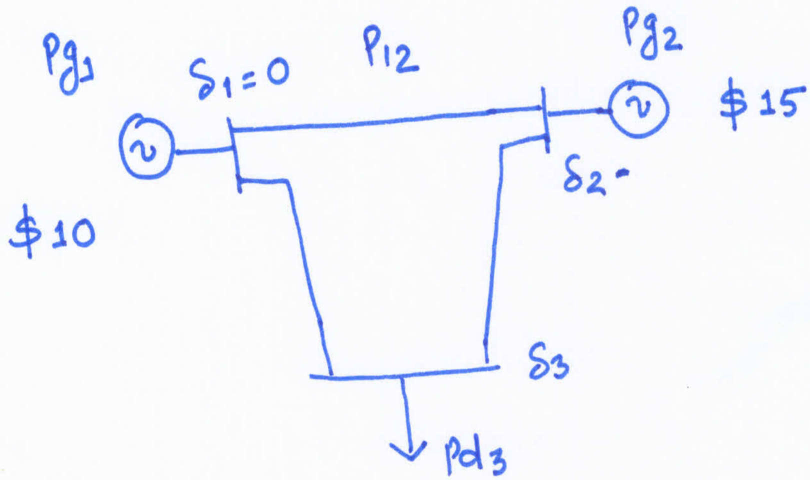
A

Dc. power Flow (LP Example)



Solve.

δ_2
δ_3
Pg_1
Pg_2

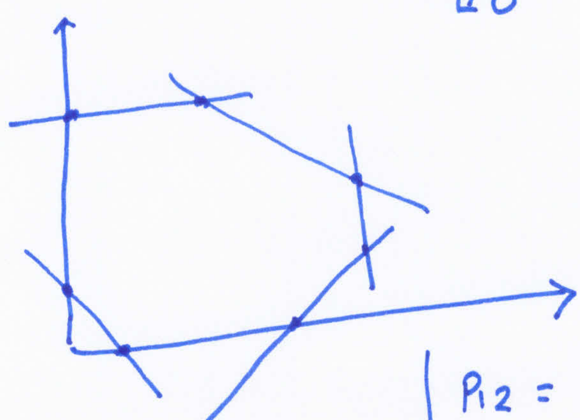
Min $\quad 10\,Pg_1 + 15\,Pg_2$

s.t.

$$Pg_1 - 0 = B_{11}\cancel{(\delta_1-\delta_1)} + B_{12}(\underset{0}{\delta_1}-\delta_2) + B_{13}(\underset{0}{\delta_1}-\delta_3)$$

$$Pg_2 - 0 = B_{21}(\delta_2-\underset{0}{\delta_1}) + B_{22}\cancelto{0}{(\delta_2-\delta_2)} + B_{23}(\delta_2-\delta_3)$$

$$0 - Pd_3 = B_{31}(\delta_3-\underset{0}{\delta_1}) + B_{32}(\delta_3-\delta_2) + B_{33}\cancelto{0}{(\delta_3-\delta_3)}$$



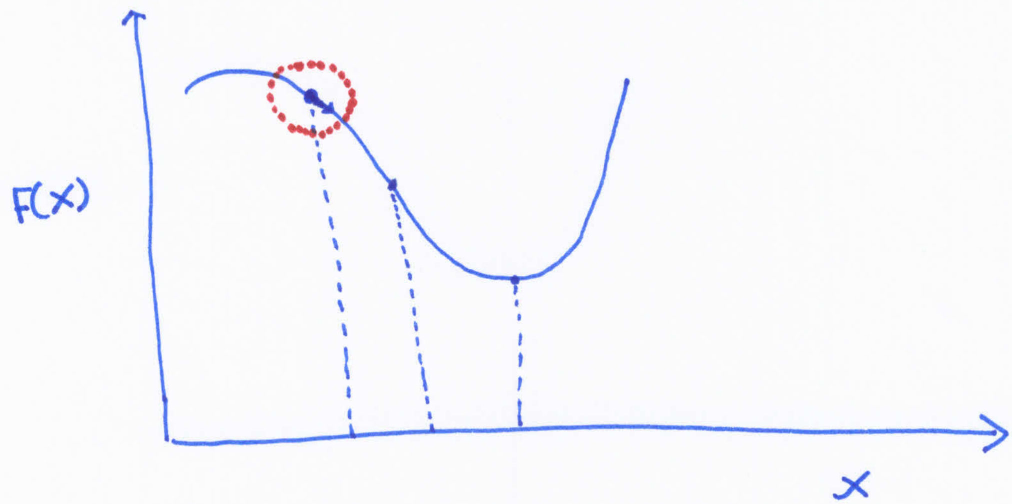
You can use Simplex method

$$\left| P_{12} = B_{12}(\delta_1-\delta_2) \right| \le P_{12}^{max}$$

$\vdots$ other lines.

$\boxed{\begin{array}{ll} \text{Min} & F(x) \\ & g(x)=0 \\ & h(x)\le 0 \end{array}}$

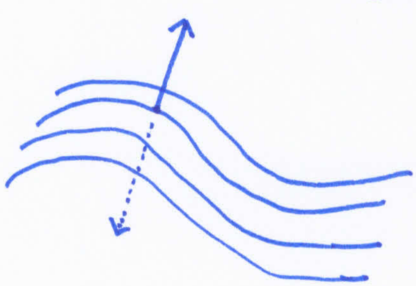
Gradient Based Method.



linear Approximation around operating point

$$\nabla f(\vec{x}) = \left[\frac{\partial f(\vec{x})}{\partial x_1}, \frac{\partial f(\vec{x})}{\partial x_2}, \dots, \frac{\partial f(\vec{x})}{\partial x_n} \right]$$

- Gradient vector always points in the direction of greatest rate of increase of $f(\vec{x})$



Min: $Z = 4x^2 + y^2 - 2xy$

$$G = \begin{bmatrix} G_X \\ G_Y \end{bmatrix} = \begin{bmatrix} 8X - 2y \\ 2y - 2X \end{bmatrix}$$

Iteration #1

$X = 6$
$y = 6$

$Z = 108$

$$G = \begin{bmatrix} 36 \\ 24 \end{bmatrix}$$

$$\begin{bmatrix} X \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ 6 \end{bmatrix} - \underbrace{\alpha}_{0.01} \begin{bmatrix} 36 \\ 24 \end{bmatrix}$$

$$= \begin{bmatrix} 5.64 \\ -5.76 \end{bmatrix}$$

$Z = 95.44$

$$x_{t+1} = x_i - \alpha \nabla f(x_i)$$

$\Downarrow$

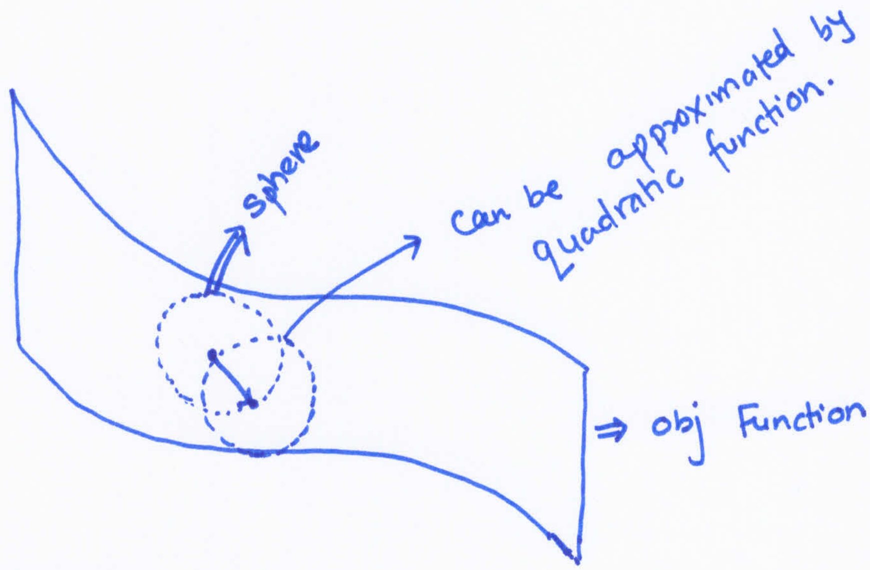
Step size you choose!!

Newton's based.

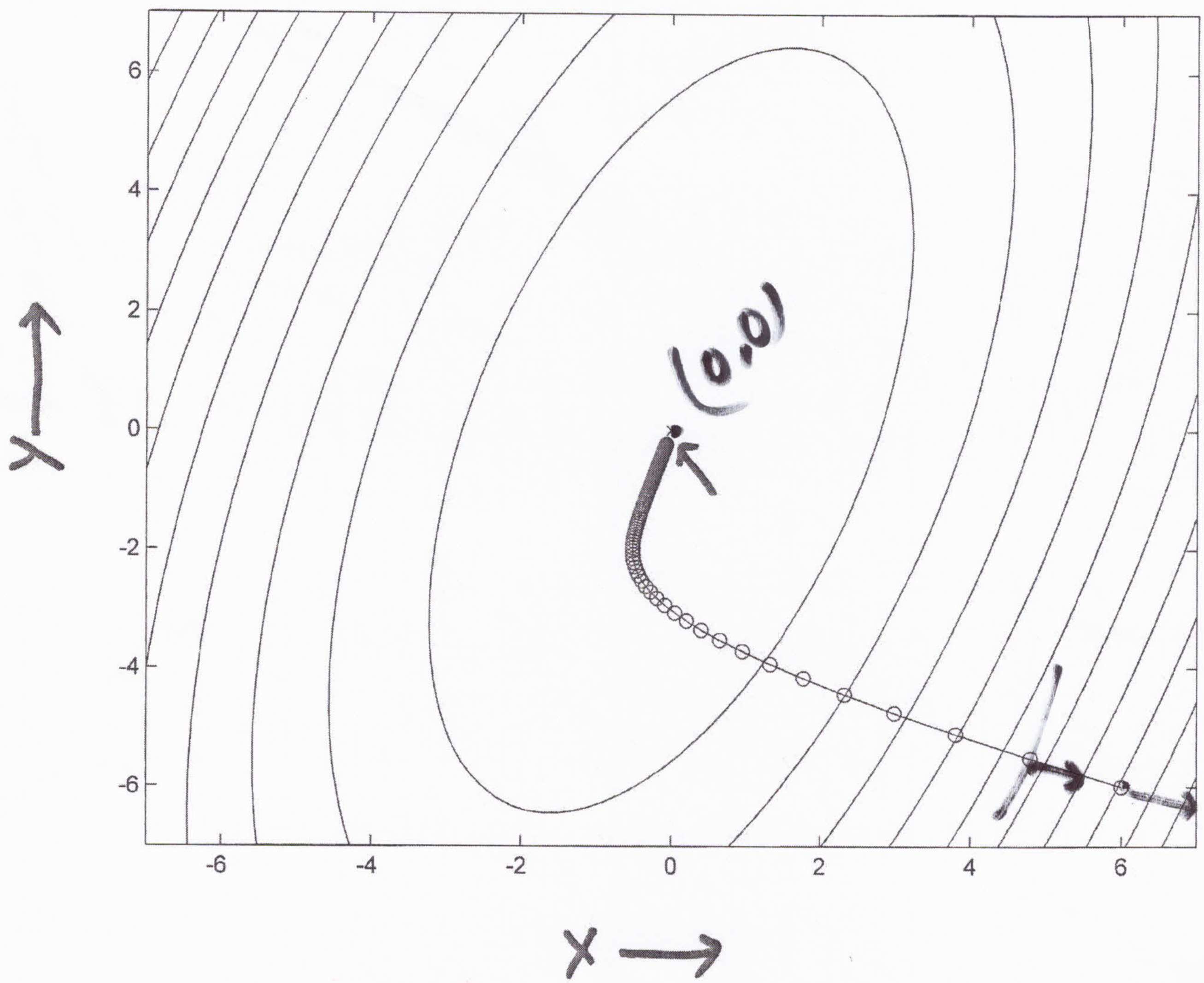
$$x_{i+1} = x_i - \frac{\nabla f(x_i)}{\nabla^2 f(x_i)}$$

$\nabla f(x_i) \rightarrow$ Gradient

$\nabla^2 f(x_i) \rightarrow$ Hessian



See Trust Region



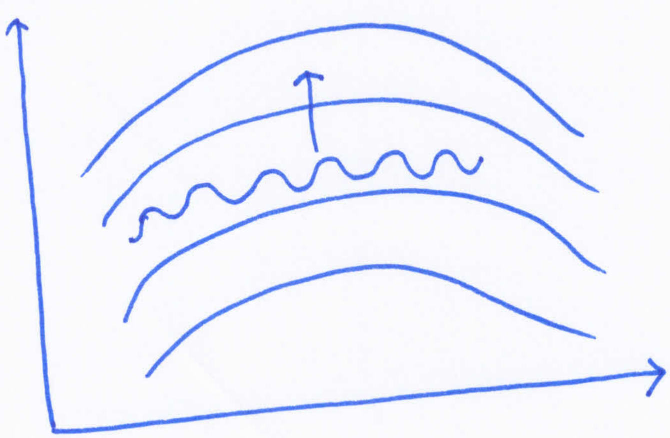
6
4
2
0
-2
-4
-6
-6
-4
-2
0
2
4
6
(0,0)
y
x

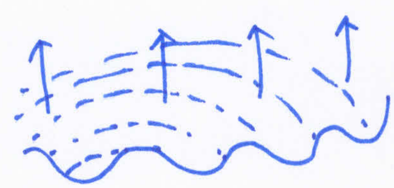
$$Z = 4x^2 + y^2 - 2xy$$

$$\begin{bmatrix}x\\y\end{bmatrix}_{i+1} = \begin{bmatrix}x\\y\end{bmatrix} - \frac{\nabla F(x,y) \Rightarrow G}{\nabla^2 F(x,y) \Rightarrow H}$$

$$G = \begin{bmatrix}8x - 2y\\ 2y - 2x\end{bmatrix} \qquad H = \begin{bmatrix}\frac{\partial G_x}{\partial x} & \frac{\partial G_x}{\partial y}\\ \frac{\partial G_y}{\partial x} & \frac{\partial G_y}{\partial y}\end{bmatrix}$$

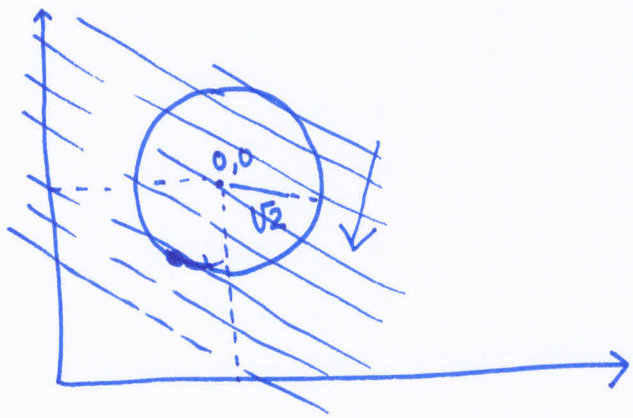
Constrained optimization





$$F(x,y) = (y+100)^2 + 0.01x^2$$

$$x_2 - \cos x_1 \geq 0$$



Min $X_1 + X_2$

S.t. $X_1^2 + X_2^2 - 2 = 0$

Lagrange multiplier

steepest decent

Min $4x^2 + y^2 - 2xy$

S.t. $y = 0.2x^2 + 0.2x - 3$

Lagrange $\mathcal{L} = 4x^2 + y^2 - 2xy - \lambda\left[y - 0.2x^2 + 0.2x - 3\right]$

$$G = \begin{bmatrix} \frac{\partial L}{\partial x} \\ \frac{\partial L}{\partial y} \\ \frac{\partial L}{\partial \lambda} \end{bmatrix} \qquad \begin{bmatrix} x \\ y \\ \lambda \end{bmatrix}_{i+1} = \begin{bmatrix} x \\ y \\ \lambda \end{bmatrix} - G\underbrace{\alpha}_{\text{step size}}$$

Newton's method

$$\mathcal{L}(x, \lambda) = f(x) + \lambda^T g(x)$$

$$G = \begin{bmatrix} \nabla f(x) + \frac{\partial[\lambda^T g(x)]}{\partial x} \Rightarrow G_1 \\ g(x) \Rightarrow G_2 \end{bmatrix}$$

$$H = \begin{bmatrix} \frac{\partial G_1}{\partial x} & \frac{\partial G_1}{\lambda} \\ \frac{\partial G_2}{\partial x} & \frac{\partial G_2}{\partial \lambda} \end{bmatrix}$$

Min $F = C_1 + C_2 + C_3$

$$C_1 = 0.0012\,P_1^2 + 6.48\,P_1 + 459$$
$$C_2 = 0.00194\,P_2^2 + 7.85\,P_2 + 310$$
$$C_3 = 0.00482\,P_3^2 + 7.97\,P_3 + 78$$

$$P_1 + P_2 + P_3 = 850$$

Ref: "Numerical optimization", J. Nocedal, S. Wright, 2nd ed.

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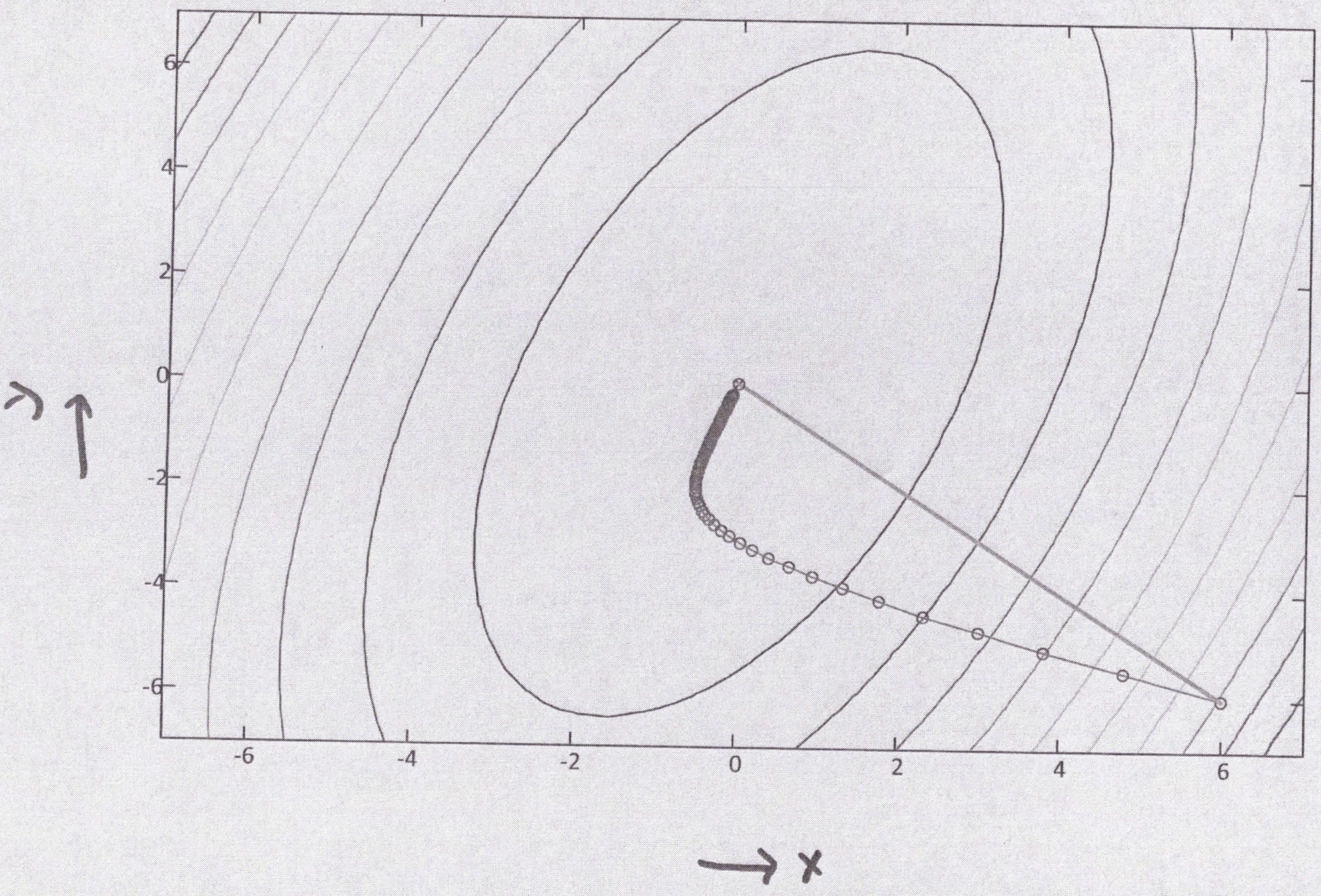
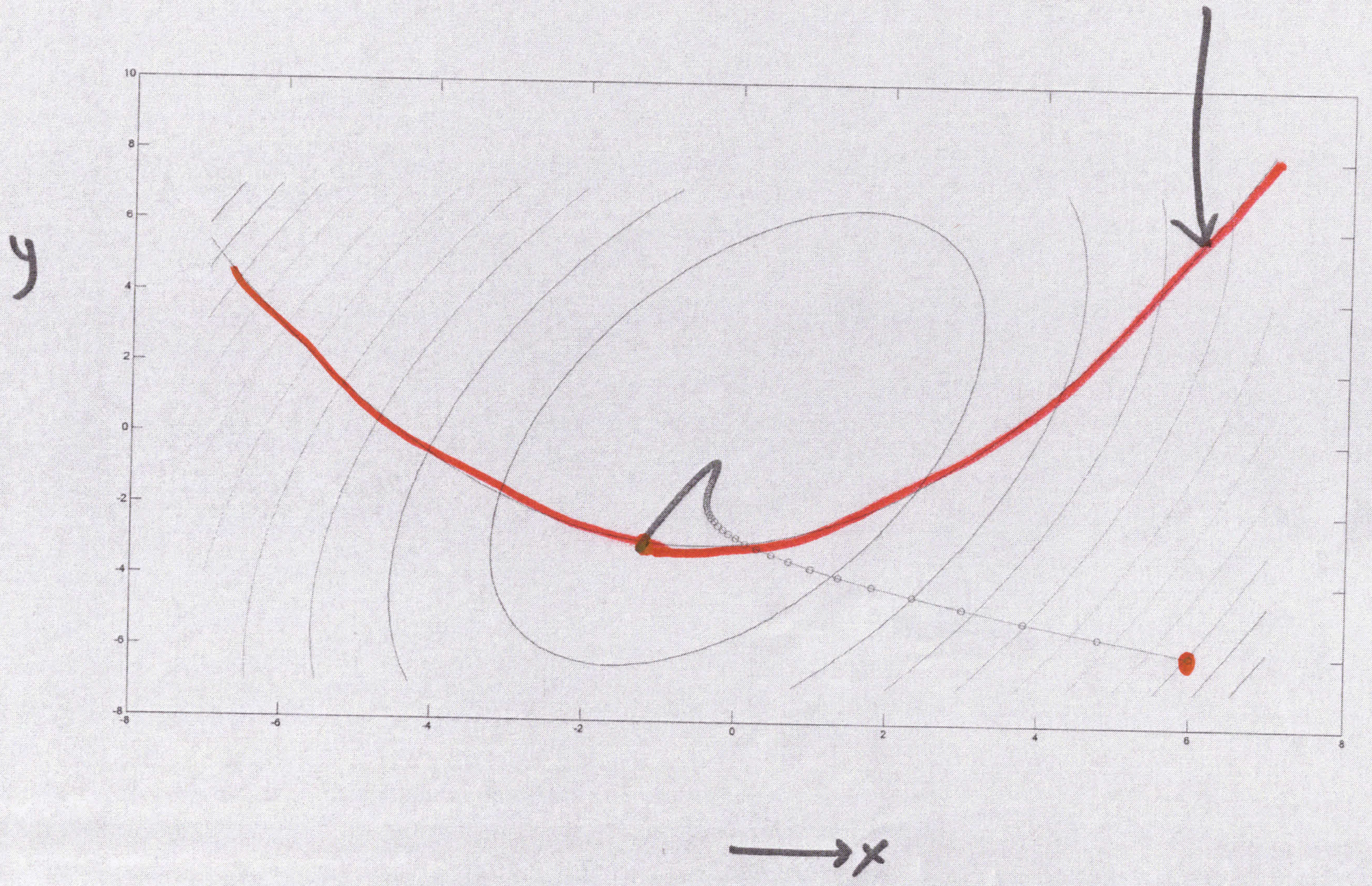


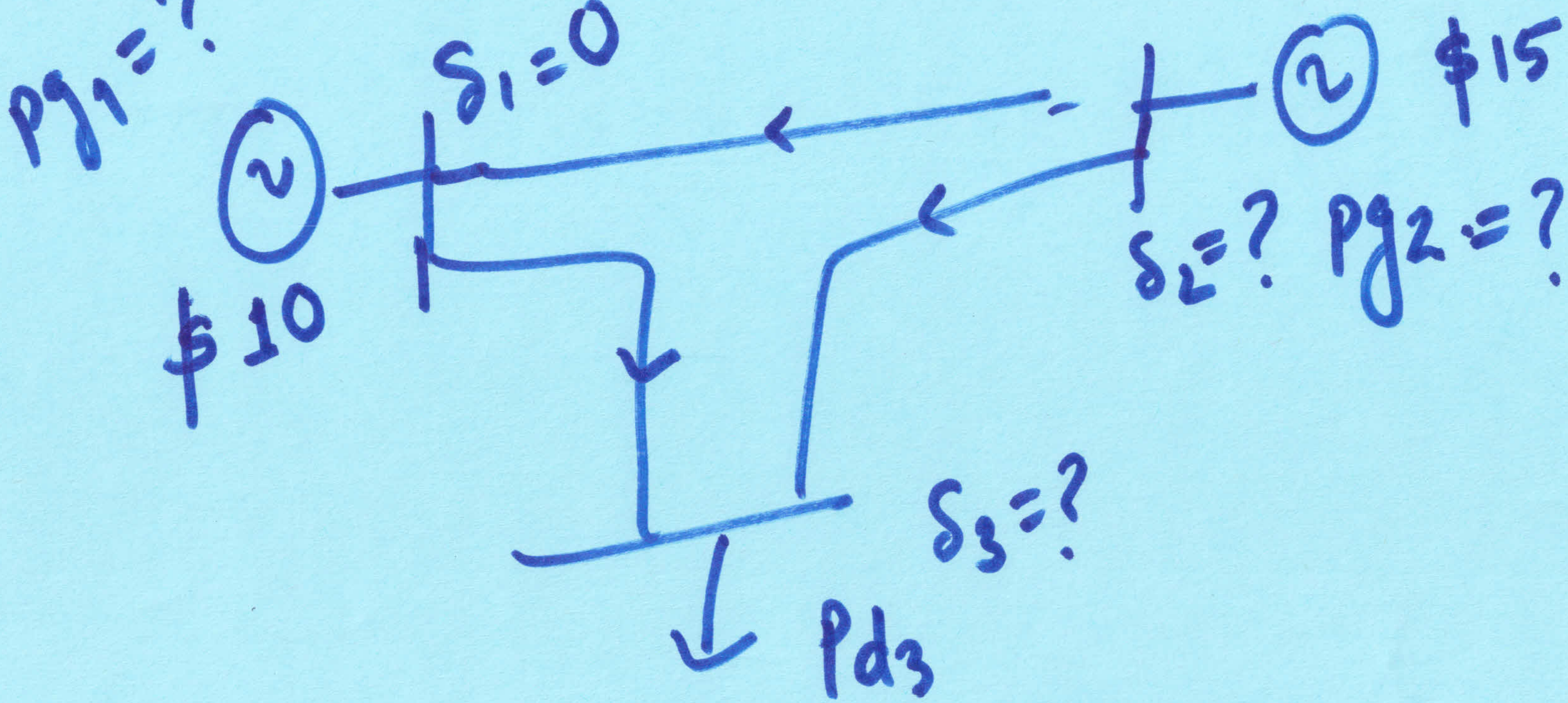
(9)

Min: $4x^2 - y^2 - xy$

s.t. $y = 0.2x^2 + 0.2x - 3$



LP example. (DC-OPF)



$$\boxed{Pg_1 + Pg_2 = Pd_3}$$

$$P_{g_1} - 0 = \sum_{j=1}^{N} B_{ij} (\delta_i - \delta_j) \quad \checkmark$$

$$\underline{\underline{i=1}}$$

$$P_{g_2} - 0 = 0 \longrightarrow \checkmark$$

$$P_{g_3} - P_{d_3} = 0 \longrightarrow \checkmark$$

$$\underline{\underline{\quad}}$$

$$\downarrow 0$$

$$\underline{\text{Cost} = 10 \times P_{g_1} + 15\, P_{g_2}}$$

(LP)

$$\min F(x)$$

$$g(x) = 0$$

$\min \quad F(x)$

$g(x) = 0$ ✓

$h(x) \leq 0$ ✓

$$\left| P_{12} = B_{12} (\delta_1 - \delta_2) \right| \leq P_{12}^{max}$$

c $\quad 4x^2+y^2-2xy$

eq. $\quad$ s.t. $\quad y = 0.2x^2 + 0.2x - 3$

$$\mathcal{L} = 4x^2+y^2-2xy - \lambda\left[y - 0.2x^2 + 0.2x - 3\right]$$

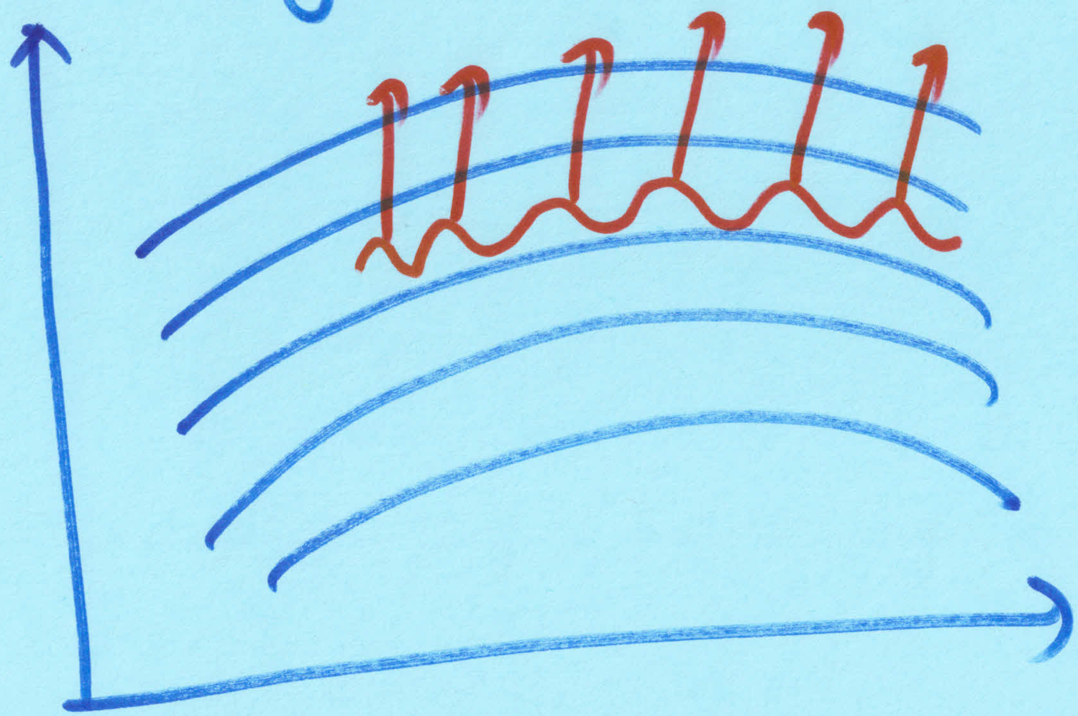
$$\frac{\partial \mathcal{L}}{\partial x} = 0 \qquad \frac{\partial \mathcal{L}}{\partial y} = 0 \qquad \frac{\partial \mathcal{L}}{\partial \lambda} = 0$$

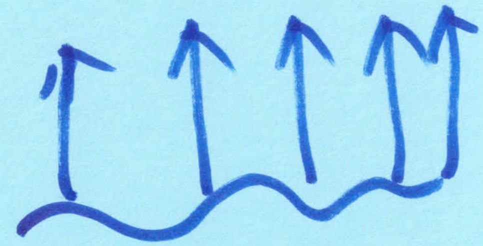
$$G = \begin{bmatrix} \dfrac{\partial \mathcal{L}}{\partial x} \\ \dfrac{\partial \mathcal{L}}{\partial y} \\ \dfrac{\partial \mathcal{L}}{\partial \lambda} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ \lambda \end{bmatrix}_{i+1} = \begin{bmatrix} x \\ y \\ \lambda \end{bmatrix}_{i} - G\overset{\text{step size}}{\alpha}$$

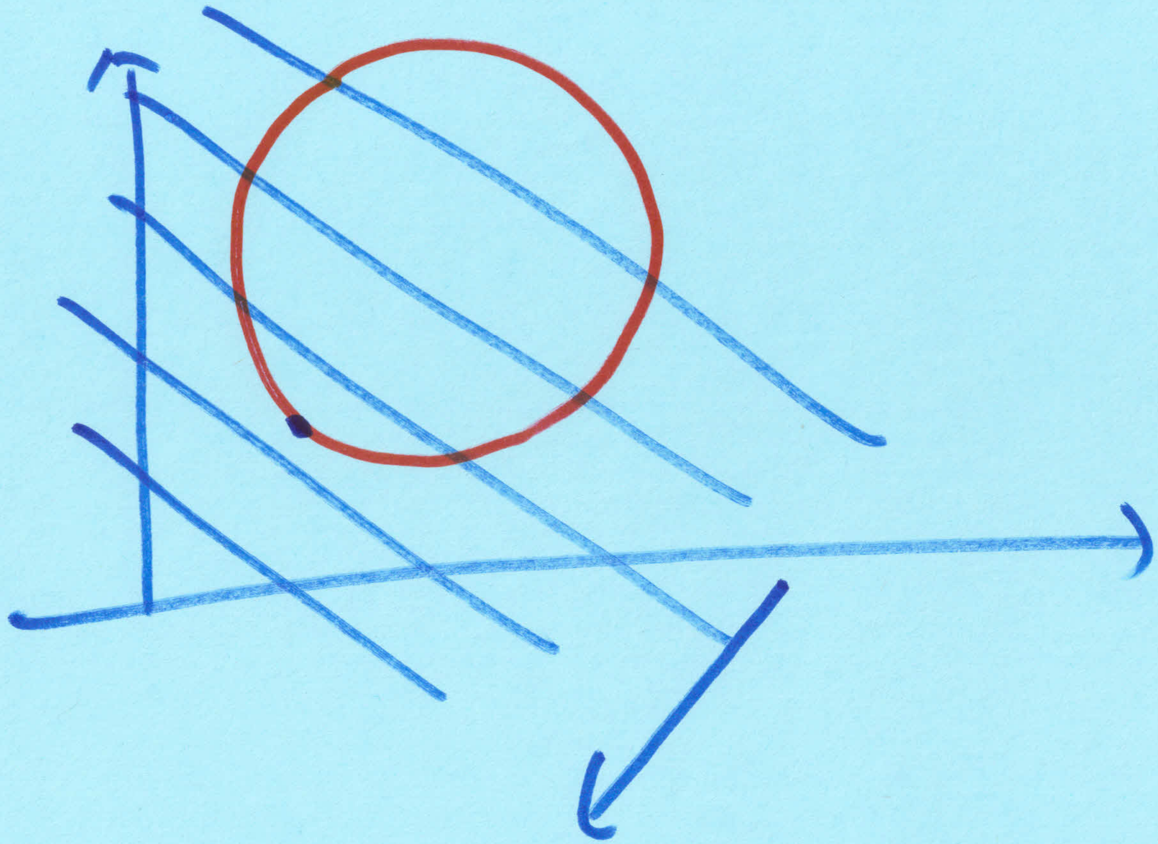
$$F(x,y) = (y+100)^2 + 0.01x^2$$

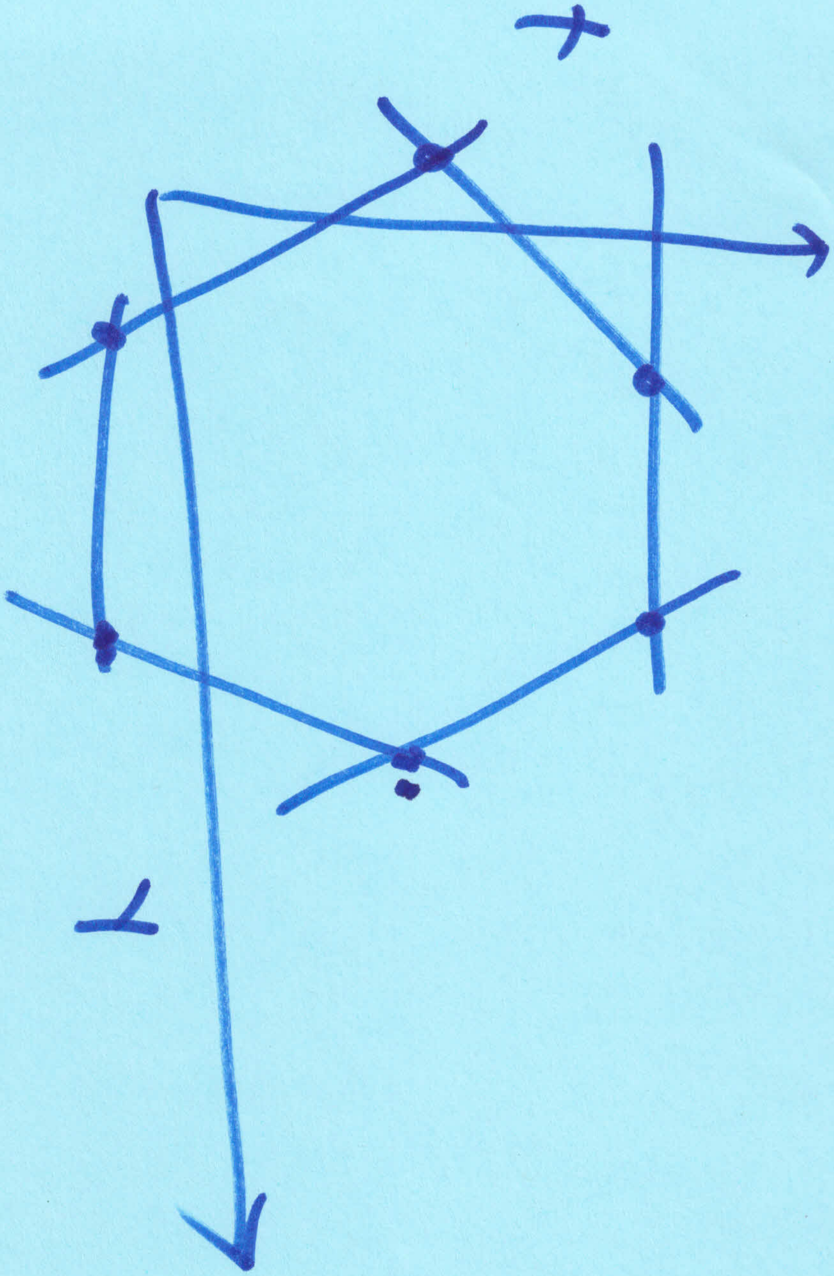
$$y - \cos x \geq 0$$

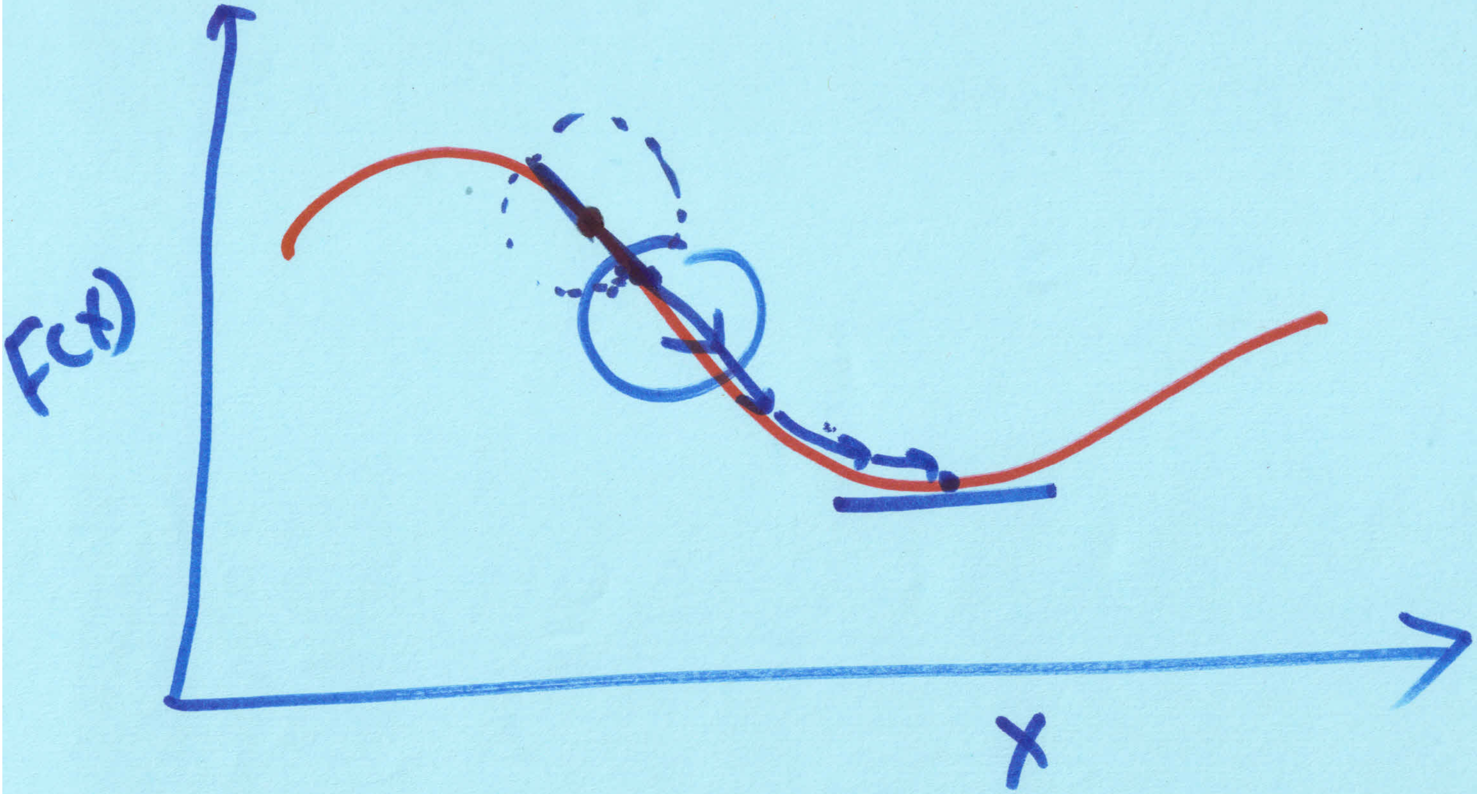




Min $\underbrace{x_1 + x_2}$ s.t. $x_1^2 + x_2^2 - 2 = 0$.

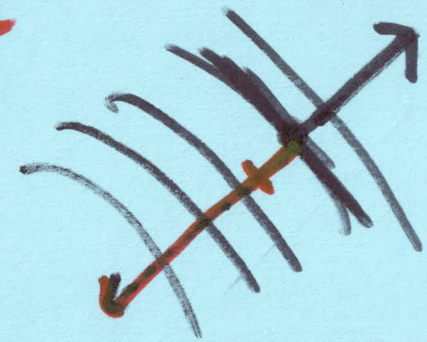




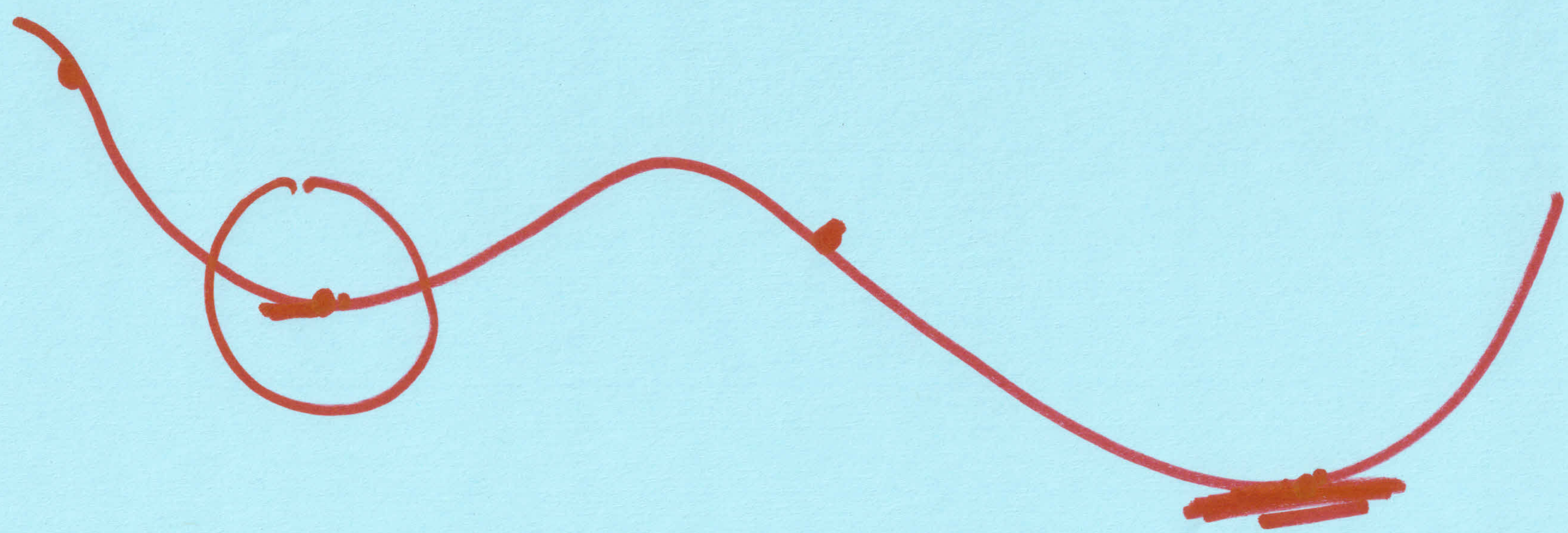


Gradient based method.

$$\nabla f(\vec{x}) = \left[\frac{\partial f(\vec{x})}{\partial x_1}, \frac{\partial f(\vec{x})}{\partial x_2} \cdots \frac{\partial f(\vec{x})}{\partial x_n} \right]$$



$$x_{i+1} = x_i - \underset{\substack{\Uparrow \\ \text{step size}}}{\alpha} \nabla f(x_i)$$



Min $Z = 4x^2 + y^2 - 2xy$.

$x = 0$
$y = 0$

$$G = \begin{bmatrix} G_x \\ G_y \end{bmatrix} = \begin{bmatrix} \underbrace{8x - 2y} \\ 2y - 2x \end{bmatrix}$$

$\Rightarrow Z = 108$

$$\begin{bmatrix} 6 \to x_0 \\ 6 \to y_0 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix}_{i+1} = \begin{bmatrix} 6 \\ 6 \end{bmatrix} - \underset{\underset{\underbrace{0.01}}{\Uparrow}}{\alpha} \begin{bmatrix} 36 \\ 24 \end{bmatrix} = \begin{bmatrix} 5.64 \\ -5.76 \end{bmatrix}$$

$\Downarrow$

$Z = 95.44$

$\nabla f \qquad \nabla^2 f$

$\Downarrow$

Hessian.

$$x_{i+1} = x_i - \frac{\nabla f \Rightarrow}{\nabla^2 f}$$