pressure leads to governing equations at zero Reynolds number that correctly include the pressure effects. The inertia-based P is more meaningful where inertia dominates; choosing the inertia-based characteristic pressure leads to governing equations at an infinite Reynolds number that correctly include the pressure effects. It appears that if we want to simplify the governing equations, it matters what we choose for characteristic values—if we make the wrong choice, we are led to the wrong simplified equations.

In addition to the confusing issue of which characteristic pressure to choose, the conundrum of potential flow remains. Dimensional analysis, even with the correct characteristic pressure chosen, leads in the high-Reynolds-number limit to results that simply do not match what is observed. It appears that dimensional analysis has failed for the case of rapid flow around a sphere: It has not led to simplified equations that predict the rich flow behavior observed (see Figure 8.22).

The failure of dimensional analysis in the case of rapid flow around a sphere is due to the choice of the sphere diameter D for the lengthscale for nondimensionalization [85]. In the boundary layer, the lengthscale over which the velocity changes is not the large lengthscale D but rather the much smaller lengthscale δ (see Example 8.22). Thus, the flow around a sphere has the property that the characteristic dimensions over which properties change are different for different regions of the flow. The choice of D as the single dimension over which to nondimensionalize leads to the difficulties experienced with the potential-flow solution [85]. When we recognize that a problem has regimes with different characteristic lengths, we can build our solution methods around the correct lengthscales. This is a technique of advanced fluid mechanics (i.e., matched asyptotic expansion). For an indepth treatment of scaling issues in fluid mechanics, see Leal [85]; see also Problem 57.

This concludes our discussion of the continuum model. The continuum model is a successful model of fluid behavior. For simple flows, with the help of calculus, we solve for the velocity and stress fields. For complex flows, with the help of dimensional analysis and advanced methods (Chapters 7, 8, and [85]), we also solve for the velocity and stress fields. In this text, we have seen how to calculate flow quantities of interest from the velocity and stress fields. In the remaining chapters of this book, we explore the origins of the macroscopic balance equations and apply these balances to more complex situations (Chapter 9) and we revisit our Chapter 2 tour of fluid behavior and see how much of that behavior is now within our modeling means.

8.4 Problems

1. The classic internal flow is pipe flow; the classic external flow is uniform flow past a sphere. Using these two examples, compare and contrast internal and external flows.
2. Compare and contrast the Fanning friction factor and the drag coefficient. What is the purpose of each?
3. Why does a skydiver reach terminal velocity? Why does the skydiver not accelerate continuously as she falls?

4. Spherical coordinates are used to solve for the velocity profile in a flow. The result is given here. Convert $\underline{v}$ from spherical coordinates to Cartesian coordinates.

$$\underline{v} = \begin{pmatrix} \left[a + b\frac{1}{r} + c\frac{1}{r^3}\right]\cos\theta \\ -\left[a + \frac{b}{2}\frac{1}{r} - \frac{c}{2}\frac{1}{r^3}\right]\sin\theta \\ 0 \end{pmatrix}_{r\theta\phi}$$

5. In Example 8.2, gravity is given by $\underline{g} = -g\hat{e}_z$. Using Equations 1.271–1.273, calculate this $\underline{g}$ in the spherical coordinate system (the answer is given in Equation 8.13).
6. From intuition, sketch the velocity field for flow around a sphere at modest flow rates. Make your arrows proportional to what you believe the velocity magnitude should be at each point.
7. In the creeping flow around a sphere problem (see Example 8.1), which terms of the Navier-Stokes equation are neglected? How is this justified?
8. In creeping flow around a sphere, we calculate the final velocity profile beginning with the guess for the velocity components in Equations 8.20 and 8.21. Carry out the detailed calculation of the final velocity profile. [Lengthy]
9. In calculating forces on the sphere in creeping flow around a sphere, we use the fact that $\tau_{rr}|_{r=R}$ at the surface of the sphere is equal to zero. Confirm this.
10. Using plotting software, plot the pressure distribution in creeping flow around a sphere. Comment on the results.
11. For a 1.0-mm-diameter polystyrene bead falling in water, what is the expected terminal speed? Assume creeping flow. What is the Reynolds number of this flow? Would this flow represent creeping flow?
12. For a 1.0-mm-diameter ball made of stainless steel falling in glycerol, what is the expected terminal speed? Assume ~~Stokes~~ creeping flow. Will the ball fall in the creeping flow ~~Stokes~~ regime?
13. What is the largest Reynolds number that we can explore with sphere-dropping experiments? What limits this experimental technique?
14. For stainless-steel spheres of reasonable sizes, in reasonable fluids, what is the minimum fluid viscosity you may use in a terminal velocity experiment? What sizes of steel ball would you use to obtain these measurements of terminal velocity?
15. When we nondimensionalize the Navier-Stokes equation in pipe flow, two dimensionless groups appear: the Reynolds number, Re, and the Froude number, Fr. When the Navier-Stokes equation was nondimensionalized for flow around a sphere, the Froude number did not appear. Explain the difference.
16. The force on a sphere in creeping flow was found to be unidirectional: $\underline{\mathcal{F}}|_{\text{creeping}} = F_z\hat{e}_z$, whereas for noncreeping flow, the force is not unidirectional. Why?
17. What is lift? What are the dimensions of lift?
18. A cricket ball is thrown with an initial speed of 52 mph straight up in the air. How long until the ball hits the ground? With what speed will the ball hit the ground? Do not neglect air resistance.

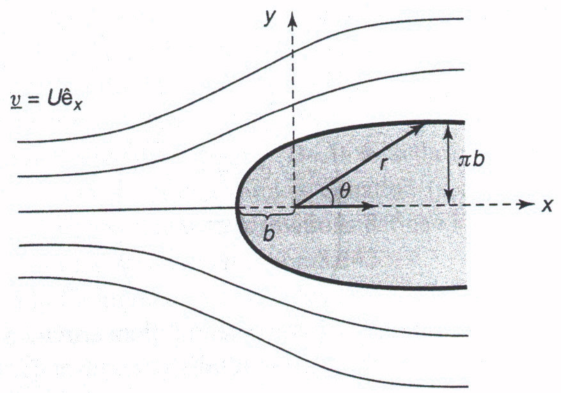

Figure 8.61 A Rankine half-body (Problem 29).

at impact

19. A smooth ball the size of a soccer ball is dropped from a bridge to a river 140 m below. Calculate the speed of the ball both with and without drag. How much error is there in the calculation if air resistance is neglected?
20. A smooth ball 4.0 cm in diameter weighing 0.25 kg is launched at an initial velocity of 40.0 mph at an angle of 34 degrees from the horizontal. What is the speed of the ball as a function of time and how far will the ball go? What is the path traced out by the ball?
21. Calculate the true pressure drag on a cylinder by numerically integrating the experimental pressure data in Figure 8.53.
22. The flow patterns behind a sphere at high Reynolds numbers are shown in Figure 8.22. Compare these flow patterns to what is observed behind a long cylinder. Discuss the comparison.
23. What is the definition and meaning of *stream function*?
24. What are the governing equations for potential flow around a sphere? Where do these equations originate?
25. Using plotting software, plot the pressure distribution in potential flow around a sphere. Comment on the results.
26. What is d'Alembert's paradox? Why is this observation important?
27. For potential flow around a sphere, calculate the pressure distribution. Start from the velocity solution given in Equation 8.238.
28. Demonstrate the error involved when the Bernoulli equation is applied inappropriately by carrying out the following calculation and comparison: Beginning with the correct velocity profile result for creeping flow around a sphere, use the Bernoulli equation (incorrectly) to calculate the pressure profile. Compare the incorrect profile obtained from the Bernoulli equation to the correct pressure profile for creeping flow around a sphere. Comment on your results.
29. The velocity field for uniform upstream flow $\underline{v} = U\hat{e}_x$ flowing in potential flow around an obstacle called a Rankine half-body is sketched in Figure 8.61. The shape of the obstacle follows the equation $r_{\text{body}}(\theta)$ given here. What is the pressure field for this flow? You may neglect gravity. The quantities b and U are constants. Plot the results for a half-body with $b = 1.0$ m and

upstream flow speed $U = 1.0$ m/s.

$$r_{\text{body}}(\theta) = \frac{(\pi - \theta)b}{\sin\theta}$$

$$\underline{v} = \begin{pmatrix} -U\cos\theta - \dfrac{Ub}{r} \\ -U\sin\theta \\ 0 \end{pmatrix}_{r\theta z}$$

30. Calculate the extra-stress tensor $\underline{\underline{\tilde{\tau}}}$ for potential flow around a sphere of an inviscid Newtonian fluid. Calculate the total-stress tensor $\underline{\underline{\tilde{\Pi}}}$. Comment on what is obtained.
31. At first glance, the streamlines for creeping flow and potential flow around a sphere (see Figures 8.9 and 8.24 ~~8.9)~~ seem similar. The arrow plots of the velocity fields for these two flow solutions, however, show the striking differences between the two scenarios (see Figures 8.8 and 8.25). Summarize the differences in velocity fields. Why do the streamline plots look similar? When looking at streamline plots, how can a viewer perceive the differences in flows?
32. Are boundary layers important in both internal and external flows? Explain your answer.
33. What type of forces dominate in the boundary layer? What type of forces dominate outside the boundary layer?
34. To solve the microscopic mass and momentum balances in the boundary layer, we make many assumptions. List the assumptions that go into developing the simplified equations of change for the boundary layer. Comment on each.
35. For a laminar boundary layer on a flat plate, how does the boundary-layer thickness vary with viscosity? How does the thickness vary with distance from the leading edge?
36. The flow in a boundary layer near a flat plate has two components: one that is large (v_1), and one that is much smaller but nonzero (v_2). For several locations x_1, plot $v_2(x_1)$. Comment on your results.
37. The solution for the boundary-layer flow near a flat plate is given by Equation 8.356. Plot the velocity v_1 as a function of the distance away from the plate x_2 for various distances from the leading edge (i.e., various x_1 values).
38. For water flow at 1.5 m/s over a flat plate, at what distance downstream will the boundary-layer thickness be 1.0 inch? Assume laminar boundary layer.
39. A boundary layer is considered thin if $\delta/x < 0.1$. For these conditions, calculate whether the boundary layer is thin for the following system: water flowing over a 1.0-m-long flat plate with a free-stream velocity of 0.010 m/s.
40. What is the force (i.e., drag) on a thin plate given the following conditions? The fluid is water, the plate is 0.52 m long and 6.3 m wide, and the free-stream velocity is 1.3×10^{-2} m/s.
41. What is the thickness of the boundary layer on a golf ball driven from the tee at 145 mph? Assume that the ball is completely smooth and therefore has a laminar boundary layer. For a real golf ball, the dimples on the surface

ensure that the boundary layer is everywhere turbulent. What is the thickness of a turbulent boundary layer under these conditions?

42. For the flow in the boundary layer near a flat plate, derive the third-order, ordinary differential equation that governs the spatial variation of the principal velocity component. Begin with the continuity equation (see Equation 8.340) and the Navier-Stokes equation (see Equation 8.341) and incorporate the coordinate transformations defined in this chapter (see Equation 8.343). The final result is Equation 8.347.
43. Example 8.19 addresses the solution for the velocity field in the problem of boundary-layer flow past a flat plate. To obtain the velocity field, we need the solution to the third-order, nonlinear ODE in Equation 8.347. Solve Equation subject to the boundary conditions in Equations 8.348, 8.349, and 8.352. This can be done using Mathematica [203] or equivalent software and by using a shooting algorithm, whereby the initial value of the second derivative of the function is guessed and adjusted until the boundary condition at infinity is satisfied. The correct guess for f''(0) is 0.332 [48].
44. Derive the expression for wall shear stress on a flat plate as a function of location (see Equation 8.366). Use the empirical curve fit (see Equation 8.356) for the velocity profile.
45. What is streamlining? Why does it work?
46. Blunt objects experience drag from two sources: pressure drag and friction drag. Explain these two types of drag. Which type is eliminated through streamlining?
47. How much faster will a cyclist traveling at 40 mph go if he buys a recumbent bicycle compared to an upright posture on a standard bicycle?
48. When riding downhill on a bicycle, a cyclist can slow down by sitting up straight rather than crouching over. How much deceleration can be expected from this posture change? Make reasonable assumptions in your calculations and indicate those assumptions.
49. What would the drag coefficient have to be to obtain the correct value of the terminal speed of a skydiver? Calculate for both the head-first and the belly-to-Earth positions.
50. If a coin falls flat-side-down through water versus edge-side-down, what is the speed difference at terminal speed?
51. What is vorticity? It is mentioned only in the advanced study of fluid mechanics, yet it is a fundamental property of flow fields. Discuss the utility of vorticity.
52. The isovorticity lines in Figure 8.60 appear to be pushed downstream by the flow. Describe what is happening in the flow that results in this effect.
53. Show that uniform potential flow past an obstacle is an irrotational flow. Hint: Far upstream of the obstacle, the flow is irrotational.
54. A vector identity useful in vorticity calculation is given in Equation 8.267. Writing the vectors in Cartesian coordinates, verify this vector identity.
55. For two-dimensional flow, use matrix calculations to show that $\underline{\omega} \cdot \nabla \underline{v} = 0$, where $\underline{\omega} = \nabla \times \underline{v}$ is the vorticity.
56. Show that $\nabla \times \nabla f = 0$, where f is a scalar function.

57. In this chapter, we always nondimensionalize time with a characteristic time $T = D/V$. For this characteristic time to be appropriate, the scaled time-derivative should be $O(1)$. This is true if characteristic changes in the velocity take place over an amount of time equal to T. A second characteristic time what we could construct from various quantities in the flow is based on the viscosity:

$$\tilde{T} = \frac{\rho D^2}{\mu} = \frac{D^2}{\nu}$$

where ν is the kinematic viscosity, which takes the role of a momentum-diffusion coefficient. Also, if the flow has its own imposed characteristic time—such as an imposed frequency of oscillation—this is another potential characteristic time to adopt.

(a) Using the definition of characteristic time $\tilde{T}$ introduced in this problem, what are the two forms of the nondimensional Navier-Stokes equation that result from choosing characteristic pressure to be first $P = \rho V^2$ and then $P = \mu V/D$?

(b) The *Strouhal number* Str is defined as the dimensionless ratio of time scales in the flow:

$$Str = \frac{T}{D/V}$$

$$Str = \frac{T}{\rho D^2/\mu}$$

Incorporate the Strouhal number into the two forms of the nondimensional Navier-Stokes equation found in (a).

(c) For the nondimensional Navier-Stokes equation discussed in this chapter, what value do we implicitly assume for the Strouhal number? In unsteady and oscillating flows, the Strouhal number assumes a prominent role [85].