MODULE 4 SPRING CM3110 2021 ①

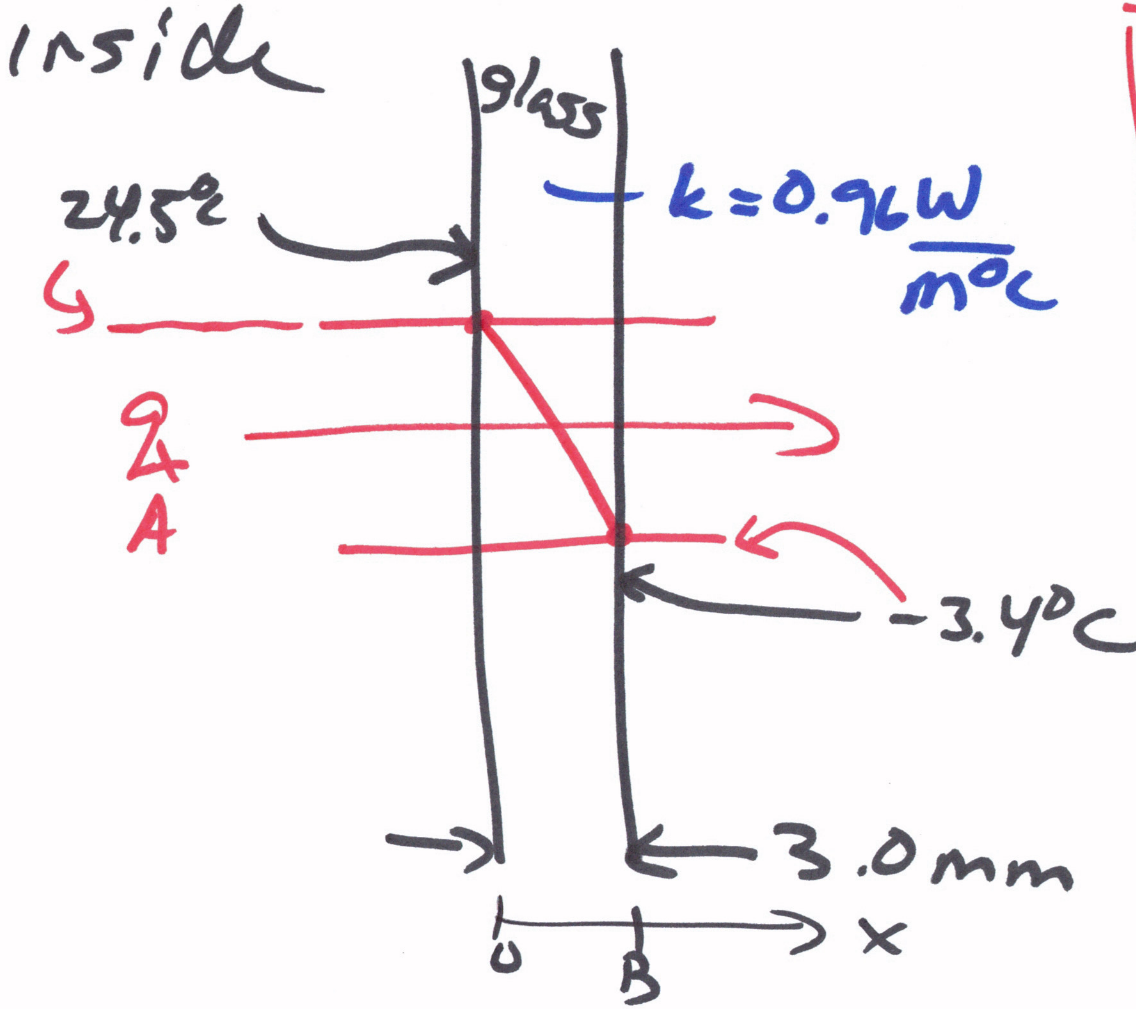
30MAR21 F. MORRISON Michigan Tech

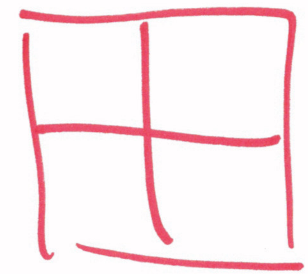
Requests for today?

- HW4 1a (23MAR)
- HW4 1b } 25 MAR)
- HW4 1c
- HW4 2 } 30 MAR
- HW4 4

MOD 4 #3 CM3110

4.2





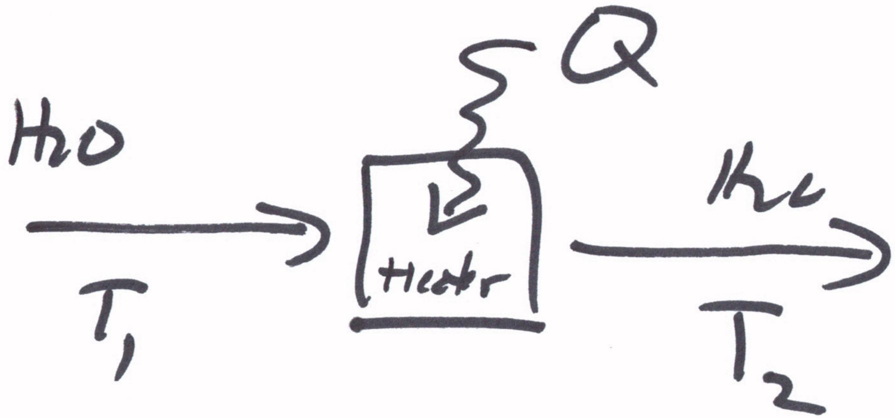
related to the issue of $T(x)$

$B = 3.0\,mm$

What is the rate of heat loss through the glass?

$$\tilde{q} = \frac{q_x}{A} = \text{FLUX} = \frac{J}{m^2\,s} \qquad W = J/s$$

Can we use our old tools?



STEADY OPEN SYSTEM MACROSCOPIC ENERGY BAL

$$\cancel{\Delta E_p} + \cancel{\Delta E_k} + \Delta H = Q_{in} + \cancel{W_{s,on}}$$

$$Q_{in} = \Delta H = \int_{T_1}^{T_2} C_p \, dT$$

$$= C_p (T_2 - T_1)$$

<u>No</u>. These tell us about capacity not rate.

③ $\frac{1}{2}$

1D rectangular

$$\frac{q}{A} = \text{const}$$

$$T = c_1 x + c_2$$

derive from micro E-bal

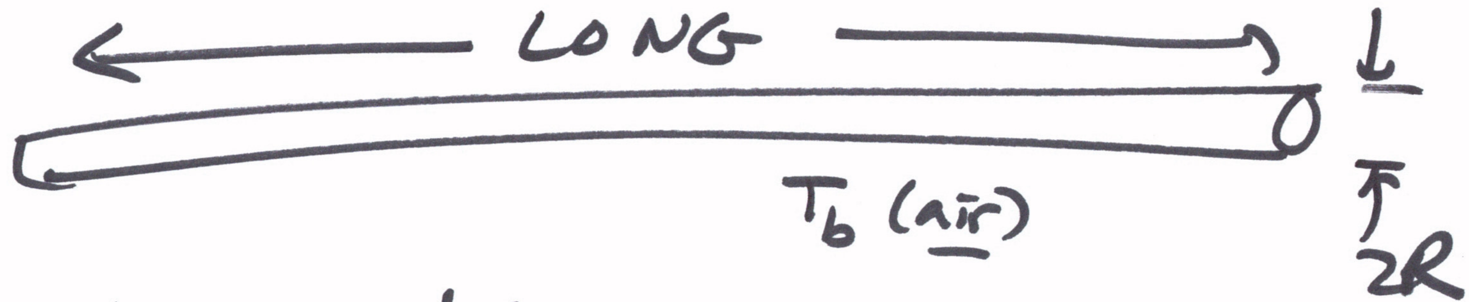
$$\frac{dT}{dx} = c_1 = \frac{-3.4°C - 24.5°C}{3\text{ mm}}$$

Fourier's Law

$$\frac{q_x}{A} = -k \frac{dT}{dx} = \boxed{8.9 \frac{kW}{m^2}}$$

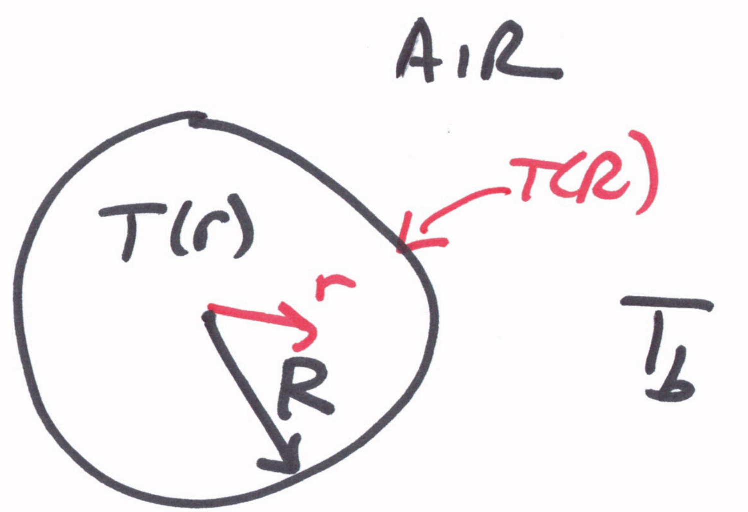
internet

4.4



- heat generated uniformly at $S_e \left(\frac{W}{m^3}\right)$
- steady

find $T(r, \theta, z)$



Newton's Law of cooling

$$\left|\left.\frac{q}{A}\right|_{wall}\right| = h\left|T_w - T_b\right|$$

$0 \leq r \leq R$

5

The **Equation of Energy** for systems with **constant k**

Microscopic energy balance, constant thermal conductivity; Gibbs notation

STEP 1

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + \underline{v} \cdot \nabla T \right) = k \nabla^2 T + S_e$$

Microscopic energy balance, constant thermal conductivity; Cartesian coordinates

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} + v_z \frac{\partial T}{\partial z} \right) = k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + S_e$$

$\underline{v} = 0$

in the domain
elec current
chem rxn?

Microscopic energy balance, constant thermal conductivity; cylindrical coordinates

$$\rho \hat{C}_p \left(\cancel{\frac{\partial T}{\partial t}} + \cancel{v_r \frac{\partial T}{\partial r}} + \cancel{\frac{v_\theta}{r} \frac{\partial T}{\partial \theta}} + \cancel{v_z \frac{\partial T}{\partial z}} \right) = k \left(\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \cancel{\frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2}} + \cancel{\frac{\partial^2 T}{\partial z^2}} \right) + S_e$$

Microscopic energy balance, constant thermal conductivity; spherical coordinates

steady

0

long

symmetry

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} + \frac{v_\phi}{r \sin \theta} \frac{\partial T}{\partial \phi} \right)$$
$$= k \left(\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial T}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 T}{\partial \phi^2} \right) + S_e$$

$$k \left(\frac{1}{r} \frac{d}{dr} \left(r \frac{dT}{dr} \right) \right) + S_e = 0$$

Reference: F. A. Morrison, "Web Appendix to *An Introduction to Fluid Mechanics*," Cambridge University Press, New York, 2013 (*pages.mtu.edu/~fmorriso/cm310/IFMWebAppendixDMicroEBalanceMorrison.pdf*). This worksheet is on the web at *pages.mtu.edu/~fmorriso/cm310/energy.pdf*.

⑥

$$k \frac{1}{r} \frac{d}{dr}\left(r \frac{dT}{dT}\right) = -S_e$$

$$\frac{d}{dr}\underbrace{\left(r \frac{dT}{dr}\right)}_{=\Phi} = \left(\frac{-S_e}{k}\right) r$$

$$\frac{d\Phi}{dr} = \left(-\frac{S_e}{k}\right) r$$

$$r \frac{dT}{dr} = \Phi = \left(-\frac{S_e}{k}\right) \frac{r^2}{2} + \cancel{c_1}$$

BC
$r=0$
$\frac{dT}{dr}=0$

$$\frac{dT}{dr} = \left(\frac{-S_e}{2k}\right) r + \cancel{\frac{c_1}{r}}$$

$$\boxed{T = \left(\frac{-S_c}{2k}\right)\frac{r^2}{2} + \cancel{c_1 \ln r} + C_2}$$

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BC:

$r = R$

$$\cancel{\frac{q}{A}} = \underbrace{\frac{q_r}{A}}_{\text{positive}} = h\left(\underbrace{T(R) - T_b}_{\text{positive}}\right)$$

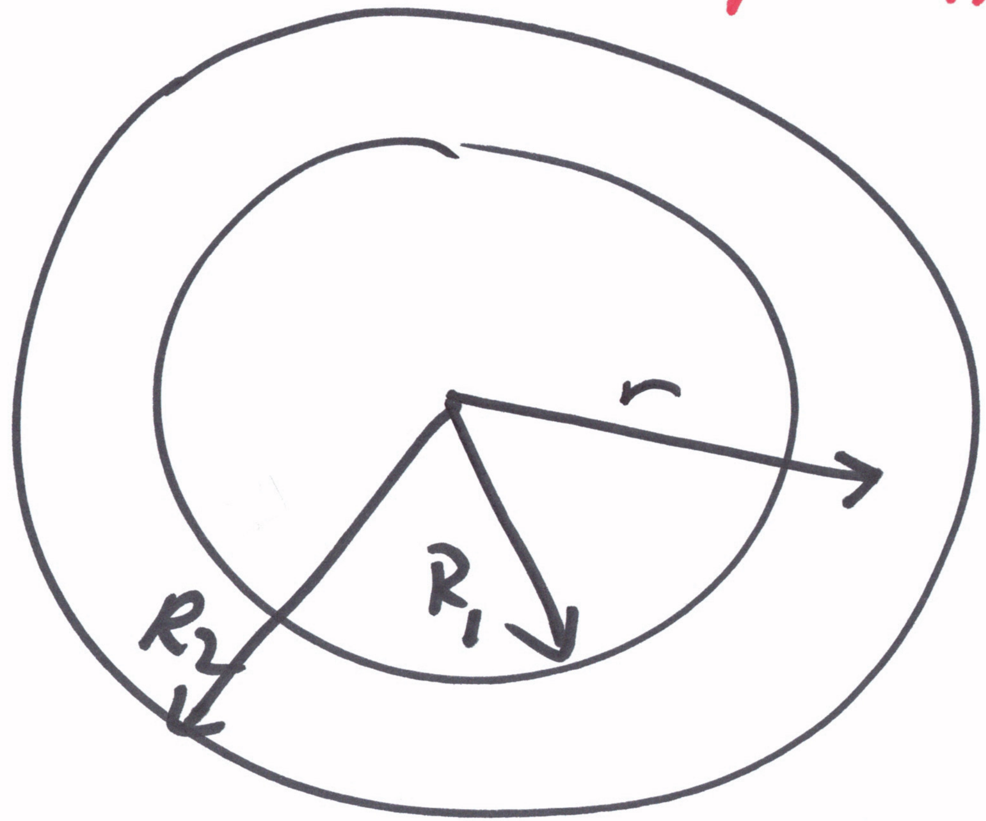
$$-\frac{S_c}{2k}R + \cancel{\frac{c_1}{R}}$$

$r = 0$ $\quad \frac{dT}{dr} = 0$ (Temp goes thru max)

$\Rightarrow c_1 = 0$

note: $r=0$ is not ALWAYS a problem: $7\frac{1}{2}$

Pipe



$R_1 \le r \le R_2$

$r=0$ is not in the domain!

Solu

$$T = \left(-\frac{S_e}{4k}\right) r^2 + C_2$$

$$-\frac{S_e R}{2k} = h\left(\left(-\frac{S_e}{4k} R^2 + C_2\right) - T_b\right)$$

check pls

$$\underbrace{\frac{1}{h}\left(\left(-\frac{S_e R}{2k}\right) + T_b\right)} = -\frac{S_e}{4k} R^2 + C_2$$

$$\boxed{C_2 = (\qquad\qquad) + \frac{S_e}{4k} R^2}$$

$\therefore$

Answer: $$\boxed{(T - T_b) = \frac{S_e}{4k}(R^2 - r^2) + \frac{S_e R}{2h}}$$

What is flux?

Fourier's LAW

(see next pg)

$$\left.\frac{q_r}{A}\right|_{r=R} = -k \underbrace{\left.\frac{dT}{dr}\right|_{r=R}}_{-\frac{S_e}{2k}R}$$

$$\frac{q_r}{A} = +\cancel{k}\frac{S_e}{2\cancel{k}}R = \boxed{\frac{S_e}{2}R}$$

10

The **Equation of Energy** in Cartesian, cylindrical, and spherical coordinates for Newtonian fluids of constant density, with source term S_e. Source could be electrical energy due to current flow, chemical energy, etc. Two cases are presented: the general case where thermal conductivity may be a function of temperature (vector flux $\underline{\tilde{q}} = \underline{q}/area$ appears in the equations); and the more usual case, where thermal conductivity is constant (on the reverse).

Spring 2020 Faith A. Morrison, Michigan Technological University

Microscopic energy balance, in terms of flux; Gibbs notation

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + \underline{v} \cdot \nabla T \right) = -\nabla \cdot \underline{\tilde{q}} + S_e$$

Microscopic energy balance, in terms of flux; Cartesian coordinates

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} + v_z \frac{\partial T}{\partial z} \right) = -\left(\frac{\partial \tilde{q}_x}{\partial x} + \frac{\partial \tilde{q}_y}{\partial y} + \frac{\partial \tilde{q}_z}{\partial z} \right) + S_e$$

Microscopic energy balance, in terms of flux; cylindrical coordinates

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} + v_z \frac{\partial T}{\partial z} \right) = -\left(\frac{1}{r} \frac{\partial (r \tilde{q}_r)}{\partial r} + \frac{1}{r} \frac{\partial \tilde{q}_\theta}{\partial \theta} + \frac{\partial \tilde{q}_z}{\partial z} \right) + S_e$$

Microscopic energy balance, in terms of flux; spherical coordinates

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} + \frac{v_\phi}{r \sin\theta} \frac{\partial T}{\partial \phi} \right) = -\left(\frac{1}{r^2} \frac{\partial (r^2 \tilde{q}_r)}{\partial r} + \frac{1}{r \sin\theta} \frac{\partial (\tilde{q}_\theta \sin\theta)}{\partial \theta} + \frac{1}{r \sin\theta} \frac{\partial \tilde{q}_\phi}{\partial \phi} \right) + S_e$$

step 2

Fourier's law of heat conduction, Gibbs notation: $\underline{\tilde{q}} = \underline{q}/A = -k\nabla T$

Fourier's law of heat conduction, Cartesian coordinates: (constant thermal conductivity k)

$$\begin{pmatrix} \tilde{q}_x \\ \tilde{q}_y \\ \tilde{q}_z \end{pmatrix}_{xyz} = \begin{pmatrix} q_x/A \\ q_y/A \\ q_z/A \end{pmatrix}_{xyz} = \begin{pmatrix} -k\frac{\partial T}{\partial x} \\ -k\frac{\partial T}{\partial y} \\ -k\frac{\partial T}{\partial z} \end{pmatrix}_{xyz}$$

$$\frac{q_r}{A} = -k\frac{dT}{dr}$$

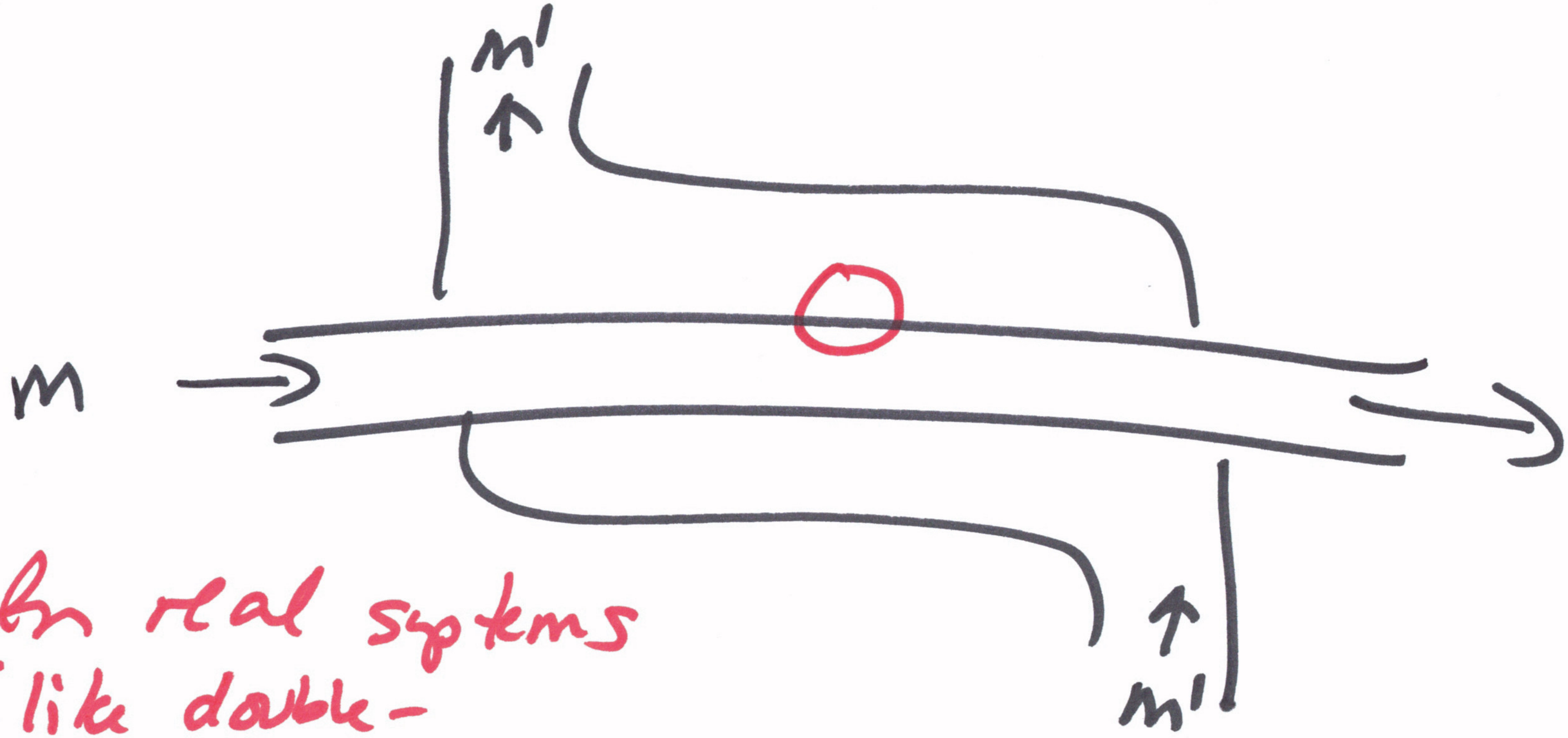
Fourier's law of heat conduction, cylindrical coordinates: (constant thermal conductivity k)

$$\begin{pmatrix} \tilde{q}_r \\ \tilde{q}_\theta \\ \tilde{q}_z \end{pmatrix}_{r\theta z} = \begin{pmatrix} q_r/A \\ q_\theta/A \\ q_z/A \end{pmatrix}_{r\theta z} = \begin{pmatrix} -k\frac{\partial T}{\partial r} \\ -\frac{k}{r}\frac{\partial T}{\partial \theta} \\ -k\frac{\partial T}{\partial z} \end{pmatrix}_{r\theta z}$$

Fourier's law of heat conduction, spherical coordinates: (constant thermal conductivity k)

$$\begin{pmatrix} \tilde{q}_r \\ \tilde{q}_\theta \\ \tilde{q}_\phi \end{pmatrix}_{r\theta\phi} = \begin{pmatrix} q_r/A \\ q_\theta/A \\ q_\phi/A \end{pmatrix}_{r\theta\phi} = \begin{pmatrix} -k\frac{\partial T}{\partial r} \\ -\frac{k}{r}\frac{\partial T}{\partial \theta} \\ -\frac{k}{r\sin\theta}\frac{\partial T}{\partial \phi} \end{pmatrix}_{r\theta\phi}$$

Why does h exist? Why invented? (11)



In real systems (like double-pipe heat exchangers) we have liquids. Let's see the effect. $\Rightarrow$

What about the $\underline{v} \neq 0$ case? 12
e.g. pipe flow

The **Equation of Energy** for systems with **constant k**

LIQUID in Pipe

Microscopic energy balance, constant thermal conductivity; Gibbs notation

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + \underline{v} \cdot \nabla T \right) = k \nabla^2 T + S_e$$

Microscopic energy balance, constant thermal conductivity; Cartesian coordinates

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} + v_z \frac{\partial T}{\partial z} \right) = k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + S_e$$

Microscopic energy balance, constant thermal conductivity; cylindrical coordinates

$$\rho \hat{C}_p \left(\cancel{\frac{\partial T}{\partial t}} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} + v_z \frac{\partial T}{\partial z} \right) = k \left(\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \cancel{\frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2}} + \cancel{\frac{\partial^2 T}{\partial z^2}} \right) + \cancel{S_e}$$

no rxn
no elec

Microscopic energy balance, constant thermal conductivity; spherical coordinates

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} + \frac{v_\phi}{r \sin\theta} \frac{\partial T}{\partial \phi} \right)$$
$$= k \left(\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial \theta} \left(\sin\theta \frac{\partial T}{\partial \theta} \right) + \frac{1}{r^2 \sin^2\theta} \frac{\partial^2 T}{\partial \phi^2} \right) + S_e$$

steady

θ symmetry

long

laminar flow:

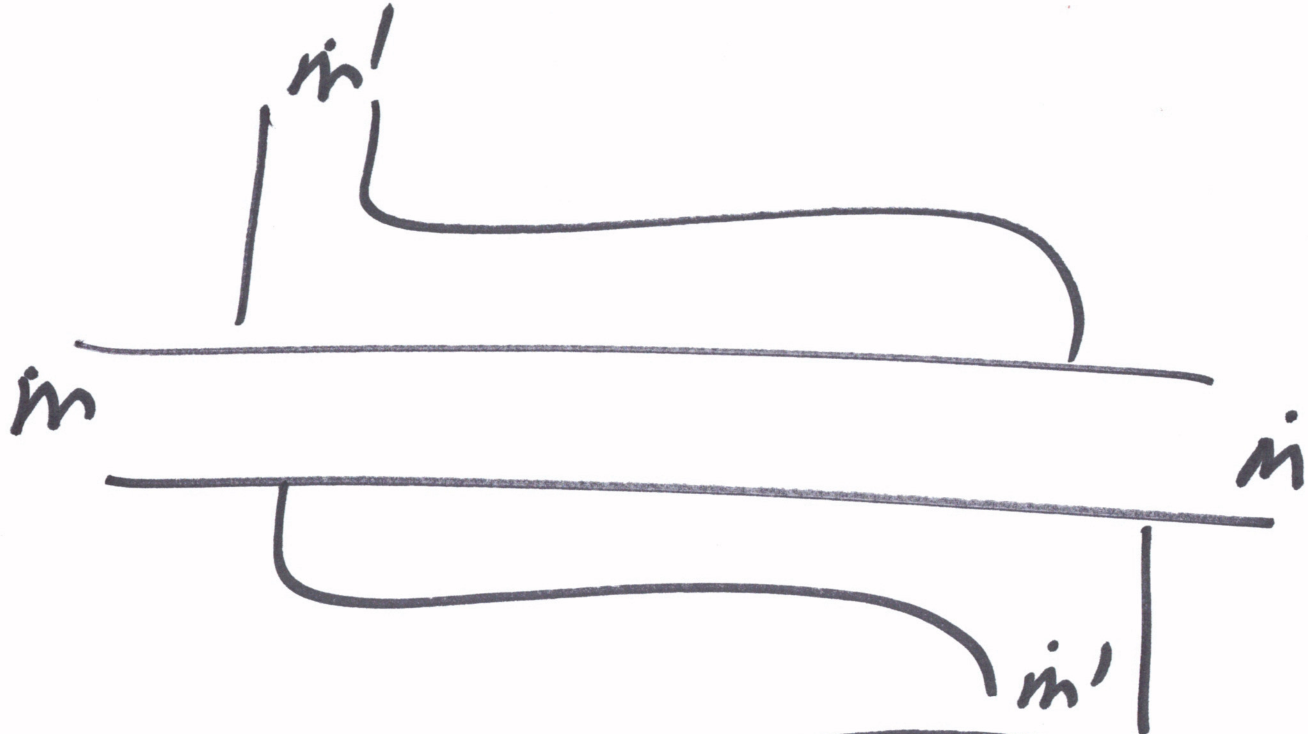
$$\underline{v} = \begin{pmatrix} 0 \\ 0 \\ v_z \end{pmatrix}_{r\theta z}$$

$$\frac{dT}{dz} = 0$$
(long)

turbulent flow

$$\underline{v} = \underbrace{\begin{pmatrix} v_r \\ v_\theta \\ v_z \end{pmatrix}_{r\theta z}}_{\text{defeats slash + burn!!}}$$

Reference: F. A. Morrison, "Web Appendix to *An Introduction to Fluid Mechanics*," Cambridge University Press, New York, 2013 (*pages.mtu.edu/~fmorriso/cm310/IFMWebAppendixDMicroEBalanceMorrison.pdf*). This worksheet is on the web at *pages.mtu.edu/~fmorriso/cm310/energy.pdf*.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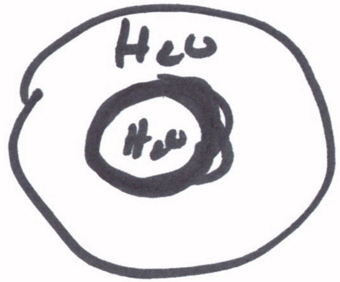

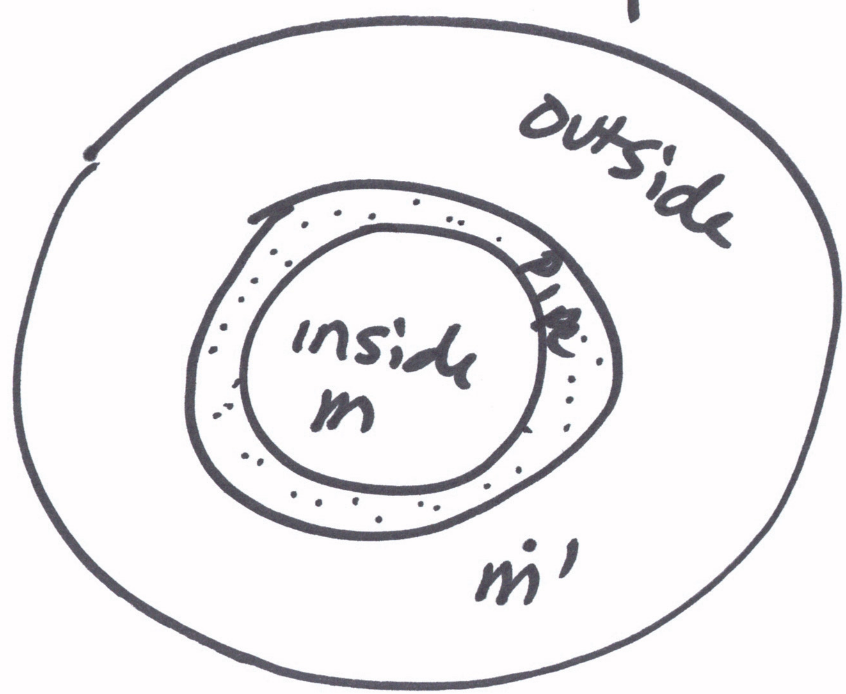
outside
pipe
inside
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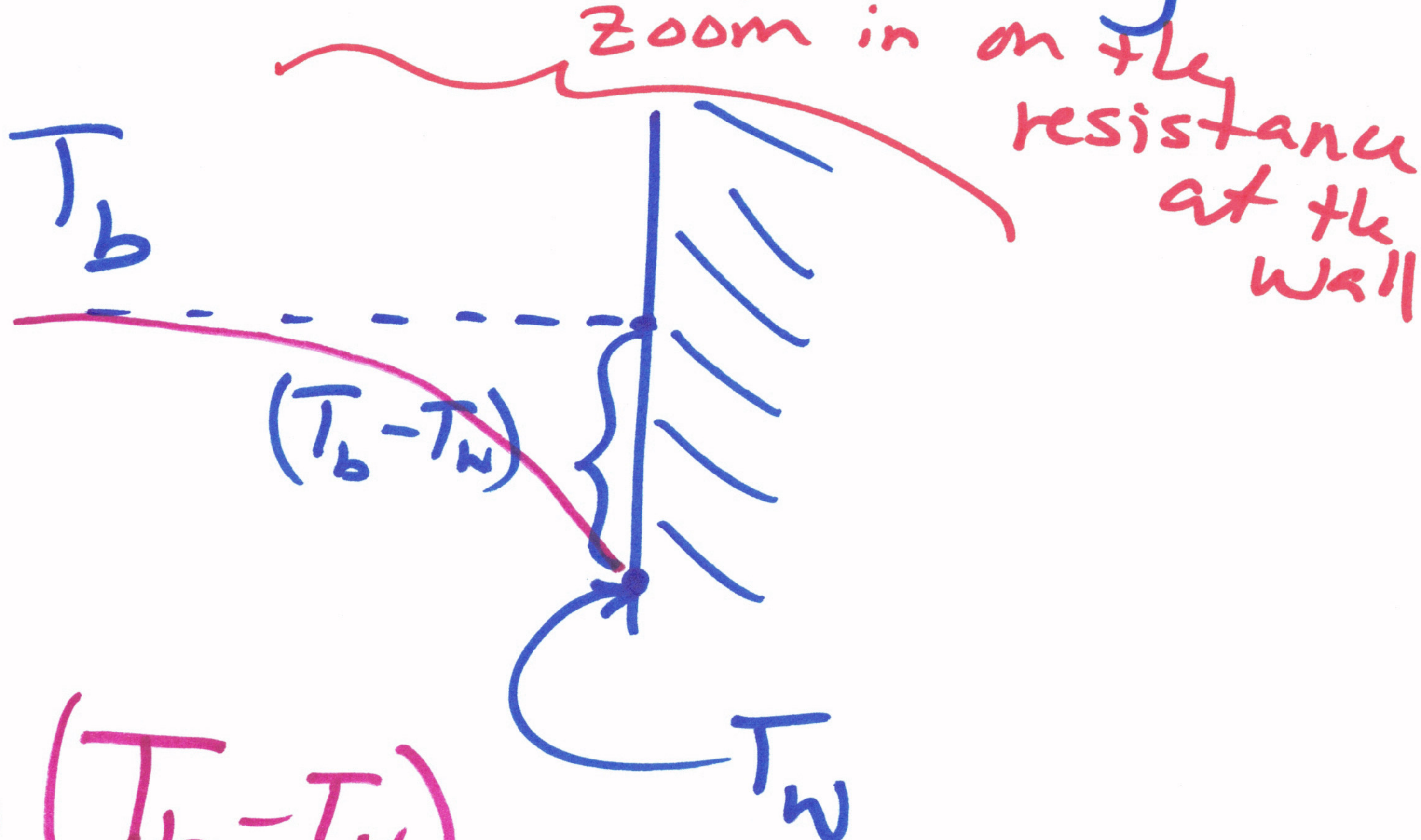
Engineering soln:

Newton's Law of cooling



Define:

$$\frac{q_x}{A} = h\,(T_b - T_w)$$

then take a TON of data

$$\frac{q_x/A}{T_b - T_w} = h$$

tabulate

group by the physics:

FORCED CONVECTION IN PIPES

$$Re = \frac{\rho V D}{\mu}$$

$$Pr = \frac{C_p \mu}{k}$$

organize the data

do dimensional analyses

$$Nu = \frac{hD}{k}$$

D — characteristic lenthscale (pipe inner diameter)

$$Nu = f(Re, Pr)$$

See EXAM 4 Handout!

- take data
- $\underbrace{\text{fit to a function}}_{\text{data correlation}}$ //