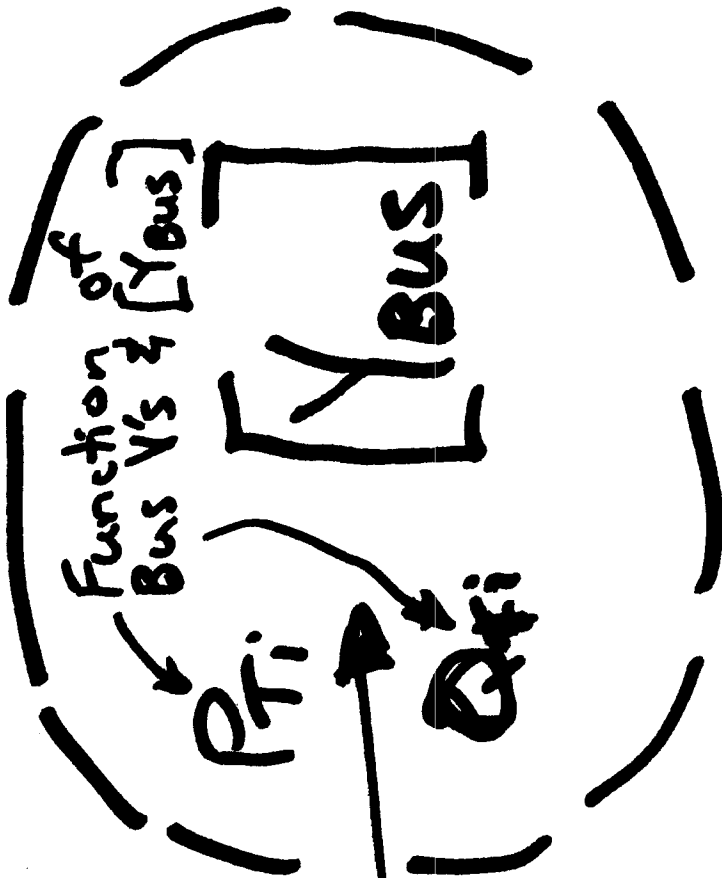
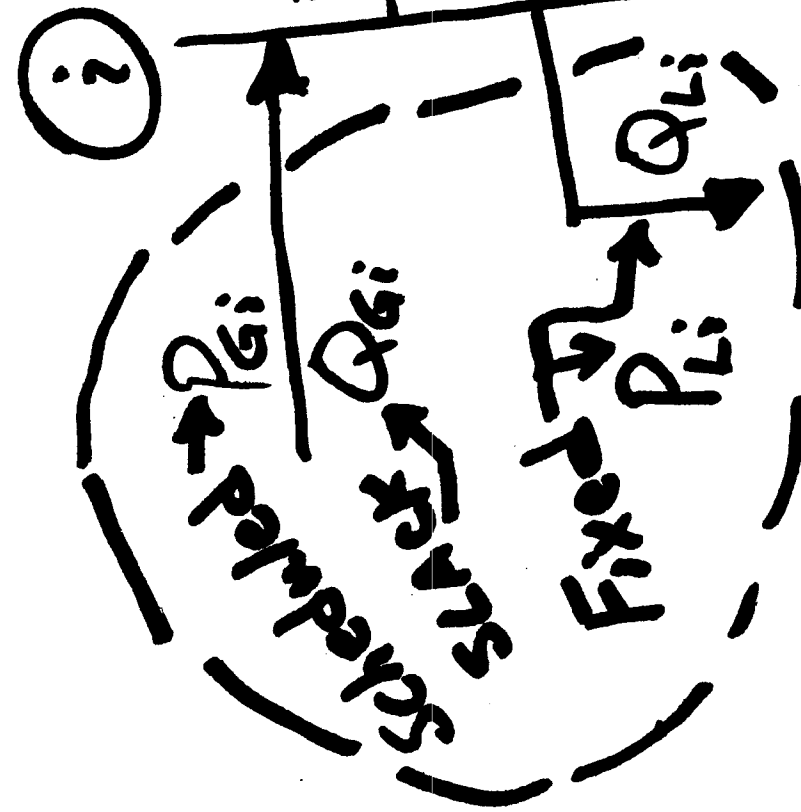


SHUNT CAP BANK
OR REACTOR
(IF THERE)
(Add into y_{ii})

$[Y_{Bus}]$

j

At each bus,



$P_i = -P_{Gi} + P_{Li} + P_{Ti}$	$Q_i = -Q_{Gi} + Q_{Li} + Q_{Ti}$
-----------------------------------	-----------------------------------

$\left. \begin{matrix} Gen\ Buses \\ Load\ Buses \end{matrix} \right\}$

$\left. \begin{matrix} - \\ - \end{matrix} \right\}$ Only at Load Buses

$\sum P_{out} = 0 \Rightarrow$

$\sum Q_{out} = 0 \Rightarrow$

4

No. Eqs
 $2 \times N_{LOAD} + N_{GEN}$

$$\begin{bmatrix} P_i \\ \vdots \\ Q_i \end{bmatrix} = \begin{bmatrix} \frac{\partial P}{\partial \delta_i} \\ \vdots \\ \frac{\partial P}{\partial \delta_i} + \frac{\partial Q}{\partial \delta_i} \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \vdots \\ \Delta |V| \end{bmatrix}$$

From $[Y_{bus}]$ $[I_{inj}] = [Y_{bus}] [V]$

$$\begin{bmatrix} \tilde{I}_1 \\ \vdots \\ \tilde{I}_i \\ \vdots \\ \tilde{I}_N \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} & \dots & y_{1N} \\ \vdots & \vdots & \ddots & \vdots \\ y_{n1} & y_{n2} & \dots & y_{nN} \\ \vdots & \vdots & \ddots & \vdots \\ y_{N1} & y_{N2} & \dots & y_{NN} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \end{bmatrix}$$

$$\tilde{I}_i = \sum_{n=1}^N y_{in} \tilde{V}_n$$

$$\begin{aligned} \tilde{S}_{Ti} &= \tilde{V}_i \tilde{I}_i^* \\ &= \tilde{V}_i \left(\sum_{n=1}^N y_{in} \tilde{V}_n \right)^* \end{aligned}$$

$$P_{Ti} =$$

$$\sum_{n=1}^N |\tilde{V}_i| |\tilde{V}_n| y_{in} \cos(\delta_n - \delta_i - \gamma_{in})$$

$$Q_{Ti} = \sum_{n=1}^N |\tilde{V}_i| |\tilde{V}_n| y_{in} \sin(\delta_i - \delta_n - \gamma_{in})$$

Newton Iteration

Nonlinear Systems of Equations

- If closed form then we can use symbolic solvers (Mathematica)
- If not in closed form or if there are many variables, then an iterative method is better. (Numerical).

Newton Iteration:

Based on Taylor Expansion

If $f(x) = 0$, then if a slightly different value of the dependent variable, x^0 , is evaluated, then $f(x)$ can be approximated as

$$f(x) = f(x_0) + \frac{df(x_0)}{dx} (x-x_0) + \frac{1}{2} \frac{d^2 f(x_0)}{dx^2} (x-x_0)^2 + \dots$$

Depending on severity of the nonlinearity, the series can be truncated at some point and thus yield an approximation of $f(x)$.

Newton Method - Truncate after first term.

$$f(x) \cong f(x_0) + \frac{df(x_0)}{dx} (x-x_0)$$

Truncation
Error = ?

$$f(x) - f(x_0) - \frac{df(x_0)}{dx} (x - x_0)$$

Ex:

$$I_f(E_f) = E_f + A e^{B E_f}$$

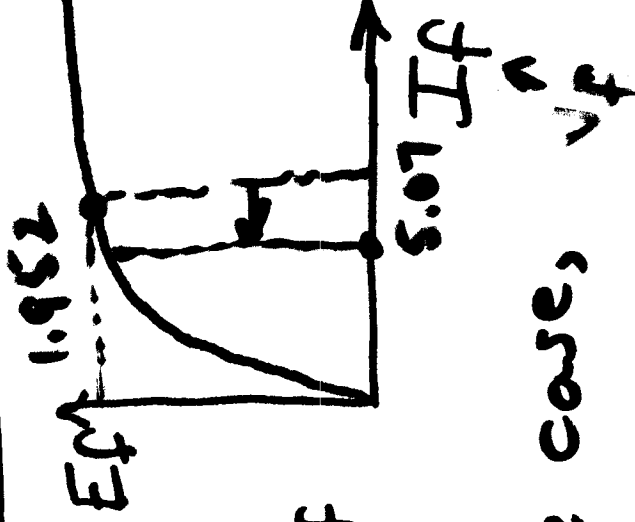
Find E_f given $I_f = 4.3 E_f$

$$I_f(E_f) = E_f + .0008 e^{4.3 E_f}$$

I_f has dropped 20% from base case, ∇f
find E_f . Base case is defined as

$$E_f = 1.952 \text{ p.u.} \Rightarrow I_f = 5.07 \text{ p.u.}$$

$$\therefore \text{find } E_f \text{ for } I_f = 5.07(.8) = 4.06 \text{ p.u.}$$



If we isolate first deriv. on right side,

$$f(x) = 0 \Rightarrow f(x) - f(x_0) \approx f'(x_0) \Delta x$$

$$-f(x) + f(x_0) \approx f'(x_0) \Delta x$$

$$\approx f'(x_0)(x - x_0)$$

$$\Delta x \approx -\frac{f(x_0)}{f'(x_0)} \quad \text{stop when } \Delta x \leq \epsilon$$

ITERATE

$$x_{i+1} = -\frac{f(x_i)}{f'(x_i)} + x_i$$

$$E_f^1 = - \frac{E_f^0 + 0.008 e^{4.3 E_f^0} - I_f}{1 + 4.3 \times 0.008 e^{4.3 E_f^0}} + E_f^0$$

1st Iteration:

$$E_f^1 = \frac{1.5 + 0.008 e^{4.3 \cdot 1.5} - 4.06}{1 + 4.3 \times 0.008 e^{4.3 \cdot 1.5}} + 1.5$$

Guess $E_f = 1.5$

$$= 2.146$$

2nd Iter.

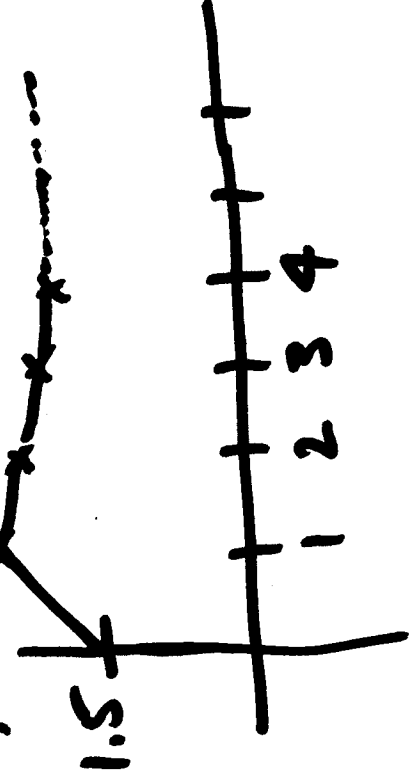
$$\text{Repeat, } \Rightarrow E_f^2 = 1.973$$

$$E_f^3 = 2.146$$

$$E_f^3 = 1.872$$

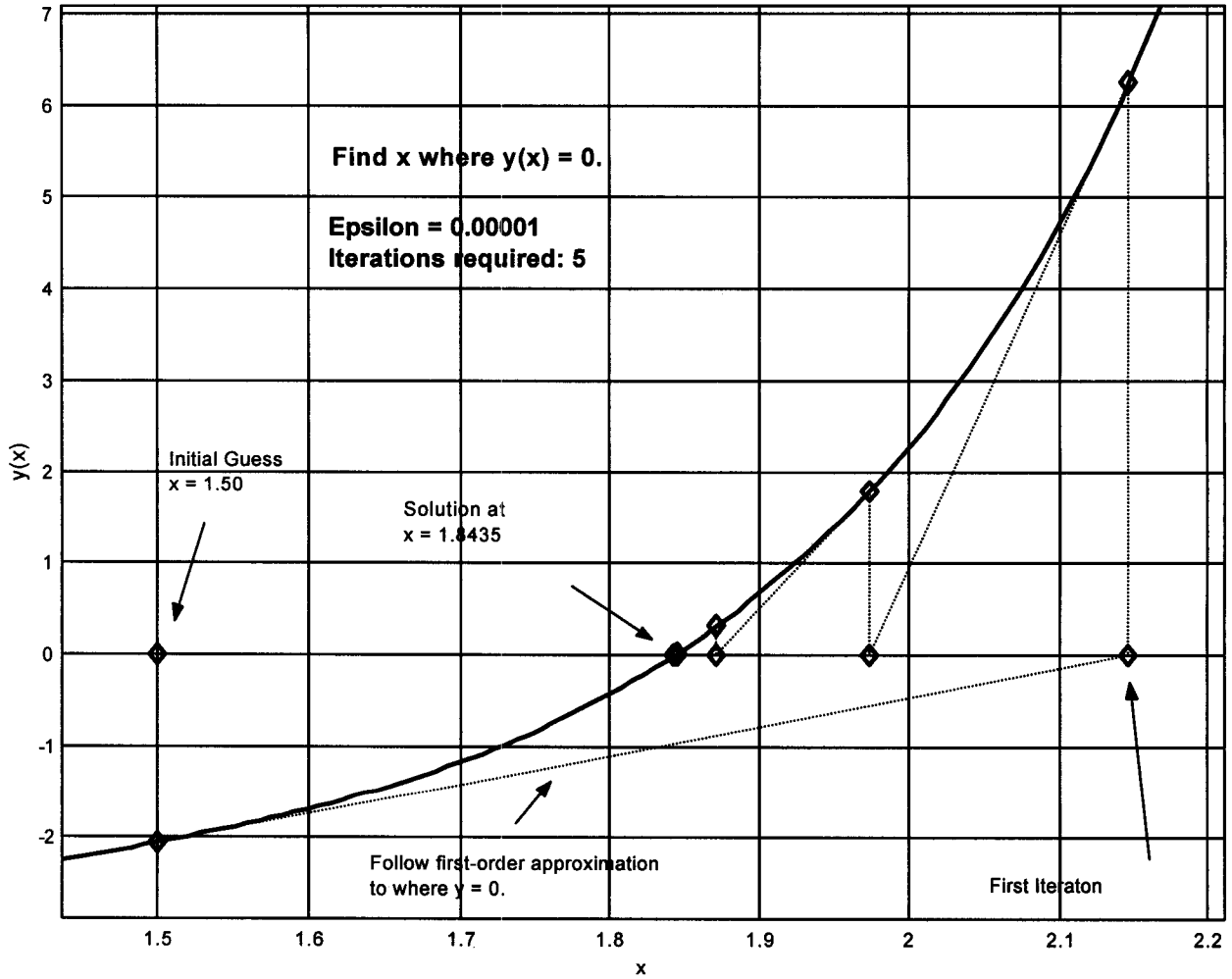
$$E_f^4 = 1.845$$

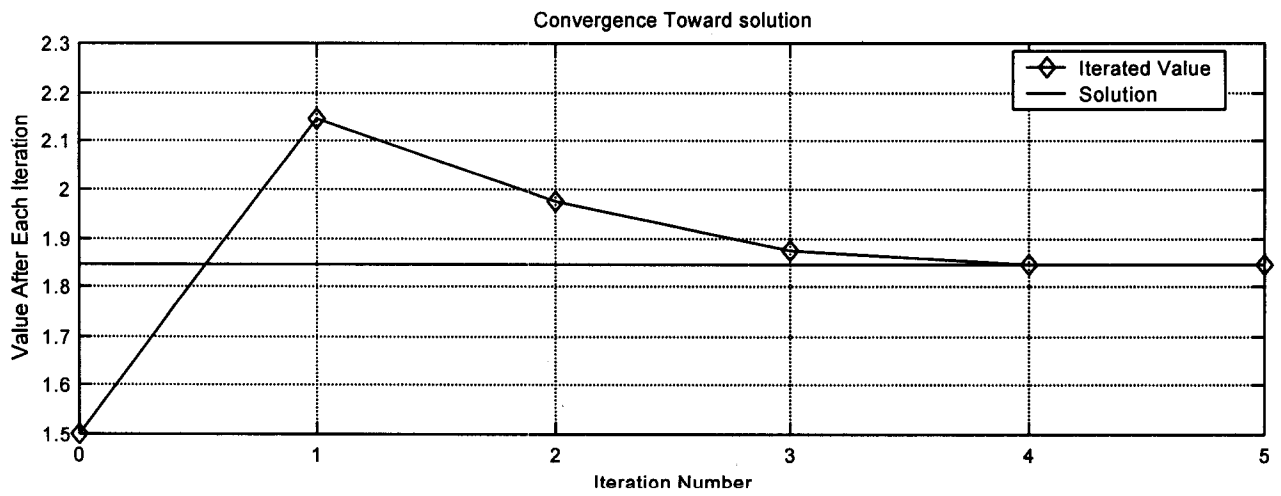
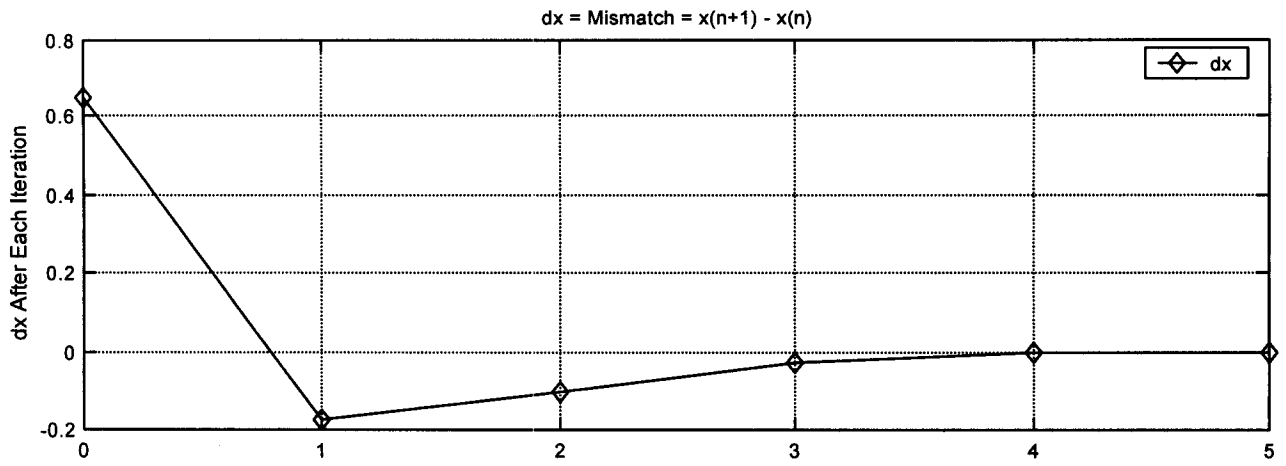
$$E_f^5 = 1.843$$



Newton Iteration - Path Toward Convergence

— $y(x) = x + 0.0008 \cdot \exp(4.5x) - 4.06$
◆ Iteration Path

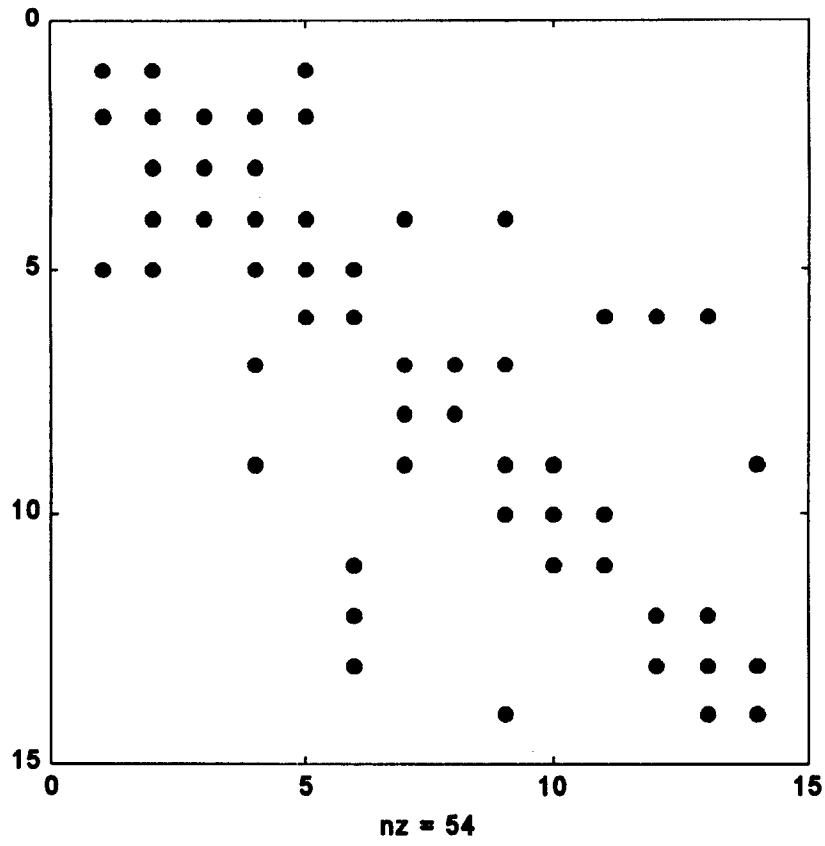




(1,1) 6.0250 -19.4471i
(2,1) -4.9991 +15.2631i
(5,1) -1.0259 + 4.2350i
(1,2) -4.9991 +15.2631i
(2,2) 9.5213 -30.2721i
(3,2) -1.1350 + 4.7819i
(4,2) -1.6860 + 5.1158i
(5,2) -1.7011 + 5.1939i
(2,3) -1.1350 + 4.7819i
(3,3) 3.1210 - 9.8224i
(4,3) -1.9860 + 5.0688i
(2,4) -1.6860 + 5.1158i
(3,4) -1.9860 + 5.0688i
(4,4) 10.5130 -38.6542i
(5,4) -6.8410 +21.5786i
(7,4) 0 + 4.8895i
(9,4) 0 + 1.8555i
(1,5) -1.0259 + 4.2350i
(2,5) -1.7011 + 5.1939i
(4,5) -6.8410 +21.5786i
(5,5) 9.5680 -35.5336i
(6,5) 0 + 4.2574i
(5,6) 0 + 4.2574i
(6,6) 6.5799 -17.3407i
(11,6) -1.9550 + 4.0941i
(12,6) -1.5260 + 3.1760i
(13,6) -3.0989 + 6.1028i
(4,7) 0 + 4.8895i
(7,7) 0 -19.5490i
(8,7) 0 + 5.6770i
(9,7) 0 + 9.0901i
(7,8) 0 + 5.6770i
(8,8) 0 - 5.6770i
(4,9) 0 + 1.8555i
(7,9) 0 + 9.0901i
(9,9) 5.3261 -24.0925i
(10,9) -3.9020 +10.3654i
(14,9) -1.4240 + 3.0291i
(9,10) -3.9020 +10.3654i
(10,10) 5.7829 -14.7683i
(11,10) -1.8809 + 4.4029i
(6,11) -1.9550 + 4.0941i
(10,11) -1.8809 + 4.4029i
(11,11) 3.8359 - 8.4970i
(6,12) -1.5260 + 3.1760i
(12,12) 4.0150 - 5.4279i
(13,12) -2.4890 + 2.2520i
(6,13) -3.0989 + 6.1028i
(12,13) -2.4890 + 2.2520i
(13,13) 6.7249 -10.6697i
(14,13) -1.1370 + 2.3150i
(9,14) -1.4240 + 3.0291i
(13,14) -1.1370 + 2.3150i
(14,14) 2.5610 - 5.3440i

Network topology, spy(Ybus):

2



Sparsity = $54/14 \times 14 = 27.55\%$

$$\{A\} \{x\} = \{B\}$$

2-Variable NR

$$\textcircled{1} \quad 2x + y = 4$$

$$\textcircled{2} \quad 2x + y^2 = 6$$

$$\textcircled{2} - \textcircled{1} \quad y^2 - y = 2 \Rightarrow$$

$$\begin{matrix} x = 1 \\ y = 2 \end{matrix}$$

define $f(x, y) = 2x + y - 4 = 0$

$$g(x, y) = 2x + y^2 - 6 = 0$$

Taylor Expansion iteration index

$$0 = f(x, y) = f(x^m, y^m) + \frac{\partial f(x^m, y^m)}{\partial x} (x - x^m) + \frac{\partial f(x^m, y^m)}{\partial y} (y - y^m)$$

$$0 = g(x, y) = g(x^m, y^m) + \frac{\partial g(x^m, y^m)}{\partial x} (x - x^m) + \frac{\partial g(x^m, y^m)}{\partial y} (y - y^m)$$

Rearranging terms:

$$\underbrace{f(x, y) - f(x^m, y^m)}_{=0} \rightarrow f^m$$

$$\frac{\partial f^m}{\partial x} \Delta x + \frac{\partial f^m}{\partial y} \Delta y = -f^m$$

$$\frac{\partial g^m}{\partial x} \Delta x + \frac{\partial g^m}{\partial y} \Delta y = -g^m$$

$$\Delta x = \frac{x - x^m}{}$$

In matrix form

$$\begin{bmatrix} \frac{\partial f^m}{\partial x} & \frac{\partial f^m}{\partial y} \\ \frac{\partial g^m}{\partial x} & \frac{\partial g^m}{\partial y} \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} = - \begin{bmatrix} f^m \\ g^m \end{bmatrix}$$

$$\Delta y = y - y^m$$

JACOBIAN

$$\begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} = - \begin{bmatrix} J^m \end{bmatrix}^{-1} \begin{bmatrix} f^m \\ g^m \end{bmatrix} \approx \begin{bmatrix} x - x^m \\ y - y^m \end{bmatrix}$$

Estimate of (x, y) for next iteration:

$$\begin{aligned} \underline{\Delta x} \approx x - x^m &\Rightarrow x^{m+1} = x^m + \Delta x \\ \underline{\Delta y} \approx y - y^m &\Rightarrow y^{m+1} = y^m + \Delta y \end{aligned}$$

use (x^{m+1}, y^{m+1}) for next iteration,

Continue until "converged".

Common tests: ① $\Delta x \approx \Delta y$ both $< \epsilon$

② $\| \Delta x, \Delta y \| < \epsilon$

Norm: $\sqrt{\Delta x_1^2 + \Delta x_2^2 + \dots + \Delta x_n^2}$ (for n -variable system)

for this case,

$$\text{Norm is } \|\Delta x, \Delta y\| = \sqrt{\Delta x^2 + \Delta y^2}$$

Back to example:

$$\begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} = - \begin{bmatrix} J^m \end{bmatrix}^{-1} \begin{bmatrix} f^m \\ g^m \end{bmatrix}$$

Start: $m=0$ (0th iteration is initial guess)

$$[J] = \begin{bmatrix} 2 & 1 \\ 2 & 2y \end{bmatrix}$$

$$\text{guess: } \begin{bmatrix} 0 \\ 3 \end{bmatrix} = \begin{bmatrix} x^0 \\ y^0 \end{bmatrix}$$

$$[J^0] = \begin{bmatrix} 2 & 1 \\ 2 & 6 \end{bmatrix} \Rightarrow [J^0]^{-1} = \begin{bmatrix} -1.6 & .1 \\ .2 & -.2 \end{bmatrix}$$

$$\begin{bmatrix} f^0 \\ g^0 \end{bmatrix} = \begin{bmatrix} f(0) \\ g(3) \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} = \begin{bmatrix} -1.6 & .1 \\ .2 & -.2 \end{bmatrix} \begin{bmatrix} -1 \\ 3 \end{bmatrix} = \begin{bmatrix} .9 \\ -.8 \end{bmatrix} = \begin{bmatrix} x-x^0 \\ y-y^0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x^0 \\ y^0 \end{bmatrix} + \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} = \begin{bmatrix} x^1 \\ y^1 \end{bmatrix} = \begin{bmatrix} +0.9 \\ +2.2 \end{bmatrix}$$

- Next, repeat iteration using $\begin{bmatrix} x^1 \\ y^1 \end{bmatrix}$
- Test $\| \Delta x, \Delta y \| \leq \epsilon$ at each iteration.

After 3 iterations, $\epsilon < 0.001$

$$x = 1.000$$

$$y = 2.000$$

↑ typical for
p.u. load flow.

Notes: - Convergence rate not dependent
on no. of variables.
- Important to make intelligent guess
of initial values.

e.g. $\left\{ \begin{array}{l} |\bar{V}_{bus}| = 1.0 \text{ p.u.} \\ \delta = 0^\circ \end{array} \right. \quad (\delta = \angle V_{bus})$

Flat Start ↗

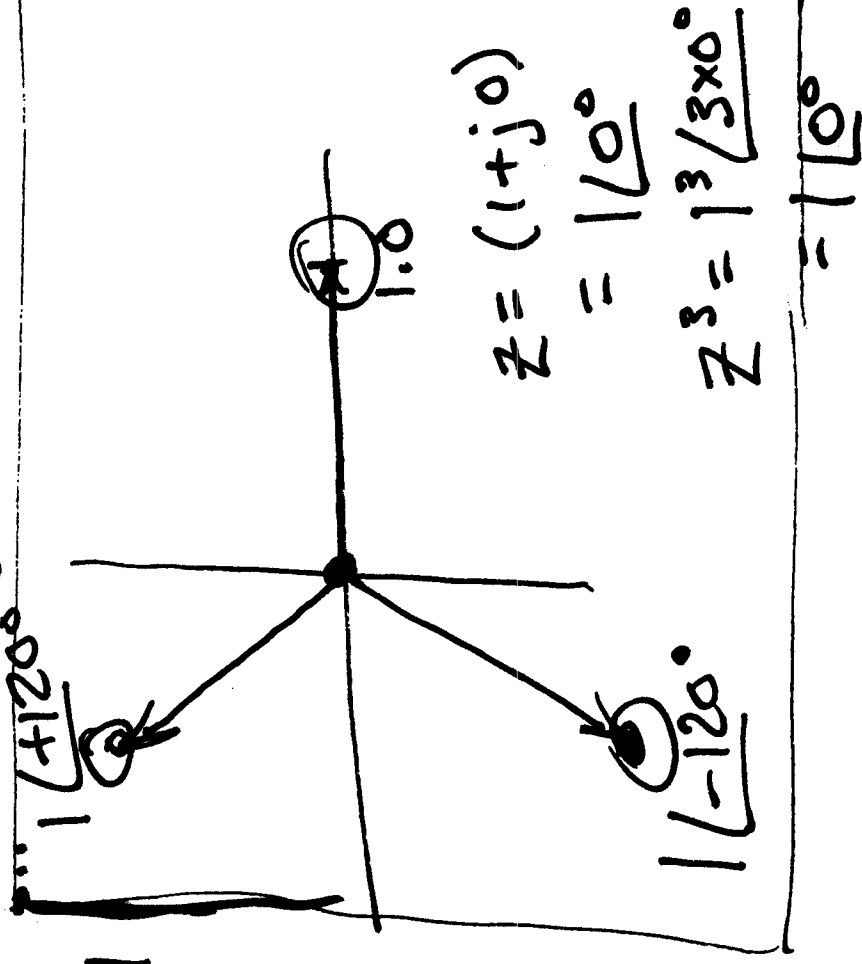
- NR can fail to converge. Important
to have correct system data.

- Possible to converge to wrong solution.

Ex:

$$z^3 = 1$$

Which of the 3 roots will solution converge to for an arbitrary initial value? Your guess is as good as mine. Must let NR iteration converge to find out.



Note: We can generate some beautiful fractal patterns in this way. Let

each initial condition on the rectangular area be assigned a color depending on which root it converges to. (Each pixel = initial condition).

$$\underline{S}_{Ti} = P_{Ti} + j Q_{Ti}$$

$$P = |\underline{V}_i| \sum_{n=1}^N |\underline{V}_n| |y_{in}| \cos(\delta_i - \delta_n - \theta_{in})$$

$$Q = |\underline{V}_i| \sum_{n=1}^N |\underline{V}_n| |y_{in}| \sin(\delta_i - \delta_n - \theta_{in})$$

Assume:

$$\underline{V}_i = |\underline{V}_i| \angle \delta_i$$

$$\underline{V}_n = |\underline{V}_n| \angle \delta_n$$

$$y_{in} = |y_{in}| \angle \theta_{in}$$

For referenced current, P, Q , flow directions,

See Lectur 8 p.4.

Newton-Raphson Power Flow

- Active and Reactive Power Flowing INTO bus k:

$$I_k = \sum_{n=1}^N Y_{kn} V_n$$

$$S = V I^*$$

$$\begin{cases}
 P_k = |V_k| \left| \sum_{n=1}^N |Y_{kn}| |V_n| \cos(\delta_k - \delta_n - \theta_{kn}) \right| \\
 Q_k = |V_k| \left| \sum_{n=1}^N |Y_{kn}| |V_n| \sin(\delta_k - \delta_n - \theta_{kn}) \right|
 \end{cases}
 \quad k=1,2,\dots,N$$

$$\begin{aligned}
 V_k &= |V_k| e^{j\delta_k} \\
 Y_{kn} &= |Y_{kn}| e^{j\theta_{kn}}
 \end{aligned}$$

- Let:

$$\begin{aligned}
 x &= \begin{bmatrix} \delta \\ V \end{bmatrix} = \begin{bmatrix} \delta_2 \\ \vdots \\ \delta_N \\ V_2 \\ \vdots \\ V_N \end{bmatrix} &
 y &= \begin{bmatrix} P \\ Q \end{bmatrix} = \begin{bmatrix} P_2 \\ \vdots \\ P_N \\ Q_2 \\ \vdots \\ Q_N \end{bmatrix} &
 f(x) &= \begin{bmatrix} P(x) \\ Q(x) \end{bmatrix} = \begin{bmatrix} P_2(x) \\ \vdots \\ P_N(x) \\ Q_2(x) \\ \vdots \\ Q_N(x) \end{bmatrix}
 \end{aligned}$$

where V, P, and Q terms are in per unit and δ terms are in radians.

$$J = \begin{array}{c} \text{I} \qquad \qquad \qquad \text{II} \\ \begin{bmatrix} \frac{\partial P_2}{\partial \delta_2} & \cdot & \cdot & \frac{\partial P_2}{\partial \delta_N} & \frac{\partial P_2}{\partial V_2} & \cdot & \cdot & \frac{\partial P_2}{\partial V_N} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \frac{\partial P_N}{\partial \delta_2} & \cdot & \cdot & \frac{\partial P_N}{\partial \delta_N} & \frac{\partial P_N}{\partial V_2} & \cdot & \cdot & \frac{\partial P_N}{\partial V_N} \\ \hline \frac{\partial Q_2}{\partial \delta_2} & \cdot & \cdot & \frac{\partial Q_2}{\partial \delta_N} & \frac{\partial Q_2}{\partial V_2} & \cdot & \cdot & \frac{\partial Q_2}{\partial V_N} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \frac{\partial Q_N}{\partial \delta_2} & \cdot & \cdot & \frac{\partial Q_N}{\partial \delta_N} & \frac{\partial Q_N}{\partial V_2} & \cdot & \cdot & \frac{\partial Q_N}{\partial V_N} \\ \frac{\partial Q_N}{\partial \delta_2} & \cdot & \cdot & \frac{\partial Q_N}{\partial \delta_N} & \frac{\partial Q_N}{\partial V_2} & \cdot & \cdot & \frac{\partial Q_N}{\partial V_N} \end{bmatrix} \\ \text{B} \qquad \qquad \qquad \text{III} \end{array}$$

- Jacobian Entries:



For n ≠ k:

$$\begin{aligned}
 J_{1_{kn}} &= \frac{\partial P_k}{\partial \delta_n} = |V_k| |Y_{kn}| |V_n| \sin(\delta_k - \delta_n - \theta_{kn}) \\
 \left\{ \begin{aligned}
 J_{2_{kn}} &= \frac{\partial P_k}{\partial V_n} = |V_k| |Y_{kn}| \cos(\delta_k - \delta_n - \theta_{kn}) \\
 J_{3_{kn}} &= \frac{\partial Q_k}{\partial \delta_n} = -|V_k| |Y_{kn}| |V_n| \cos(\delta_k - \delta_n - \theta_{kn}) \\
 J_{4_{kn}} &= \frac{\partial Q_k}{\partial V_n} = |V_k| |Y_{kn}| \sin(\delta_k - \delta_n - \theta_{kn})
 \end{aligned} \right.
 \end{aligned}$$

For n = k:

$$\begin{aligned}
 J_{1_{kk}} &= \frac{\partial P_k}{\partial \delta_k} = -|V_k| \left[\sum_{\substack{n=1 \\ n \neq k}}^N |Y_{kn}| |V_n| \sin(\delta_k - \delta_n - \theta_{kn}) \right] \\
 \left\{ \begin{aligned}
 J_{2_{kk}} &= \frac{\partial P_k}{\partial V_k} = |V_k| |Y_{kk}| \cos \theta_{kk} + \sum_{n=1}^N |Y_{kn}| |V_n| \cos(\delta_k - \delta_n - \theta_{kn}) \\
 J_{3_{kk}} &= \frac{\partial Q_k}{\partial \delta_k} = |V_k| \left[\sum_{\substack{n=1 \\ n \neq k}}^N |Y_{kn}| |V_n| \sin(\delta_k - \delta_n - \theta_{kn}) \right] \\
 J_{4_{kk}} &= \frac{\partial Q_k}{\partial V_k} = -|V_k| |Y_{kk}| \sin \theta_{kk} + \sum_{n=1}^N |Y_{kn}| |V_n| \sin(\delta_k - \delta_n - \theta_{kn})
 \end{aligned} \right.
 \end{aligned}$$

*

PV Buses:

Omit V_k term from x vector.

Omit the Q_k term from y vector.

In [J] omit columns corresp to $\frac{\partial}{\partial V_k}$

and omit row corresponding to partials of row corresponding V_k

- Use Newton Raphson to solve:

1) Use P & Q equations to calculate $\Delta y(i)$:

$$\Delta y(i) = \begin{bmatrix} \Delta P(i) \\ \Delta Q(i) \end{bmatrix} = \begin{bmatrix} P - P[x(i)] \\ Q - Q[x(i)] \end{bmatrix}$$

where:

$$x(i) = \begin{bmatrix} \delta(i) \\ V(i) \end{bmatrix}$$

If Convergence criteria is met, quit.

2) Calculate the Jacobian.

3) Use LU Factorization to solve:

$$\begin{bmatrix} J1(i) & J2(i) \\ J3(i) & J4(i) \end{bmatrix} \begin{bmatrix} \Delta \delta(i) \\ \Delta V(i) \end{bmatrix} = \begin{bmatrix} \Delta P(i) \\ \Delta Q(i) \end{bmatrix}$$

4) Compute:

$$x(i+1) = \begin{bmatrix} \delta(i+1) \\ V(i+1) \end{bmatrix} = \begin{bmatrix} \delta(i) \\ V(i) \end{bmatrix} + \begin{bmatrix} \Delta \delta(i) \\ \Delta V(i) \end{bmatrix}$$

5) Goto 1.

Previous
New/Current

iteration index = i

Calc Q at each voltage controlled bus.

- Compare to min/max limits
- If limit exceeded, set Q to limit & change to PQ bus.
- ~~Check Q against limit~~
- Check Q on later iterations to see if PQ can be changed back to PV bus.