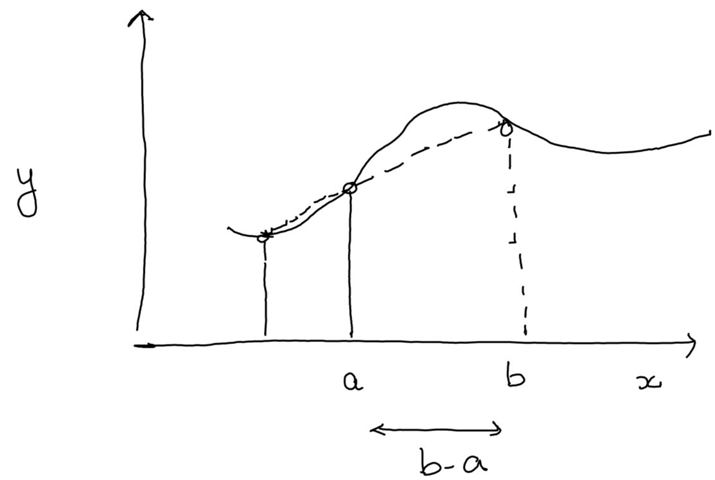
Lecture on 2/15

Numerical Integration Techniques

Trapezoidal Method



$$\int_a^b f(x) \approx \left[\frac{f(x)\big|_a + f(x)\big|_b}{2}\right](b-a)$$

Euler's Method

$$\frac{dx}{dt} = f(x,t)$$

$$\Rightarrow \dot{x} = f(x,t)$$

Taylor's Series

$$\boxed{x_1 = x_0 + \Delta t\,(\dot{x}_0)} + \frac{\Delta t^2}{2!}(\ddot{x}_0) + \frac{\Delta t^3}{3!}(\dddot{x}_0) + \cdots$$

$\Downarrow$

Euler's Method

Modified Euler's Method

predictor step

$$x_1^p = x_0 + \dot{x}_0 \Delta t$$

Corrector step

$$x_1^c = x_0 + \frac{1}{2}\left(\dot{x}_0 + \dot{x}_p\right)\Delta t$$

Runge- Kutta Method (RK Method)

- is equivalent of Taylor's series expansion upto the second order
- It does not require evaluating the second order differential equation.

RK-2 (Second order)

$$x_1 = x_0 + \Delta x = x_0 + \frac{K_1 + K_2}{2}$$

$$K_1 = f(x_0, t_0)\,\Delta t$$

$$K_2 = f(x_0 + K_1,\ t_0 + \Delta t)\,\Delta t$$

RK-4

$$x_{n+1} = x_n + \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4)$$

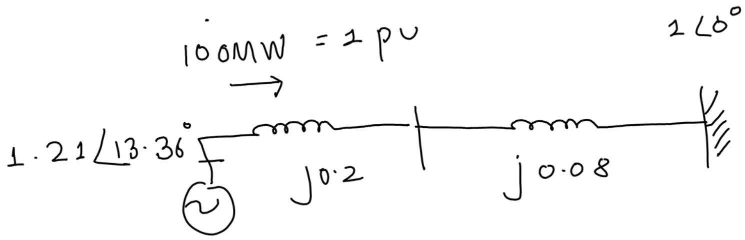
$$K_1 = f(x_n, t_n)\,\Delta t$$

$$K_2 = f\left(x_n + \frac{K_1}{2},\ t_n + \frac{\Delta t}{2}\right)\Delta t$$

$$K_3 = f\left(x_n + \frac{K_2}{2},\ t_n + \frac{\Delta t}{2}\right)\Delta t$$

$$K_4 = f(x_n + K_3,\ t_n + \Delta t)\,\Delta t$$

Example



$$\frac{\pi f_0}{H} = 47.123 = \frac{1}{M}$$

$$H \Downarrow 4$$

$$P = \frac{1.21 \times 1}{0.28} \sin\delta$$

$$= 4.321 \sin\delta$$

Swing equation

$$\frac{d^2\delta}{dt} = \frac{P_a}{M} = \frac{P_m - P_{max}\sin\delta}{M}$$

$$\frac{d\delta}{dt} = \omega - \omega_{sy}$$

$$\frac{d\omega}{dt} = \frac{P_m - P_{max}\sin\delta}{M}$$

$\delta_0 = 13.365^\circ$ $\qquad \omega_0 = 2\pi f_0 = 376.99 \text{ rad s}^{-1}$

$\Delta t = 0.02$ seconds

predictor

$$\delta^{p}_{0.02} = \delta_0 + \left.\frac{\partial \delta}{\partial t}\right|_0 \Delta t$$

$$= 13.365^\circ + 0$$

$$= 13.365^\circ$$

$$\omega^{p}_{0.02} = 376.99 + 0.02 \times \left.\frac{P_m - P_{max}\sin\delta}{M}\right|_{t=0}$$

$$= 376.99 + 0.02 \times (1 - 4.321 \sin 13.365^\circ) \times 47.123$$

$$= 377.932 \text{ rad/second}$$

$$\left.\frac{d\delta}{dt}\right|_0 = 0$$

$$\left.\frac{d\omega}{dt}\right|_0 = 47.123$$

<u>Corrector</u>

$$\left.\frac{d\delta}{dt}\right|_{0.02} = \left.\omega - 2\pi f_0\right|_{0.02} = 0.95$$

$$\left.\frac{d\omega}{dt}\right|_{0.02} = \left.\frac{1}{M}(P_m - P_e)\right|_{0.02} = 47.123 \times (1 - 4.321 \sin 13.365)$$

$$= -0.0558$$

$$\delta^{c}_{0.02} = \delta_0 + \frac{\Delta t}{2}\left[\left.\frac{d\delta}{dt}\right|_0 + \left.\frac{d\delta}{dt}\right|_{0.02}\right]$$

$$= 13.365^{\circ} + \frac{0.02}{2}\left[0 + 0.95\right] \times \frac{180^{\circ}}{\pi} = 13.815^{\circ}$$

$$\omega^{c}_{0.02} = \omega(0) + \frac{\Delta t}{2}\left[\left.\frac{d\omega}{dt}\right|_0 + \left.\frac{d\omega}{dt}\right|_{0.02}\right]$$

$$= 376.99 + \frac{0.02}{2}\left[47.123 - 0.0538\right]$$

$$\vdots$$

$$= 377.46 \text{ rad/second}$$

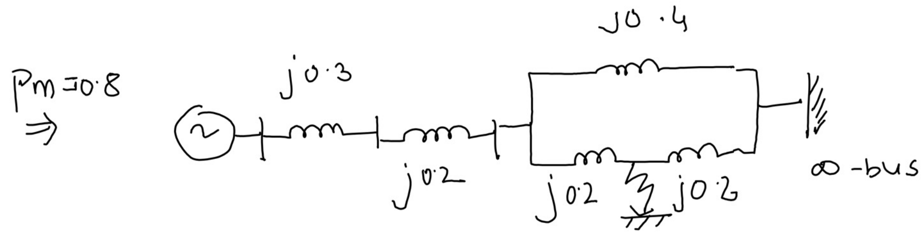
Multi-machine Swing Equation

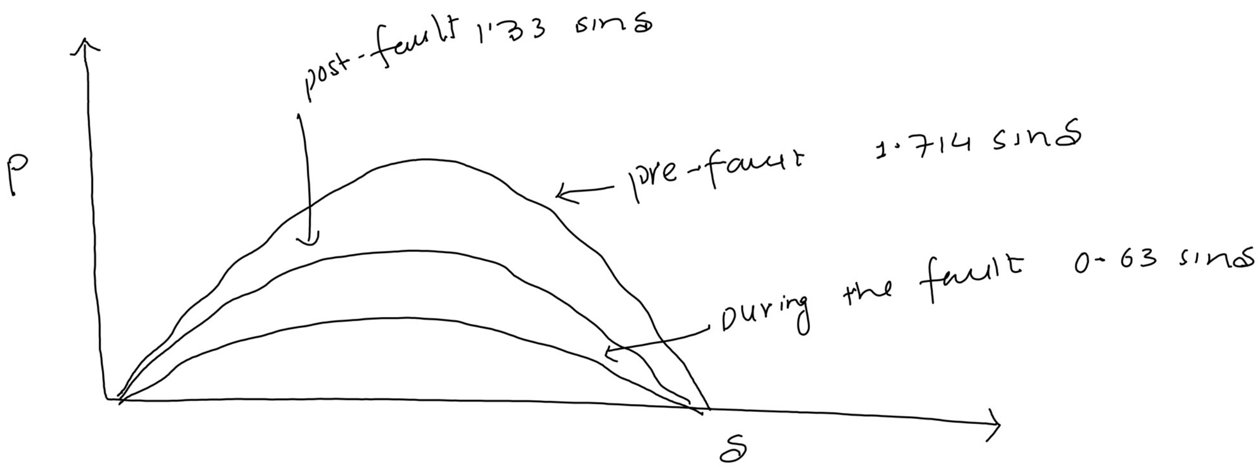
$$\dot{\omega}_i = \frac{1}{M_i}\left(P_{mi} - E_i^2 G_{ii} - E_i \sum_{j \neq i} E_j (B_{ij} \sin \delta_{ij} + G_{ij} \cos \delta_{ij}\right)$$

$$\dot{\delta}_i = \omega_i - \omega_s \qquad i = 1, \dots, n$$

Lecture on 2/17

Effect of Time step and integration techniques on stability decisions





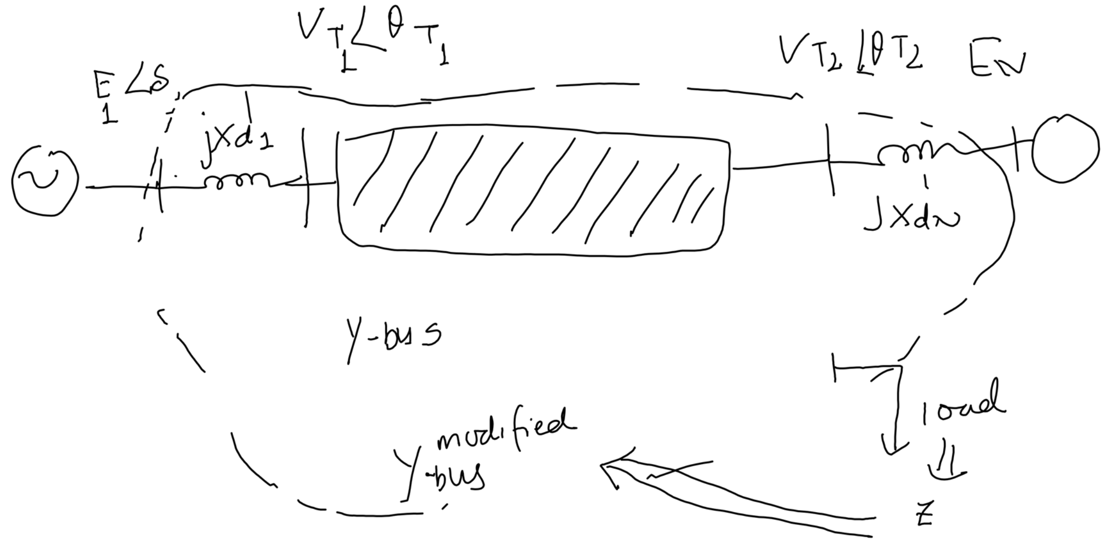
$$\frac{M\, d^2\delta}{dt^2} = P_m - P_e$$

$$\frac{M\, d^2\delta}{dt^2} + \underbrace{D(\delta)\frac{d\delta}{dt}}_{\text{Damping}} = P_m - P_e(\delta)$$

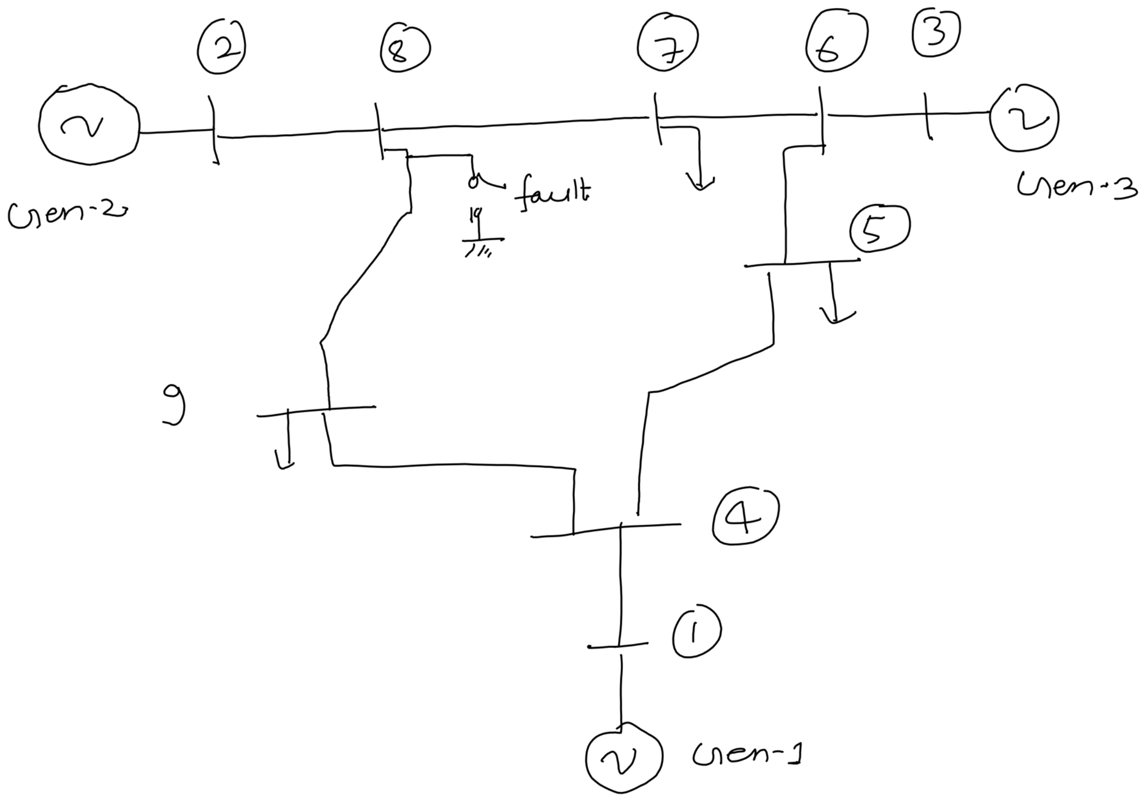
Multi-machine swing Equations

$$\dot{\omega}_i = \frac{1}{M_i}\left(P_{mi} - E_i^2 G_{ii} - E_i \sum_{j \neq i}^{n} E_j \left(B_{ij} \sin\delta_{ij} + G_{ij}\cos\delta_{ij} \right) \right)$$

$$\dot{\delta}_i = \omega_i - \omega_s \qquad i = 1 \ldots n$$



pre-fault, During fault and post fault Y_{bus}^{mod} could be different.



2
8
7
6
3
fault
Gen-2
Gen-3
5
9
4
1
Gen-1