<u>Economic Dispatch</u> – Optimum allocation of generation among system generators.

<u>Goal:</u> Maximize system efficiency
Minimize system losses (can't bill customers)

<u>Specifics</u>:

<u>Control voltage</u>/vars
- Adjust generator exciter
- Reactors, caps (shunt)
- Tap-Changining transformers

<u>Control Power Flow</u>
- Control P_{gen} at each generator
- Phase-shifting transformers
- Line Switching

<u>Frequency</u> – (**later**)
- Prime mover control (droop controller)
- Load management.

$$P_{GEN} = P_{LOADS} + P_{\substack{TRANS/DIST \\ LOSSES}}$$

$$P_G = \sum_{i=1}^{n} P_{Gi}$$

How should P_G be divvied up among the n units?

Constraints:

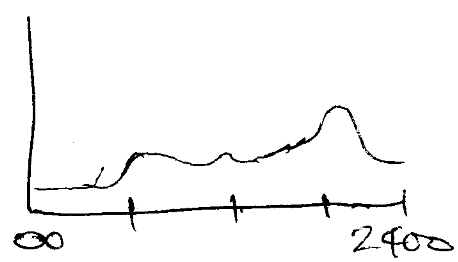
UNIT COMMITMENT
- On-line time reqmnts - Coal 8 hrs+
- Some units down for maintenance
- Should have rolling/spinning reserve in case units fail.
- $P_{min_i} < P_{G_i} < P_{max_i}$

↑ Therme constraints of turbine.

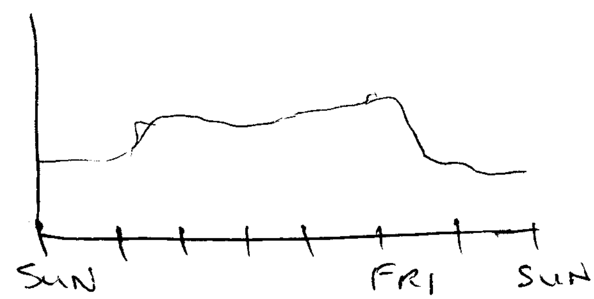
↑ I^2R of stator

Lingo - Load Characteristics:

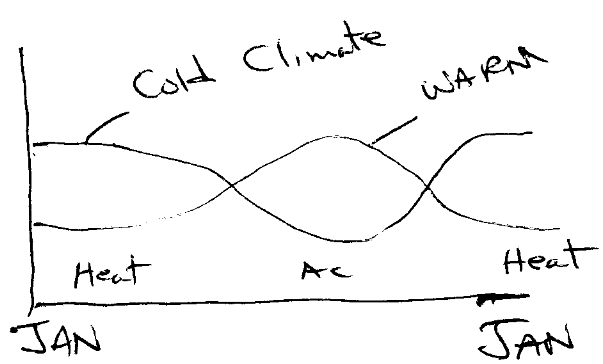
A) Daily -



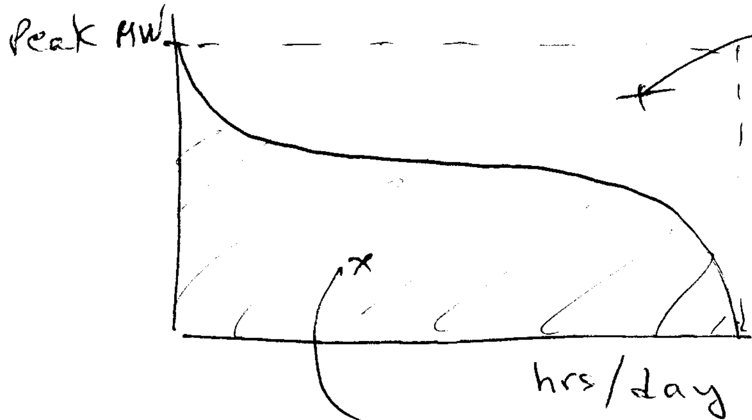
B) Weekly -



C) ~~Monthly~~ - Annual



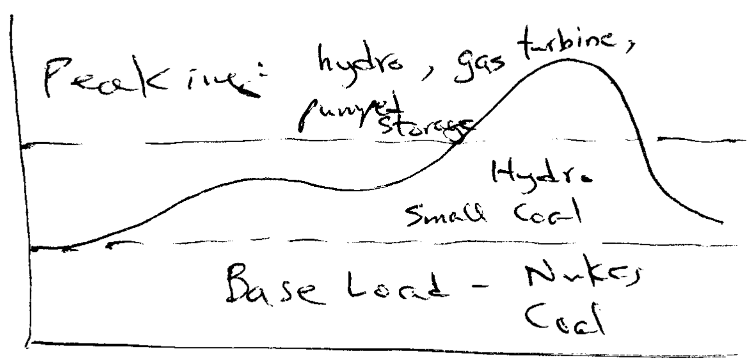
Load Duration Curve

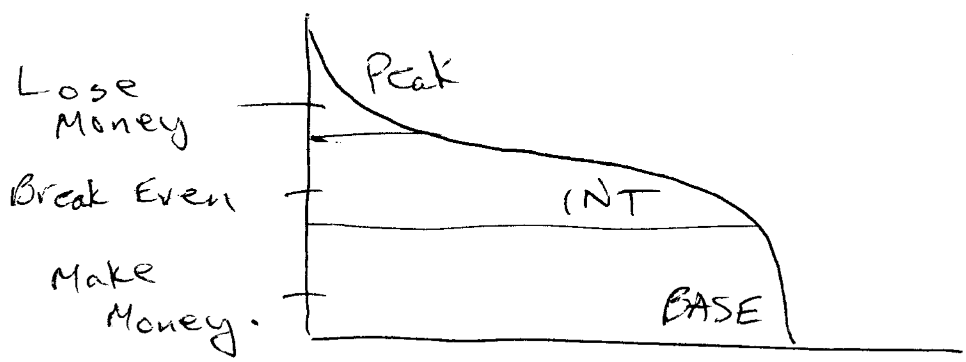


Load Factor: $LF = \dfrac{\text{Energy used}}{\text{Peak Power} \times \text{hrs}}$

0.4 - Bad
0.85 - Good

Strategy:

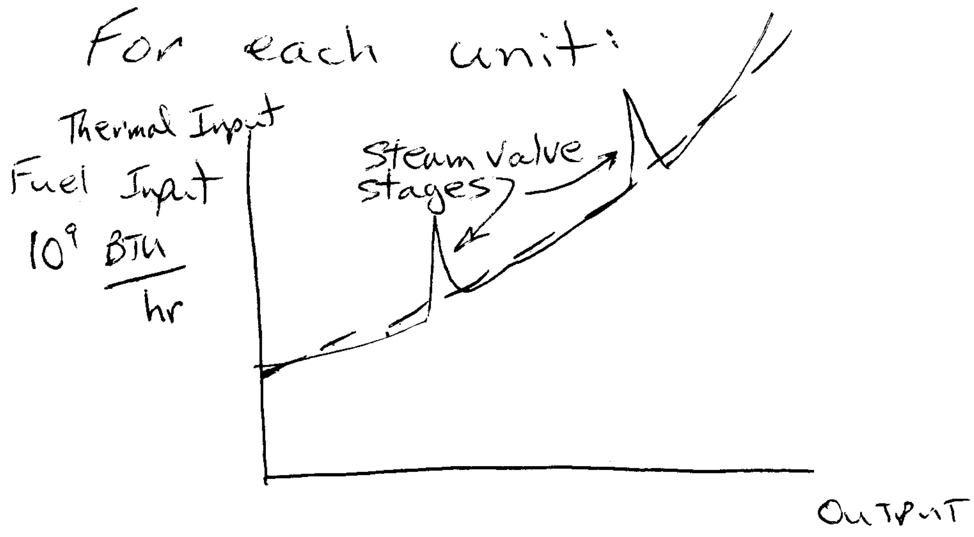




Ways a Utility can make money:

- Raise rates PUC / PSC must approve
- Sell more Base Load MWH
- Reduce Peak load (Load management)
  - Interruptable loads - water heaters, etc
  - Time of day rates
- Increase efficiency
  - Reduce Aux use in plant (10-15%)
  - Improve thermal efficiency (Net Heat Rate)
  - * Economic Dispatch

~~Back~~ For each unit:



$$HR = \frac{\text{Input Thermal power, BTU/hr}}{\text{Electrical output}}$$

Typical: 10.5×10^6 BTU/MWH

Recognize form as $1/\eta$

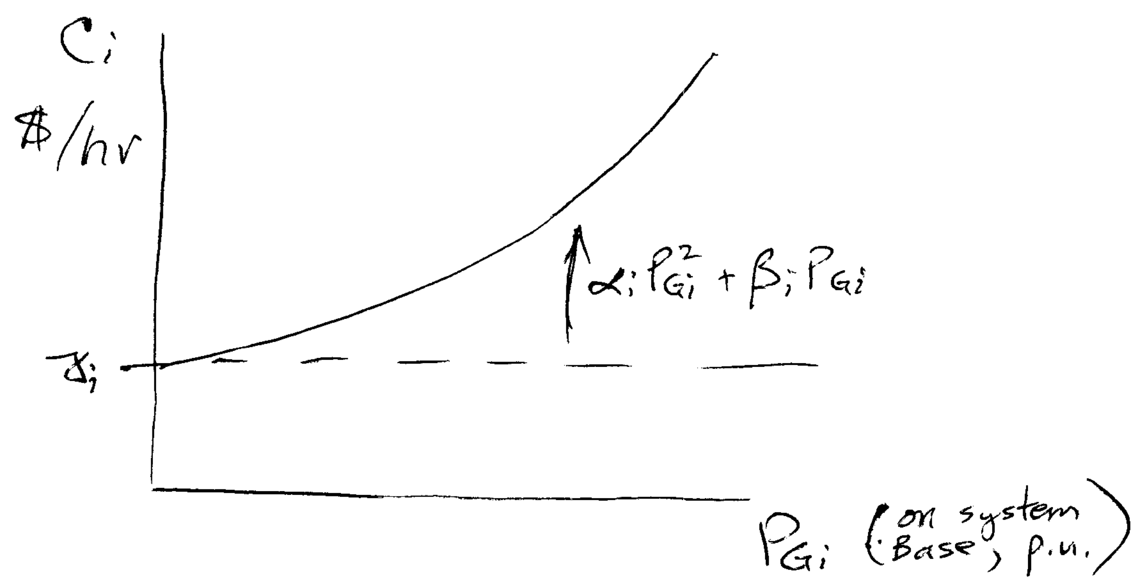
But one $\text{BTU}/\text{hr} = 0.293 \ \text{W}$

$$\eta = \frac{1}{HR \times 0.293 \times 10^{-6}} = \boxed{\frac{3.413 \times 10^{6}}{HR}}$$

Operating cost of unit i

$$C_i = F_i P_i$$

- Input power in MBTU
- Fuel cost in \$/MBTU (Order of Mag. \$1.50) (+ labor, supplies, maint).



Empirically, $C_i = \alpha_i P_{Gi}^2 + \beta_i P_{Gi} + \gamma_i$

α, β, γ in \$/hr

~~The curve is empirically described as:~~

$$\cancel{C_i = \alpha_i P_{Gi}^2 + \beta_i P_{Gi} + \gamma_i}$$

Again,
$$P_G = P_L + P_{TL}$$

Problem: solve for n P_{Gi}'s subject to constraints.

Simplest mathematical formulation is to use Lagrange Multipliers.

~~Example~~

Objective function:

$$\text{Min} \quad F(x_1, x_2, x_3 \cdots x_n)$$

$$\text{Constraints:} \quad G_1(x_1, x_2 \cdots x_n) = 0$$
$$\vdots$$
$$G_m(x_1, x_2 \cdots x_n) = 0$$

Usually (for our purposes) $m = 1$

1) Form the Lagranian:

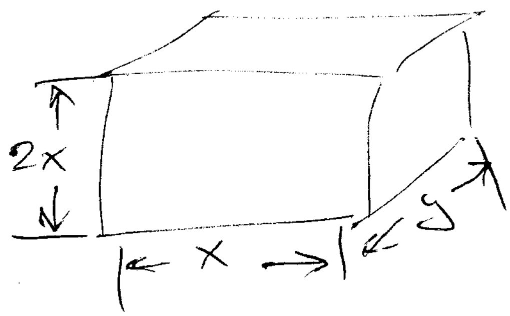
$$\mathcal{L} = F(x_1, x_2 \cdots x_n) - \lambda_1 G(x_1, x_2 \cdots x_n)$$

2) Find all partial derrivates of $\mathcal{L}$ wrt $x_1, x_2 \cdots x_n$ and set them $= 0$

3) Solve for $(x_1, x_2 \cdots x_n)$, λ
from partial derivatives & $G(x_1, x_2 \cdots x_n)$

4) Establish whether solution is min/max or saddle point. (Evaluate Hessian Matrix)
Min if pos definite.
Local Max if neg def, Saddle if indef.

ex: 1 Box of dimensions x, y, z



Maximize Volume for $S = 432\ cm^2$

Objective function: $V = xyz$
$= 2x^2y$

Constraint:
$2(xy + 2x^2 + 2xy) - 432 = 0$

$$\mathcal{L} = 2x^2y - \lambda(4x^2 + 6xy - 432)$$

$$\frac{\partial \mathcal{L}}{\partial x} = 4x - 8\lambda x \overset{\text{neg}}{\cancel{+}} 6y = 0$$

$$\frac{\partial \mathcal{L}}{\partial y} = 2x^2 - 6\lambda x = 0$$

Constraint: $2(xy + 2x^2 + 2xy) - 432 = 0$

solve simultaneously.
$\lambda = 2$
$x = 6\,cm$
$y = 8\,cm$
$V = 576\,cm^3$

Applying to Economic Dispatch:

Objective: $C = \sum_{i=1}^{n} C_i = \sum_{i=1}^{n} \alpha_i P_{Gi}^2 + \beta_i P_{Gi} + \gamma_i$

Constraints: $G = P_G - P_L = 0 = \underbrace{\sum_{i=1}^{n} P_{Gi}}_{\text{gen}} - \underbrace{P_L}_{\text{Total Loads.}}$

(ignore line losses for now.)

① $\mathcal{L} = C - \lambda\left(\sum_{i=1}^{n} P_{Gi} - P_L\right)$

② Partial Derivatives:

$$\frac{\partial \mathcal{L}}{\partial P_{Gi}} = \frac{\partial C}{\partial P_{Gi}} - \lambda(1) = \frac{\partial C}{\partial P_{Gi}} - \lambda$$

since λ is the same in every term, one way to satisfy conditions is:

$$\frac{\partial C}{\partial P_{G1}} - \lambda = 0, \quad \frac{\partial C}{\partial P_{G2}} - \lambda = 0 \quad \cdots \quad \frac{\partial C}{\partial P_{Gn}} - \lambda = 0$$

Therefore, each plant must be at same incremental cost, λ_i $(\lambda_1 = \lambda_2 = \cdots \lambda_n = \lambda)$

For each unit,

$$\boxed{\lambda_i = \frac{\partial C_i}{\partial P_{Gi}} = 2\alpha_i P_{Gi} + \beta_i}$$

Example 8.2

Unit 1: $25MW < P_{G1} < 150MW$

$$C_1 = \underset{\alpha_1 \,\$/hr/MW^2}{0.01} P_{G1}^2 + \underset{\beta_1 \,\$/hr/MW}{2} P_{G1} + \underset{\gamma_1 \,\$/hr}{100}$$

Unit 2: $30MW < P_{G2} < 200MW$

$$C_2 = \underset{\alpha_2}{0.004} P_{G2}^2 + \underset{\beta_2}{2.6} P_{G2} + \underset{\gamma_2}{80}$$

How to divide P_{G1} & P_{G2} within range $55MW \le P_L \le 350$ MW ?

For ex, $\underline{P_L = 282 \text{ MW}}$

First, select $S_{BASE} = 100$ MVA & convert data to p.u.

$$\alpha_1 = (100)^2(.01) = 100 \qquad \alpha_2 = 40$$
$$\beta_1 = (100)(2) = 200 \qquad \beta_2 = 260$$
$$\gamma_1 = 100 \qquad \gamma_2 = 80$$

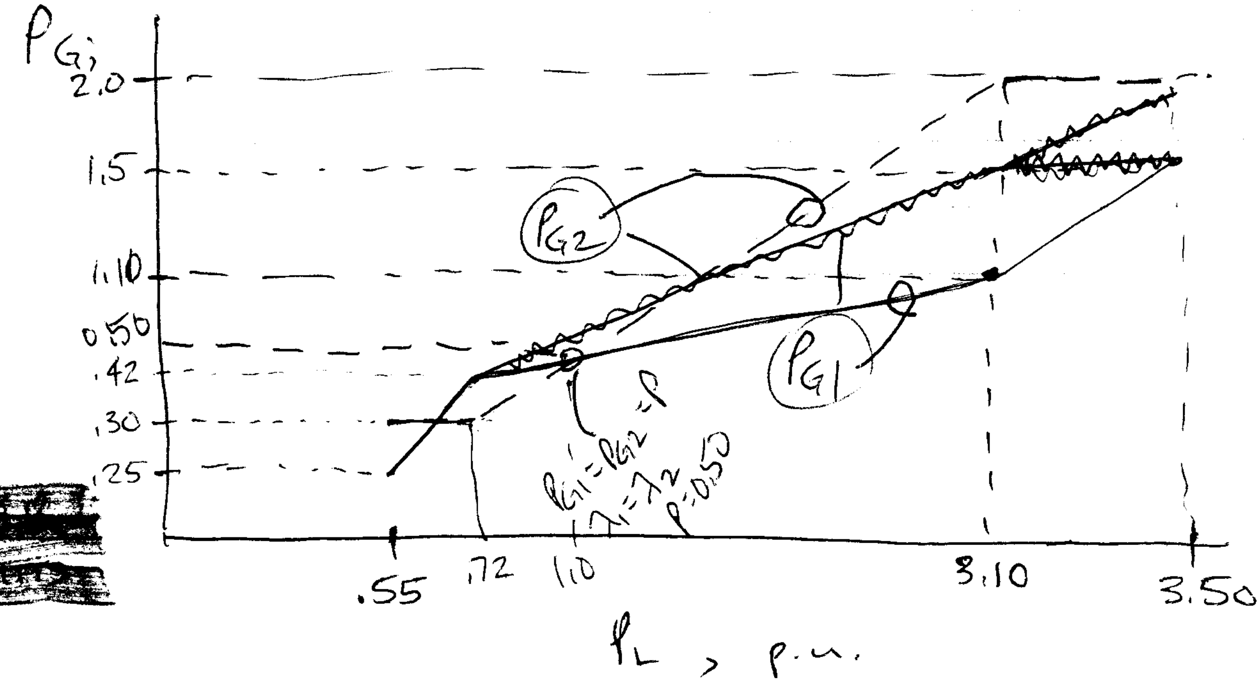
$$0.25 \le P_{G1} \le 1.50 \text{ p.u}$$
$$0.30 \le P_{G2} \le 2.00 \text{ p.u.}$$
$$0.55 \le P_L \le 3.50 \text{ p.u.}$$

$$\boxed{\begin{aligned} \lambda_1 &= \frac{\partial C_1}{\partial P_{G1}} = 200 P_{G1} + 200 \\ \lambda_2 &= \frac{\partial C_2}{\partial P_{G2}} = 80 P_{G2} + 260 \\ P_{G1} + P_{G2} &= 2.82 \text{ p.u.} \end{aligned}}$$

Solving, setting $\lambda_1 = \lambda_2 = \lambda$

$P_{G1} = 1.02$ p.u. (102 MW)
$P_{G2} = 1.80$ p.u. (180 MW)

Looking at complete range, $55\,MW \le P_L \le 350\,MW$,



@ .55 p.u. $\Rightarrow$ $P_{G1} = 0.25$, $P_{G2} = 0.30$
$\lambda_1 = 250$, $\lambda_2 = 284$

→ ∴ must increase unit 1 first, until $\lambda_1 = 284$. This happens at $P_{G1} = \frac{284-200}{200} = \underline{0.42\text{ p.u.}}$

→ Then λ_1 & λ_2 can be equal until one unit hits P_{max}. @ 3.5 p.u.,
$\lambda_1 = 500$ @ $P_{G1} = 1.5$ p.u. $\lambda_2 = 420$ @ $P_{G2} = 2.0$ p.u.
∴ P_{G2} limits out first. $P_{G1} = \frac{420-200}{200} = 1.1$ p.u.

→ From $P_L = 3.10$ and up, only P_{G1} increases.