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CM3120

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Requests for today?

- HW3 #3
- HW3 #11
- HW #5

PREP FOR EXAM 3 (Mod 3)

(a) Show: $\underline{N}_A + \underline{N}_B = c\underbrace{\underline{v}^*}_{\text{bulk velocity, m/s, averaged over moles}}$

What is $\underline{N}_A$?

$$\text{molar flux of Species A} = \frac{\text{moles A}}{\text{AREA} \quad \text{time}}$$

$$N_A \longrightarrow \qquad \longleftarrow N_B$$

$$c\underline{v}^* [=] \frac{\text{moles mix}}{\text{volume mix}} \cdot \frac{m}{s}$$

$$[=] \frac{\text{moles}}{m^{\cancel{3}2}} \cdot \frac{\cancel{m}}{s}$$

$$\frac{Q}{A} = \langle V \rangle$$

Velocity of A $\frac{m}{s}$

$$\underline{N}_A = C_A \underline{V}_A = \frac{\text{Moles}}{\text{Volume mix}} \cdot$$

$$= \frac{\text{Moles}}{\text{Volume}} \frac{\text{vol}}{\text{time } A}$$

$$\underline{N}_B = C_B \underline{V}_B$$

$$\underline{N}_A + \underline{N}_B = C_A \underline{V}_A + C_B \underline{V}_B$$

$$= C X_A \underline{V}_A + C X_B \underline{V}_B$$

$$= C\,\underbrace{(X_A \underline{V}_A + X_B \underline{V}_B)}_{\underline{v}^*}$$

$$= C \underline{v}^*$$

(b) (3)

$$\underline{n}_A + \underline{n}_B = \rho \underline{v} \quad ?$$

ρw_A

$$\underline{n}_A = \rho_A \underline{v}_A$$

$$\underline{n}_B = \rho_B \underline{v}_B$$

ρw_B

$$\underline{n}_A + \underline{n}_B = \rho \left(\underbrace{w_A \underline{v}_A + w_B \underline{v}_B}_{\underline{v}} \right)$$

$$= \rho \underline{v}$$

c) Show: $\underline{j}_A + \underline{j}_B = 0$

$$\underline{j}_A = \rho w_A (\underline{v}_A - \underline{v})$$

$$\underline{j}_B = \rho w_B (\underline{v}_B - \underline{v})$$

$$\underline{j}_A + \underline{j}_B = \rho \left(w_A \underline{v}_A - w_A \underline{v} + w_B \underline{v}_B - w_B \underline{v} \right)$$

$$= \rho \Big(\underbrace{w_A \underline{v}_A + w_B \underline{v}_B}_{\underline{v}} - \left(w_A \underline{v} + w_B \underline{v} \right) \Big)$$

$$= \rho \Big(\underline{v} - \underline{v} \big(\overbrace{w_A + w_B}^{1} \big) \Big) = 0 \,//$$

3.11

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$$C_A v_A = N_A = 5.0 \times 10^{-5} \frac{kmol\,A}{m^2 s} \qquad N_B = 0 = C_B \underline{v}_B$$

$\xrightarrow[z]{}$

$$C_A = 0.0050 \frac{kmol}{m^3}$$

$$C_B = 0.036 \frac{kmol}{m^3}$$

$\underline{v}_A$

$\underline{v}_B$

$\underline{v}^*$

$$v^* = X_A \underline{V}_A + V_B X_B = 1.2 \times 10^{-3}\ m/s.$$

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$$C = \frac{\text{moles total}}{\text{volume total}} = C_A + C_B$$

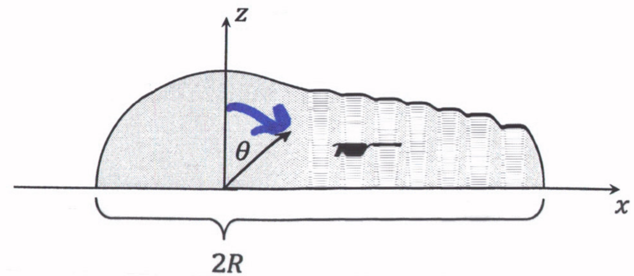
$$x_A = \frac{C_A}{C}$$

$$x_B = \frac{C_B}{C}$$

//

3.5

5. A hemispherical drop of liquid water lies on a flat surface (see figure below). The water evaporates from the surface through the still air adjacent to the droplet. The temperature and pressure are constant, and the diffusion is slow, so the size of the droplet is almost unchanged during the evaporation. Model this problem in such a way that we can determine the concentration distribution near the surface of the droplet. How can we calculate the evaporation rate? Answer for concentration distribution in the film: $\left(\frac{1-x_A}{1-x_{A\infty}}\right) = \left(\frac{1-x_{AR}}{1-x_{A\infty}}\right)^{r/R}$; see also notes.



(recently updated the question)

combined (1)

The **Equation of Species Mass Balance in Terms of Combined Molar quantities** in Cartesian, cylindrical, and spherical coordinates for binary mixtures of A and B.

The general case, where the combined molar flux with respect to molar velocity ($\underline{N}_A$), is given on page 1.

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In terms of total mo[redacted]ar flux, $\underline{N}_A$

Microscopic species mass balance, in terms of molar flux; Gibbs notation

$$\frac{\partial c_A}{\partial t} = -\nabla \cdot \underline{N}_A + R_A \qquad \text{WRF 25-11}$$

Microscopic species mass balance, in terms of combined molar flux; Cartesian coordinates

$$\frac{\partial c_A}{\partial t} = -\left(\frac{\partial N_{A,x}}{\partial x} + \frac{\partial N_{A,y}}{\partial y} + \frac{\partial N_{A,z}}{\partial z}\right) + R_A$$

Microscopic species mass balance, in terms of combined molar flux; cylindrical coordinates

$$\frac{\partial c_A}{\partial t} = -\left(\frac{1}{r}\frac{\partial (rN_{A,r})}{\partial r} + \frac{1}{r}\frac{\partial N_{A,\theta}}{\partial \theta} + \frac{\partial N_{A,z}}{\partial z}\right) + R_A$$

Microscopic species mass balance, in terms of combined molar flux; spherical coordinates

STEP 1

$$\cancel{\frac{\partial c_A}{\partial t}} = -\left(\frac{1}{r^2}\frac{\partial (r^2 N_{A,r})}{\partial r} + \cancel{\frac{1}{r\sin\theta}\frac{\partial (N_{A,\theta}\sin\theta)}{\partial \theta}} + \cancel{\frac{1}{r\sin\theta}\frac{\partial N_{A,\phi}}{\partial \phi}}\right) + \cancel{R_A}$$

steady

1D

no homog rxn

Fick's law of diffusion, Gibbs notation: $\underline{N}_A = x_A(\underline{N}_A + \underline{N}_B) - cD_{AB}\nabla x_A$ WRF 24-22

$$= c_A \underline{v}^* - cD_{AB}\nabla x_A$$

Fick's law of diffusion, Cartesian coordinates:
$$\begin{pmatrix} N_{A,x} \\ N_{A,y} \\ N_{A,z} \end{pmatrix}_{xyz} = \begin{pmatrix} x_A(N_{A,x} + N_{B,x}) - cD_{AB}\frac{\partial x_A}{\partial x} \\ x_A(N_{A,y} + N_{B,y}) - cD_{AB}\frac{\partial x_A}{\partial y} \\ x_A(N_{A,z} + N_{B,z}) - cD_{AB}\frac{\partial x_A}{\partial z} \end{pmatrix}_{xyz}$$

Fick's law of diffusion, cylindrical coordinates:
$$\begin{pmatrix} N_{A,r} \\ N_{A,\theta} \\ N_{A,z} \end{pmatrix}_{r\theta z} = \begin{pmatrix} x_A(N_{A,r} + N_{B,r}) - cD_{AB}\frac{\partial x_A}{\partial r} \\ x_A(N_{A,\theta} + N_{B,\theta}) - \frac{cD_{AB}}{r}\frac{\partial x_A}{\partial \theta} \\ x_A(N_{A,z} + N_{B,z}) - cD_{AB}\frac{\partial x_A}{\partial z} \end{pmatrix}_{r\theta z}$$

B stagnant

STEP 2

Fick's law of diffusion, spherical coordinates:
$$\begin{pmatrix} N_{A,r} \\ \cancel{N_{A,\theta}} \\ \cancel{N_{A,\phi}} \end{pmatrix}_{r\theta\phi} = \begin{pmatrix} x_A(N_{A,r} + \cancel{N_{B,r}}) - cD_{AB}\frac{\partial x_A}{\partial r} \\ x_A(\cancel{N_{A,\theta}} + \cancel{N_{B,\theta}}) - \cancel{\frac{cD_{AB}}{r}\frac{\partial x_A}{\partial \theta}} \\ x_A(\cancel{N_{A,\phi}} + \cancel{N_{B,\phi}}) - \cancel{\frac{cD_{AB}}{r\sin\theta}\frac{\partial x_A}{\partial \phi}} \end{pmatrix}_{r\theta\phi}$$

$$0 = \cancel{+\frac{1}{r^2}}\left(\frac{d}{dr}\left(\underbrace{r^2 N_{Ar}}_{\equiv \Phi}\right)\right)$$

$$\frac{d\Phi}{dr} = 0$$

$$\Phi = c_1 = r^2 N_{A,r}$$

$$\boxed{N_{A,r} = \frac{c_1}{r^2}}$$

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FICK'S LAW

$$N_{A,r} = x_A N_{A,r} - cD_{AB}\frac{dx_A}{dr}$$

$$\underbrace{N_{Ar}}_{\frac{c_1}{r^2}}(1 - x_A) = -cD_{AB}\frac{dx_A}{dr}$$

$$\frac{c_1}{r^2}(1 - x_A) = -cD_{AB}\frac{dx_A}{dr}$$

What's c constant?

$$-\frac{c_1}{cD_{AB}}\frac{1}{r^2}dr = \frac{dx_A}{1 - x_A}$$

gas: $PV = nRT$

$$\frac{P}{RT} = \frac{n}{V} = c$$

BC:

(c_{LIQ} is constant if it's dilute)

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$$\int \left(\frac{-c_1}{cD_{AB}}\right) \frac{1}{r^2}\,dr = \frac{dx_A}{1-x_A}$$

c_1 is a constant

D_{AB} – we will assume is constant

c . . . ?

$$c = \frac{\text{moles mix}}{\text{volume mix}} = \frac{n}{V} = \underbrace{\frac{P}{RT}}_{\text{IDEAL GAS}}$$

Ideal gas,
const T, P
$\Rightarrow c = \text{constant}$

$$\left(-\frac{c_1}{cD_{AB}}\right)\int \frac{dr}{r^2} = {}^{-1}\left(\frac{-dx}{1-x_A}\right)$$

$$\left(\frac{-c_1}{cD_{AB}}\right)\left(-r^{-1}\right) = \ln(1-x_A) + c_2$$

BC: $r = R \quad x_A = x_A^*$

$r = \infty \quad x_A = x_{A_b}$

Solve for c_1, c_2