Final Exam
CM3110
Fall 2019

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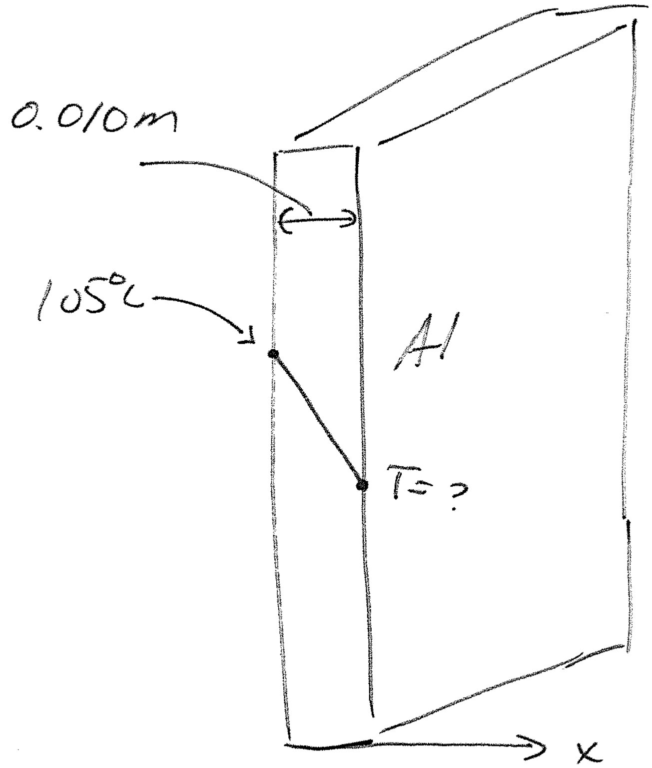


Fourier's Law

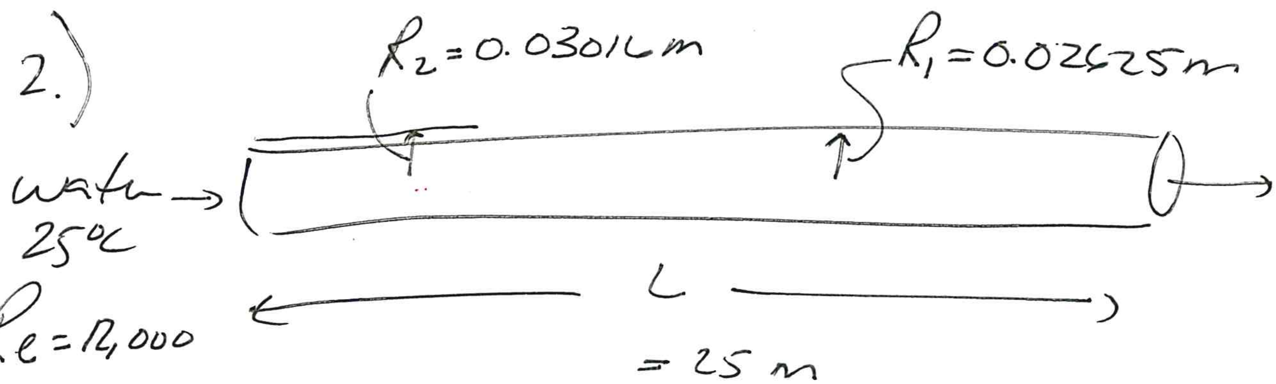
$$\frac{q_x}{A} = 3.4 \times 10^3 \; W/m^2 = -k \frac{dT}{dx} = -k\left(\frac{T_w - 105}{0.010\,m}\right)$$

$$k = 206 \frac{W}{mK}$$

$$34 \frac{\cancel{W}}{\cancel{m}} = (105 - T_w)\left(206 \frac{\cancel{W}}{\cancel{m}K}\right)$$

$$\boxed{T_w = 104.8\,°C} = 378.0\,K$$

2.)



$Re = 12{,}000$

a) $\langle v \rangle$
b) Δp

a) $$Re = \frac{\rho \langle v \rangle D}{\mu}$$

$$12000 = \frac{\left(997.08 \frac{kg}{m^3}\right)\langle v \rangle (2)(0.02625\,m)}{8.937 \times 10^{-4} \frac{kg}{ms}}$$

$$\boxed{\langle v \rangle = 0.204873 \ \frac{m}{s}} = 0.20 \ \frac{m}{s}$$

(2 sig figs)

b) Δp ? ① find f from correlation
② then $$f = \frac{\Delta p D}{2L\rho\langle v \rangle^2}$$

What is f? Use simplified correlation

$$f = \frac{1.02}{4}\left(\log(Re)\right)^{-2.5}$$

$$= \frac{1.02}{4}\left(\log(12{,}000)\right)^{-2.5}$$

$$= 7.5876 \times 10^{-3}$$

$$\boxed{f = 7.6 \times 10^{-3}}$$

(looks reasonable from Moody plot)

0.0076
~0.01 if read from chart

$$7.5876 \times 10^{-3} = \frac{\Delta P\,(\cancel{2})(0.02625\,\cancel{m})}{(\cancel{2})(25\,\cancel{m})(997.08\,\frac{kg}{m^3}) \times (0.204873\,\frac{m}{s})^2}$$

$$\Delta P = 302.423\,\frac{\cancel{kg}}{\cancel{m}\,s^2}\;\frac{Pa\,m^2}{\cancel{N}}\;\frac{\cancel{N}\,s^2}{\cancel{kg}\,m}\left(\frac{14.696\,psi}{1.01325 \times 10^5\,Pa}\right)$$

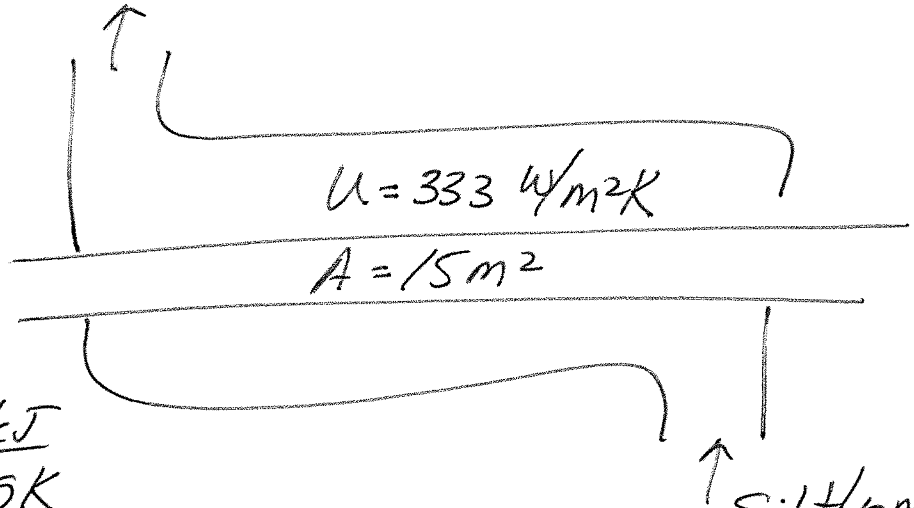
$$\boxed{\Delta P = 0.044\ psi}$$

3)

$\dot{m} = 1.3 \frac{kg}{s}$

$C_p = 4.092 \frac{kJ}{kg K}$

$25°C$



Siltherm

$C_p' = 12.2 \frac{kJ}{kg K}$

$142°C$

$\dot{m}' = 2.1 \frac{kg}{s}$

Heat Exchanger Effectiveness
(inlet temperatures known)

$$(\dot{m} C_p)_{COLD} = \left(1.3 \frac{\cancel{kg}}{\cancel{s}}\right)\left(4.092 \frac{\cancel{kJ}}{\cancel{kg} K}\right) \frac{kW}{\cancel{kJ/s}}$$

$$= 5.32 \ kW/K \leftarrow \text{minimum fluid}$$

$$(\dot{m} C_p)_{HOT} = \left(2.1 \frac{kg}{\cancel{s}}\right)\left(12.2 \frac{\cancel{kJ}}{kg K}\right)\left(\frac{kW}{\cancel{kJ/s}}\right)$$

$$= 25.62 \ kW/K$$

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Obtain ε from chart

$$NTU = \frac{UA}{(\dot{m}C_p)_{min}}$$

$$= \frac{\left(333 \frac{\cancel{W}}{\cancel{m^2}\cancel{K}}\right)(15\ \cancel{m^2})}{5.32 \times 10^3 \frac{\cancel{W}}{\cancel{K}}}$$

$$\boxed{NTU = 0.94}$$

$$\frac{(\dot{m}C_p)_{min}}{(\dot{m}C_p)_{max}} = \frac{5.32}{25.62} = 0.21$$

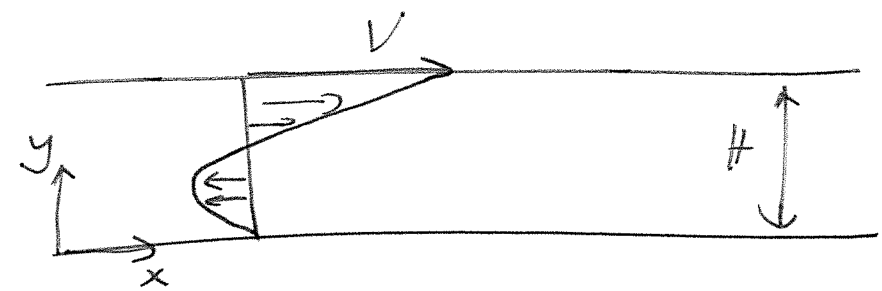
Reading chart: $\varepsilon = 0.58$

$$Q = (\dot{m}C_p)_{min}\, \varepsilon\, (T_{hi} - T_{ci})$$

$$= \left(5.32 \frac{kW}{\cancel{K}}\right)(0.58)(142 - 25)\cancel{K}$$

$$= \boxed{360\ kW}$$

4) a.



$v_y = 0$

$v_z = 0$

steady

incompressible $\Rightarrow \rho =$ constant

$$\underline{g} = \begin{pmatrix} 0 \\ -g \\ 0 \end{pmatrix}_{xyz} \approx \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}_{xyz}$$

(neglect)

$$\boxed{\begin{array}{l} 0 = -\dfrac{\partial P(x)}{\partial x} + \mu \dfrac{d^2 v_x}{dy^2} \\ \\ \text{BC: } \; x = 0 \quad P = P_0 \qquad y = 0 \quad v_x = 0 \\ \qquad x = L \quad P = P_L \qquad y = H \quad v_x = V \end{array}}$$

The Equation of Continuity and the Equation of Motion in Cartesian, cylindrical, and spherical coordinates

CM3110 Fall 2011 Faith A. Morrison

Continuity Equation, Cartesian coordinates

$$\frac{\partial \rho}{\partial t}+\left(v_x\frac{\partial \rho}{\partial x}+v_y\frac{\partial \rho}{\partial y}+v_z\frac{\partial \rho}{\partial z}\right)+\rho\left(\frac{\partial v_x}{\partial x}+\frac{\partial v_y}{\partial y}+\frac{\partial v_z}{\partial z}\right) = 0$$

Continuity Equation, cylindrical coordinates

$$\frac{\partial \rho}{\partial t}+\frac{1}{r}\frac{\partial(\rho r v_r)}{\partial r}+\frac{1}{r}\frac{\partial(\rho v_\theta)}{\partial \theta}+\frac{\partial(\rho v_z)}{\partial z} = 0$$

Continuity Equation, spherical coordinates

$$\frac{\partial \rho}{\partial t}+\frac{1}{r^2}\frac{\partial(\rho r^2 v_r)}{\partial r}+\frac{1}{r\sin\theta}\frac{\partial(\rho v_\theta\sin\theta)}{\partial \theta}+\frac{1}{r\sin\theta}\frac{\partial(\rho v_\phi)}{\partial \phi} = 0$$

ρ constant

$v_y = 0$

$v_z = 0$

$\Rightarrow \boxed{\frac{\partial v_x}{\partial x} = 0}$

Equation of Motion for an incompressible fluid, 3 components in Cartesian coordinates

$$\rho\left(\frac{\partial v_x}{\partial t}+v_x\frac{\partial v_x}{\partial x}+v_y\frac{\partial v_x}{\partial y}+v_z\frac{\partial v_x}{\partial z}\right) = -\frac{\partial P}{\partial x}+\left(\frac{\partial \tilde\tau_{xx}}{\partial x}+\frac{\partial \tilde\tau_{yx}}{\partial y}+\frac{\partial \tilde\tau_{zx}}{\partial z}\right)+\rho g_x$$

$$\rho\left(\frac{\partial v_y}{\partial t}+v_x\frac{\partial v_y}{\partial x}+v_y\frac{\partial v_y}{\partial y}+v_z\frac{\partial v_y}{\partial z}\right) = -\frac{\partial P}{\partial y}+\left(\frac{\partial \tilde\tau_{xy}}{\partial x}+\frac{\partial \tilde\tau_{yy}}{\partial y}+\frac{\partial \tilde\tau_{zy}}{\partial z}\right)+\rho g_y$$

$$\rho\left(\frac{\partial v_z}{\partial t}+v_x\frac{\partial v_z}{\partial x}+v_y\frac{\partial v_z}{\partial y}+v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial P}{\partial z}+\left(\frac{\partial \tilde\tau_{xz}}{\partial x}+\frac{\partial \tilde\tau_{yz}}{\partial y}+\frac{\partial \tilde\tau_{zz}}{\partial z}\right)+\rho g_z$$

Equation of Motion for an incompressible fluid, 3 components in cylindrical coordinates

$$\rho\left(\frac{\partial v_r}{\partial t}+v_r\frac{\partial v_r}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta}-\frac{v_\theta^2}{r}+v_z\frac{\partial v_r}{\partial z}\right) = -\frac{\partial P}{\partial r}+\left(\frac{1}{r}\frac{\partial(r\tilde\tau_{rr})}{\partial r}+\frac{1}{r}\frac{\partial \tilde\tau_{\theta r}}{\partial \theta}-\frac{\tilde\tau_{\theta\theta}}{r}+\frac{\partial \tilde\tau_{zr}}{\partial z}\right)+\rho g_r$$

$$\rho\left(\frac{\partial v_\theta}{\partial t}+v_r\frac{\partial v_\theta}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta}+\frac{v_\theta v_r}{r}+v_z\frac{\partial v_\theta}{\partial z}\right) = -\frac{1}{r}\frac{\partial P}{\partial \theta}+\left(\frac{1}{r^2}\frac{\partial(r^2\tilde\tau_{r\theta})}{\partial r}+\frac{1}{r}\frac{\partial \tilde\tau_{\theta\theta}}{\partial \theta}+\frac{\partial \tilde\tau_{z\theta}}{\partial z}+\frac{\tilde\tau_{\theta r}-\tilde\tau_{r\theta}}{r}\right)+\rho g_\theta$$

$$\rho\left(\frac{\partial v_z}{\partial t}+v_r\frac{\partial v_z}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta}+v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial P}{\partial z}+\left(\frac{1}{r}\frac{\partial(r\tilde\tau_{rz})}{\partial r}+\frac{1}{r}\frac{\partial \tilde\tau_{\theta z}}{\partial \theta}+\frac{\partial \tilde\tau_{zz}}{\partial z}\right)+\rho g_z$$

Equation of Motion for an incompressible fluid, 3 components in spherical coordinates

$$\rho\left(\frac{\partial v_r}{\partial t}+v_r\frac{\partial v_r}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta}+\frac{v_\phi}{r\sin\theta}\frac{\partial v_r}{\partial \phi}-\frac{v_\theta^2+v_\phi^2}{r}\right)$$
$$= -\frac{\partial P}{\partial r}+\left(\frac{1}{r^2}\frac{\partial(r^2\tilde\tau_{rr})}{\partial r}+\frac{1}{r\sin\theta}\frac{\partial(\tilde\tau_{\theta r}\sin\theta)}{\partial \theta}+\frac{1}{r\sin\theta}\frac{\partial \tilde\tau_{\phi r}}{\partial \phi}-\frac{\tilde\tau_{\theta\theta}+\tilde\tau_{\phi\phi}}{r}\right)+\rho g_r$$

$$\rho\left(\frac{\partial v_\theta}{\partial t}+v_r\frac{\partial v_\theta}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta}+\frac{v_\phi}{r\sin\theta}\frac{\partial v_\theta}{\partial \phi}+\frac{v_r v_\theta}{r}-\frac{v_\phi^2\cot\theta}{r}\right)$$
$$= -\frac{1}{r}\frac{\partial P}{\partial \theta}+\left(\frac{1}{r^3}\frac{\partial(r^3\tilde\tau_{r\theta})}{\partial r}+\frac{1}{r\sin\theta}\frac{\partial(\tilde\tau_{\theta\theta}\sin\theta)}{\partial \theta}+\frac{1}{r\sin\theta}\frac{\partial \tilde\tau_{\phi\theta}}{\partial \phi}+\frac{\tilde\tau_{\theta r}-\tilde\tau_{r\theta}}{r}-\frac{\tilde\tau_{\phi\phi}\cot\theta}{r}\right)+\rho g_\theta$$

$$\rho\left(\frac{\partial v_\phi}{\partial t}+v_r\frac{\partial v_\phi}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_\phi}{\partial \theta}+\frac{v_\phi}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi}+\frac{v_r v_\phi}{r}+\frac{v_\phi v_\theta\cot\theta}{r}\right)$$
$$= -\frac{1}{r\sin\theta}\frac{\partial P}{\partial \phi}+\left(\frac{1}{r^3}\frac{\partial(r^3\tilde\tau_{r\phi})}{\partial r}+\frac{1}{r\sin\theta}\frac{\partial(\tilde\tau_{\theta\phi}\sin\theta)}{\partial \theta}+\frac{1}{r\sin\theta}\frac{\partial \tilde\tau_{\phi\phi}}{\partial \phi}+\frac{\tilde\tau_{\phi r}-\tilde\tau_{r\phi}}{r}+\frac{\tilde\tau_{\phi\theta}\cot\theta}{r}\right)+\rho g_\phi$$

$\frac{\partial v_x}{\partial x} = 0$

WIDE

Equation of Motion for incompressible, Newtonian fluid (Navier-Stokes equation) 3 components in Cartesian coordinates

$$\rho\left(\frac{\partial v_x}{\partial t}+v_x\frac{\partial v_x}{\partial x}+v_y\frac{\partial v_x}{\partial y}+v_z\frac{\partial v_x}{\partial z}\right) = -\frac{\partial P}{\partial x}+\mu\left(\frac{\partial^2 v_x}{\partial x^2}+\frac{\partial^2 v_x}{\partial y^2}+\frac{\partial^2 v_x}{\partial z^2}\right)+\rho g_x$$

$$\rho\left(\frac{\partial v_y}{\partial t}+v_x\frac{\partial v_y}{\partial x}+v_y\frac{\partial v_y}{\partial y}+v_z\frac{\partial v_y}{\partial z}\right) = -\frac{\partial P}{\partial y}+\mu\left(\frac{\partial^2 v_y}{\partial x^2}+\frac{\partial^2 v_y}{\partial y^2}+\frac{\partial^2 v_y}{\partial z^2}\right)+\rho g_y$$

$$\rho\left(\frac{\partial v_z}{\partial t}+v_x\frac{\partial v_z}{\partial x}+v_y\frac{\partial v_z}{\partial y}+v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial P}{\partial z}+\mu\left(\frac{\partial^2 v_z}{\partial x^2}+\frac{\partial^2 v_z}{\partial y^2}+\frac{\partial^2 v_z}{\partial z^2}\right)+\rho g_z$$

Equation of Motion for incompressible, Newtonian fluid (Navier-Stokes equation), 3 components in cylindrical coordinates

$$\rho\left(\frac{\partial v_r}{\partial t}+v_r\frac{\partial v_r}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta}-\frac{v_\theta^2}{r}+v_z\frac{\partial v_r}{\partial z}\right) = -\frac{\partial P}{\partial r}+\mu\left(\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial(rv_r)}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2}-\frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}+\frac{\partial^2 v_r}{\partial z^2}\right)+\rho g_r$$

$$\rho\left(\frac{\partial v_\theta}{\partial t}+v_r\frac{\partial v_\theta}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta}+\frac{v_r v_\theta}{r}+v_z\frac{\partial v_\theta}{\partial z}\right) = -\frac{1}{r}\frac{\partial P}{\partial \theta}+\mu\left(\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial(rv_\theta)}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2}+\frac{2}{r^2}\frac{\partial v_r}{\partial \theta}+\frac{\partial^2 v_\theta}{\partial z^2}\right)+\rho g_\theta$$

$$\rho\left(\frac{\partial v_z}{\partial t}+v_r\frac{\partial v_z}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta}+v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial P}{\partial z}+\mu\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2}+\frac{\partial^2 v_z}{\partial z^2}\right)+\rho g_z$$

Equation of Motion for incompressible, Newtonian fluid (Navier-Stokes equation), 3 components in spherical coordinates

$$\underline{g} = \begin{pmatrix} 0 \\ -g \\ 0 \end{pmatrix}$$

$$\rho\left(\frac{\partial v_r}{\partial t}+v_r\frac{\partial v_r}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta}+\frac{v_\phi}{r\sin\theta}\frac{\partial v_r}{\partial \phi}-\frac{v_\theta^2+v_\phi^2}{r}\right)$$
$$= -\frac{\partial P}{\partial r}+\mu\left(\frac{\partial}{\partial r}\left(\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 v_r\right)\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial v_r}{\partial \theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 v_r}{\partial \phi^2}\right.$$
$$\left.-\frac{2}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(v_\theta\sin\theta\right)-\frac{2}{r^2\sin\theta}\frac{\partial v_\phi}{\partial \phi}\right)+\rho g_r$$

$$\rho\left(\frac{\partial v_\theta}{\partial t}+v_r\frac{\partial v_\theta}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta}+\frac{v_\phi}{r\sin\theta}\frac{\partial v_\theta}{\partial \phi}+\frac{v_r v_\theta}{r}-\frac{v_\phi^2\cot\theta}{r}\right)$$
$$= -\frac{1}{r}\frac{\partial P}{\partial \theta}+\mu\left(\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial v_\theta}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial \theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial \theta}\left(v_\theta\sin\theta\right)\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 v_\theta}{\partial \phi^2}\right.$$
$$\left.+\frac{2}{r^2}\frac{\partial v_r}{\partial \theta}-\frac{2\cot\theta}{r^2\sin\theta}\frac{\partial v_\phi}{\partial \phi}\right)+\rho g_\theta$$

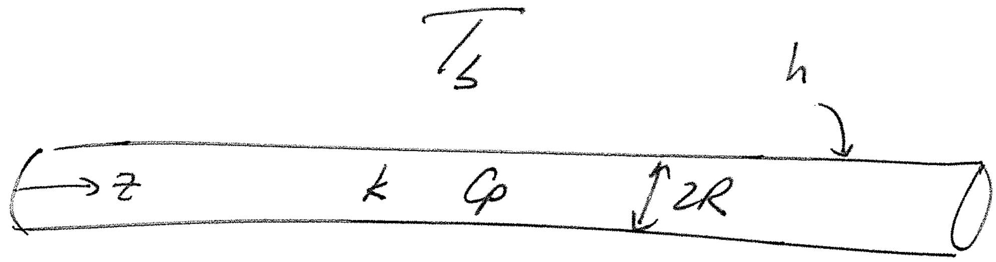
$$\rho\left(\frac{\partial v_\phi}{\partial t}+v_r\frac{\partial v_\phi}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_\phi}{\partial \theta}+\frac{v_\phi}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi}+\frac{v_r v_\phi}{r}+\frac{v_\phi v_\theta\cot\theta}{r}\right)$$
$$= -\frac{1}{r\sin\theta}\frac{\partial P}{\partial \phi}+\mu\left(\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial v_\phi}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial \theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial \theta}\left(v_\phi\sin\theta\right)\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 v_\phi}{\partial \phi^2}\right.$$
$$\left.+\frac{2}{r^2\sin\theta}\frac{\partial v_r}{\partial \phi}+\frac{2\cot\theta}{r^2\sin\theta}\frac{\partial v_\theta}{\partial \phi}\right)+\rho g_\phi$$

Note: the r-component of the Navier-Stokes equation in spherical coordinates may be simplified by adding $0 = \frac{2}{r}\nabla\cdot\underline{v}$ to the component shown above. This term is zero due to the continuity equation (mass conservation). See Bird et. al.

References:

1. R. B. Bird, W. E. Stewart, and E. N. Lightfoot, *Transport Phenomena*, 2nd edition, Wiley: NY, 2002.
2. R. B. Bird, R. C. Armstrong, and O. Hassager, *Dynamics of Polymeric Fluids: Volume 1 Fluid Mechanics*, Wiley: NY, 1987.

b)



$S_0 \frac{W}{m^3}$ due to electric current

$T(r) = ?$

steady

$\underline{v} = 0$

θ symmetry

Long $\Rightarrow \frac{\partial^2 T}{\partial z^2} = 0$

$$0 = \frac{k}{r} \frac{d}{dr}\left(r \frac{dT}{dr}\right) + S_0$$

BC: $r = 0$ T = finite or $\frac{q_r}{A} = 0$

$r = R$ Newton's law of cooling

$\Rightarrow$

$$-k \frac{dT}{dr} = \frac{q_r}{A} = (T(R) - T_b)(h)$$

The **Equation of Energy** in Cartesian, cylindrical, and spherical coordinates for Newtonian fluids of constant density, with source term S. Source could be electrical energy due to current flow, chemical energy, etc. Two cases are presented: the general case where thermal conductivity may be a function of temperature (vector flux $\tilde{\underline{q}} = \underline{q}/A$ appears in the equations); and the more usual case, where thermal conductivity is constant.

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Microscopic energy balance, in terms of flux; Gibbs notation

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + \underline{v} \cdot \nabla T \right) = -\nabla \cdot \tilde{\underline{q}} + S$$

Steady

$v = 0$

Microscopic energy balance, in terms of flux; Cartesian coordinates

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} + v_z \frac{\partial T}{\partial z} \right) = -\left(\frac{\partial \tilde{q}_x}{\partial x} + \frac{\partial \tilde{q}_y}{\partial y} + \frac{\partial \tilde{q}_z}{\partial z} \right) + S$$

Microscopic energy balance, in terms of flux; cylindrical coordinates

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} + v_z \frac{\partial T}{\partial z} \right) = -\left(\frac{1}{r} \frac{\partial (r \tilde{q}_r)}{\partial r} + \frac{1}{r} \frac{\partial \tilde{q}_\theta}{\partial \theta} + \frac{\partial \tilde{q}_z}{\partial z} \right) + S$$

Microscopic energy balance, in terms of flux; spherical coordinates

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} + \frac{v_\phi}{r \sin\theta} \frac{\partial T}{\partial \phi} \right) = -\left(\frac{1}{r^2} \frac{\partial (r^2 \tilde{q}_r)}{\partial r} + \frac{1}{r \sin\theta} \frac{\partial (\tilde{q}_\theta \sin\theta)}{\partial \theta} + \frac{1}{r \sin\theta} \frac{\partial \tilde{q}_\phi}{\partial \phi} \right) + S$$

Fourier's law of heat conduction, Gibbs notation: $\tilde{\underline{q}} = -k \nabla T$

Fourier's law of heat conduction, Cartesian coordinates: $\begin{pmatrix} \tilde{q}_x \\ \tilde{q}_y \\ \tilde{q}_z \end{pmatrix}_{xyz} = \begin{pmatrix} -k \frac{\partial T}{\partial x} \\ -k \frac{\partial T}{\partial y} \\ -k \frac{\partial T}{\partial z} \end{pmatrix}_{xyz}$

Fourier's law of heat conduction, cylindrical coordinates: $\begin{pmatrix} \tilde{q}_r \\ \tilde{q}_\theta \\ \tilde{q}_z \end{pmatrix}_{xyz} = \begin{pmatrix} -k \frac{\partial T}{\partial r} \\ -\frac{k}{r} \frac{\partial T}{\partial \theta} \\ -k \frac{\partial T}{\partial z} \end{pmatrix}_{r\theta z}$

Fourier's law of heat conduction, spherical coordinates: $\begin{pmatrix} \tilde{q}_r \\ \tilde{q}_\theta \\ \tilde{q}_\phi \end{pmatrix}_{xyz} = \begin{pmatrix} -k \frac{\partial T}{\partial r} \\ -\frac{k}{r} \frac{\partial T}{\partial \theta} \\ -\frac{k}{r \sin\theta} \frac{\partial T}{\partial \phi} \end{pmatrix}_{r\theta\phi}$

The **Equation of Energy** for systems with **constant k**

Microscopic energy balance, constant thermal conductivity; Gibbs notation

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + \underline{v} \cdot \nabla T \right) = k \nabla^2 T + S$$

Microscopic energy balance, constant thermal conductivity; Cartesian coordinates

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} + v_z \frac{\partial T}{\partial z} \right) = k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + S$$

Microscopic energy balance, constant thermal conductivity; cylindrical coordinates

$$\rho \hat{C}_p \left(\cancel{\frac{\partial T}{\partial t}} + \cancel{v_r \frac{\partial T}{\partial r}} + \cancel{\frac{v_\theta}{r} \frac{\partial T}{\partial \theta}} + \cancel{v_z \frac{\partial T}{\partial z}} \right) = k \left(\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \cancel{\frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2}} + \cancel{\frac{\partial^2 T}{\partial z^2}} \right) + S$$

LONG

θ symmetry

Microscopic energy balance, constant thermal conductivity; spherical coordinates

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} + \frac{v_\phi}{r \sin\theta} \frac{\partial T}{\partial \phi} \right)$$
$$= k \left(\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial \theta} \left(\sin\theta \frac{\partial T}{\partial \theta} \right) + \frac{1}{r^2 \sin^2\theta} \frac{\partial^2 T}{\partial \phi^2} \right) + S$$

Reference: F. A. Morrison, "Web Appendix to *An Introduction to Fluid Mechanics*," Cambridge University Press, New York, 2013. On the web at www.chem.mtu.edu/~fmorriso/IFM_WebAppendixCD2013.pdf

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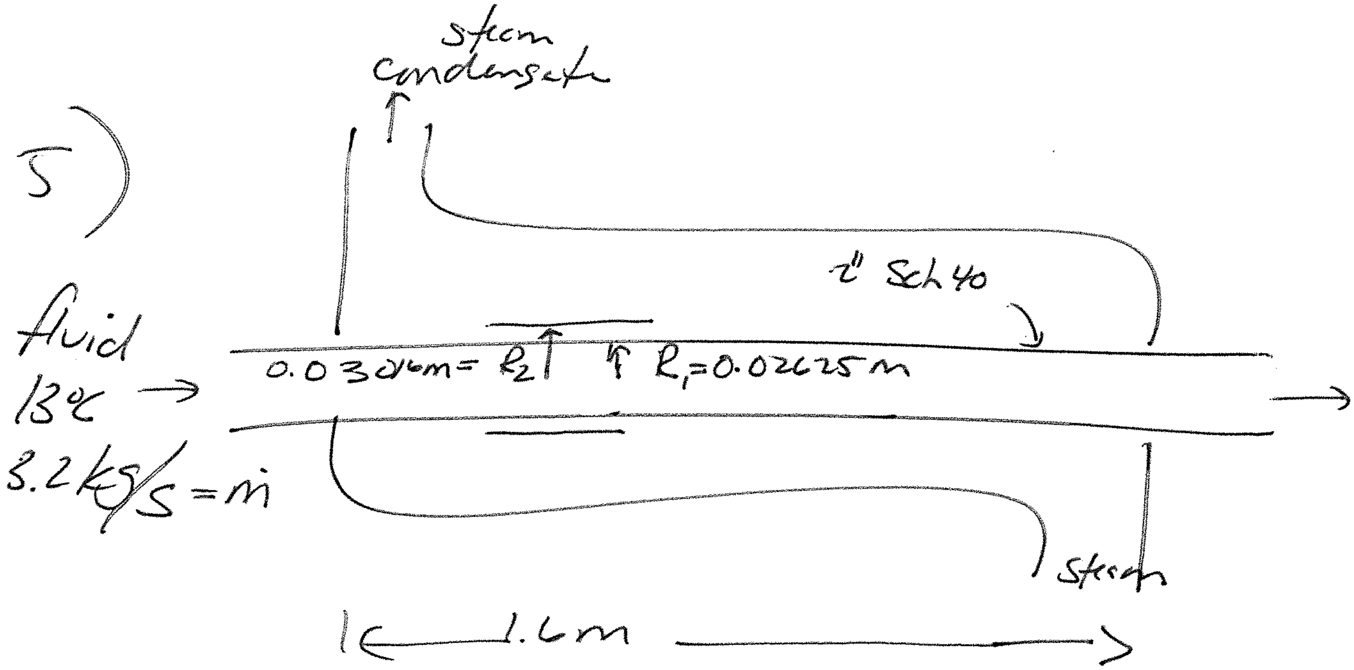
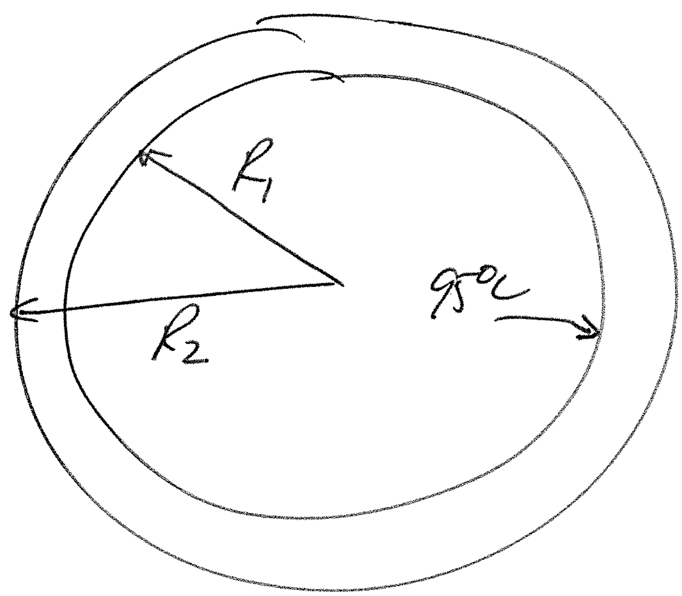




Need to obtain h_{lm} from the literature data correlations (Sieder–Tate)

Turbulent

$$Nu = \frac{h_{lm} D}{K} = 0.027\, Re^{0.8}\, Pr^{\frac{1}{3}} \underbrace{\left(\frac{\mu_b}{\mu_w}\right)^{0.14}}_{1}$$

(need to check Re)

$$Re = \frac{\rho V D}{\mu} \quad \text{need } \langle V \rangle$$

$$\langle V \rangle = \frac{Q}{\pi R^2}$$

$$Q = \frac{\dot{m}}{\rho} = \left(\frac{3.2\,\cancel{kg}}{s}\right)\left(\frac{m^3}{1022\,\cancel{kg}}\right)$$

$$= 0.00313112\ \frac{m^3}{s}$$

$$= 3.13 \times 10^{-3}\ \frac{m^3}{s}$$

$$\langle V \rangle = \frac{\left(0.00313112\ \frac{m^{\cancel{3}}}{s}\right)}{\pi\ (0.02625 m)^2}$$

$$\boxed{\langle V \rangle = 1.4464\ \frac{m}{s}}$$

$$Re = \frac{\left(1022\ \frac{\cancel{kg}}{\cancel{m^3}}\right)\left(1.4464\ \frac{\cancel{m}}{\cancel{s}}\right)(2)\left(0.02625\ \cancel{m}\right)}{\left(8.3 \times 10^{-4}\ \frac{\cancel{kg}}{\cancel{m}\,\cancel{s}}\right)}$$

$$\boxed{Re = 93{,}502} \Rightarrow \text{turbulent}$$

$$Pr = \frac{C_p \mu}{k}$$

$$= \frac{\left(4.3 \frac{\cancel{kJ}}{\cancel{kg}\,\cancel{K}}\right)\left(8.3\times10^{-4} \frac{\cancel{kg}}{m\cancel{s}}\right)\left(\frac{10^3 \cancel{J}}{\cancel{kJ}}\right)}{0.605 \frac{\cancel{J}}{\cancel{s}\,m\cancel{K}}}$$

$$\boxed{Pr = 5.899}$$

$$\frac{\mu_b}{\mu_w} = 1 \quad (\mu \text{ does not vary w/ } T)$$

$$Nu_{lm} = 0.027\, Re^{0.8}\, Pr^{\frac{1}{3}}$$

$$= (0.027)(93502)^{0.8}(5.899)^{\frac{1}{3}}$$

$$\boxed{Nu_{lm} = 462.324}$$

$$Nu = \frac{h_{lm} D}{k}$$

$$h_{lm} = \frac{k}{D} Nu$$

$$= \frac{\left(0.605 \frac{W}{mK}\right)(462.324)}{(2)(0.02625 m)}$$

$$= 5327.73 \ W/m^2K$$

$$\boxed{h_{lm} = 5300 \frac{W}{m^2K}}$$ no more than 2 sig figs

b) (BONUS)

MICRO E-BAL INSIDE:

$$\underbrace{\cancel{\Delta E_p} + \cancel{\Delta E_k}}_{\text{negligible}} + \Delta H = Q_{in} + \underbrace{\cancel{W_{s,on}}}_{\text{no shaft}}$$

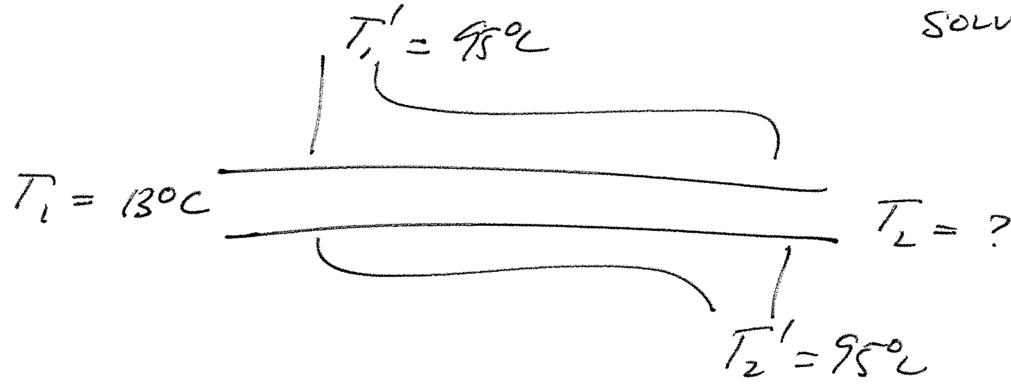
$$Q_{in} = \sum_{in} \dot{m}_i \hat{H}_i - \sum_{out} \dot{m}_i \hat{H}_i$$

$$= \dot{m} \, C_p \, (T_{out} - T_{in})$$

From part a) $\qquad$ MACRO E-BAL

$$Q_{in} = \overbrace{h_{lm} A \Delta T_{lm}} = \overbrace{\dot{m} C_p (T_2 - T_1)}$$

SOLVE FOR T_2



$$A = \pi D L = \pi (2)(0.02625\,m)(1.6\,m)$$

$$= 0.26389\ m^2$$

$$\Delta T_{lm} = \frac{82 - (95 - T_2)}{\ln\left(\frac{82}{95 - T_2}\right)} = \underbrace{\frac{\dot{m} C_p}{h_{lm} A}} (T_2 - 13°C)$$

$$\alpha \equiv \frac{\dot{m} C_p}{h_{lm} A} = \frac{\left(3.2 \frac{\cancel{kg}}{s}\right)\left(4.3 \frac{\cancel{kJ}}{\cancel{kg}\cancel{K}}\right)\left(\frac{10^3 \cancel{J}}{\cancel{kJ}}\right)}{\left(5327.73 \frac{\cancel{W}}{\cancel{m^2}\cancel{K}}\right)\left(0.26389\ \cancel{m^2}\right)}$$

$$\alpha = 9.78708$$

Algebra:

$$\frac{\cancel{(T_2 - 13)}}{\alpha\,\cancel{(T_2 - 13)}} = \ln\left(\frac{82}{95 - T_2}\right)$$

$$\frac{82}{95 - T_2} = e^{1/\alpha} = 1.1075778$$

$$95 - T_2 = 74.035$$

$$T_2 = 20.96\,^\circ C$$

$$\boxed{T_2 = 21\,^\circ C}$$