The sand pack is 38 mm in diameter and 16 mm in depth, and the mass flow rate is $5 \times 10^{-4}$ kg/s. Estimate the pressure drop through the pack if the particles have a mean diameter of 0.7 mm and $\epsilon = 0.38$.

$$v_\infty = \frac{4(5 \times 10^{-4}\ \text{kg/s})/(1300\ \text{kg/m}^3)}{\pi(38 \times 10^{-3}\ \text{m})^2} = 3.4 \times 10^{-4}\ \text{m/s}$$

$\text{Re}_p$ will be much less than 10, so the last term in Eq. (4.51) can be neglected, and we have

$$|\Delta p| = \frac{150 L v_\infty \eta (1-\epsilon)^2}{D_p^2 \epsilon^3} = \frac{150(16 \times 10^{-3}\ \text{m})(3.4 \times 10^{-4}\ \text{m/s})(600\ \text{Pa}\cdot\text{s})(1-0.38)^2}{(0.7 \times 10^{-3}\ \text{m})^2(0.38)^3}$$

$$= 7 \times 10^6\ \text{Pa}$$

### 4.4.3 Fluidized Beds

Fluidized bed reactors are used quite commonly in a number of processing applications. In a fluidized bed the solid particles move about chaotically in the gas (or liquid) stream. This includes a substantial amount of mixing and particle–particle and particle–wall contact. A fluidized bed is thus an efficient device for heat transfer, and reasonable temperature uniformity can be maintained. This is important in highly exothermic chemical reactions.

The manner in which a bed becomes fluidized can be understood by reference to Figs. 4-12 and 4-13. Gas or liquid is passed upward through a bed of solids at ever-increasing superficial velocity, $v_\infty$. The pressure drop across the bed, $|\Delta p|$, is given by Eq. (4.49). For simplicity we assume that

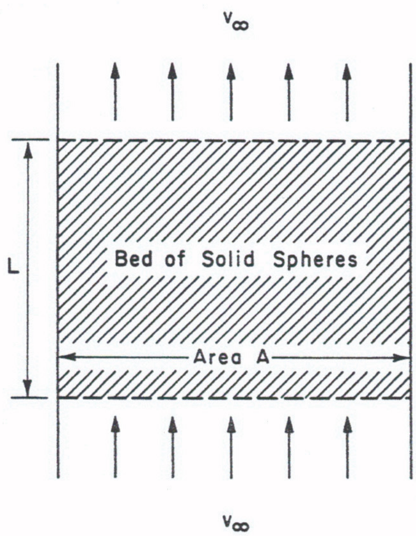


**Figure 4-12.** Schematic of a fluidized bed.

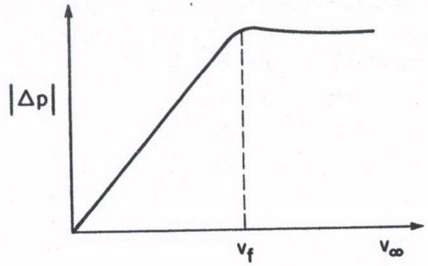


**Figure 4-13.** Typical curve of pressure drop as a function of superficial velocity for upward flow through a granular bed. $v_f$ is the point of incipient fluidization.

$Re_p \leq 10$, which is generally true, in which case we can use the Ergun equation in the form

$$Re_p \leq 10: \quad \frac{|\Delta p|}{L} = \frac{150 v_\infty \eta (1-\epsilon)^2}{D_p^2 \epsilon^3} \tag{4.52}$$

The pressure drop thus increases linearly with $v_\infty$.

Now there is a net upward force on the bed of particles equal to $|\Delta p| A$. The volume of solid particles is $(1-\epsilon)AL$, so the net gravitational and buoyant force on the solid particles is equal to $(1-\epsilon)(\rho_p - \rho)ALg$. When these two forces are equal there is no net force on the solid particles; they are in a state of "weightlessness" and are free to move about unhindered by gravity. Thus, there will be no further increase in $|\Delta p|$ as $v_\infty$ is increased. Rather, the bed will tend to expand, or become fluidized, and $|\Delta p|$ will level off.

We can calculate the minimum superficial velocity, $v_f$, at which the bed becomes fluidized by equating the two forces:

$$|\Delta p| A = (1-\epsilon)(\rho_p - \rho)ALg \tag{4.53}$$

From Eq. (4.52) we can eliminate $|\Delta p|$ and solve for $v_f$,

$$v_f = \frac{(\rho_p - \rho) g D_p^2 \epsilon^3}{150\eta(1-\epsilon)} \tag{4.54}$$

This is known as the point of *incipient fluidization*. The void fraction at incipient fluidization is a function of the material and the particle size. If data are not available, then $\epsilon^3/(1-\epsilon)$ may be roughly approximated by 0.091.

There is also an upper limiting velocity at which a fluidized bed can be operated. Individual particles will be carried up in the gas stream. Unless they settle back into the bed at a faster speed than the upward speed of the gas stream, the particles will have a net upward velocity and will be carried out of the bed. Thus, the maximum superficial gas velocity without particle entrainment, $v_{max}$, is equal to the settling velocity for a sphere. If we assume



Stokes' flow for simplicity, the maximum velocity is given by Eq. (4.11),

$$v_{max} = \frac{(\rho_p - \rho) g D_p^2}{18\eta} \tag{4.55}$$

It is readily established that $v_{max}/v_f > 1$ for all physically possible values of $\epsilon$.

The behavior of fluidized beds beyond the point of fluidization is quite complex. The bed no longer remains homogeneous, and "bubbles" of particle-free gas move through the system. For analytical purposes in reaction engineering it is often sufficient to treat the fluidized bed as a well-stirred tank containing a single homogeneous fluid phase.

***Example 4.7***

Pulverized coal is to be burned at atmospheric pressure in a fluidized bed. The density of the coal is approximately 1000 kg/m³. The mean particle diameter is 0.074 mm and the gas, mostly air, has a viscosity $\eta = 10^{-4}$ Pa·s. Estimate the minimum fluidization velocity.

The void fraction is not given, so we use the approximation $\epsilon^3/(1-\epsilon) = 0.091$. The gas density can be neglected. In that case, Eq. (4.54) is

$$v_f = \frac{(1000)(9.8)(7.4 \times 10^{-5})^2(0.091)}{(150)(10^{-4})} = 3.2 \times 10^{-4} \text{ m/s}$$

The entrainment velocity, from Eq. (4.55), is

$$v_{max} = \frac{(1000)(9.8)(7.4 \times 10^{-5})^2}{(18)(10^{-4})} = 3.0 \times 10^{-2} \text{ m/s}$$

The Reynolds numbers for the packed bed at the point of incipient fluidization and the free entrained particles are both less than unity.

## 4.5 CONCLUDING REMARKS

This concludes the introduction to the use of dimensional analysis and experimentation for the solution of engineering problems. The presentation should suggest the broad applicability of the methods to problems far beyond the scope of this chapter and the preceding one.

The drag coefficient is a quantity that appears frequently in process applications, and its definition should be committed to memory. So, too, should Stokes' law in any of its forms, and the limiting value $C_D = 0.44$ for a sphere, for it is important to be able to estimate orders of magnitude rapidly.

We turn next to the use of more fundamental principles in order to obtain quantitative descriptions that include a better understanding of the underlying physical processes. In later sections of the book we will return to some of the problems considered in this chapter and the preceding one, and we will show how certain of the results shown here as the outcome of experiments are, in fact, obtainable directly from first principles.

# *Process Fluid Mechanics*

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