Ex 6 — MACRO ABSORBER (start to finish) ①

CM3120: Module 4

PACK I

Diffusion and Mass Transfer II

Ex 6!

I. Classic diffusion and mass transfer: d) EMCD
II. Classic diffusion and mass transfer: e) Penetration model
III. Unsteady macroscopic species A mass balances (Intro)
IV. Interphase species A mass transfers—To an interface—k_x, k_c, k_p (develop tool: film coefs)
V. Unsteady macroscopic species A mass balances (Redux)
VI. Interphase species A mass transfers—Across multiple resistances—K_L, K_G (develop tool: overall coef)
VII. Dimensional analysis
VIII. Data correlations

②

Modeling practical devices involving mass transfer

Example 6: Height of a packed bed absorber

How can we use mass transfer to design a packed bed gas absorber to achieve a desired separation?

Example 6 is presented as a series of **linked examples** that navigate around apparent "dead ends" in modeling mass-transfer units

Identify a question	Invent something	Try to use it
1. How can we model a large, practical device dependent on mass transfer? III	1. Apply the species A mass balance to a macroscopic C.V. III	1. Lack a system to account for A going between phases PAUSE III
2. How an we account for A going between phases? IV	2. Invent k_x through linear driving force (LDF) model IV	2. Gets A *to* the boundary, but not *across* PAUSE IV
3. How can we improve LDF model to cross the boundary (bulk-to-bulk transfer)? VI	3. Write LDF in both phases and combine to create overall effect of multiple resistances VI	3. Working, but can we devise a convenient shorthand? II PAUSE
4. Can we model a large, practical device, incorporating K_L, K_G to account for mass xfer between phases? VI	**4. Yes** VI	

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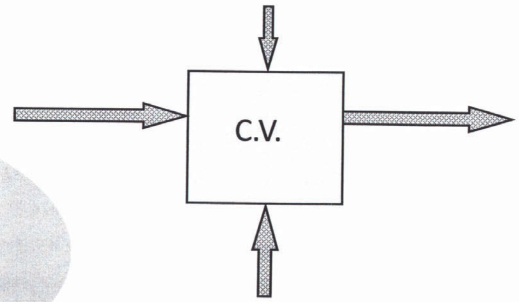


Module 4 Lecture III

Unsteady Macroscopic species A mass balances

(Introduction)

Professor Faith A. Morrison
Department of Chemical Engineering
Michigan Technological University

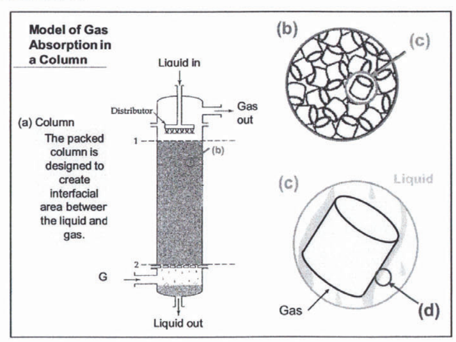
III

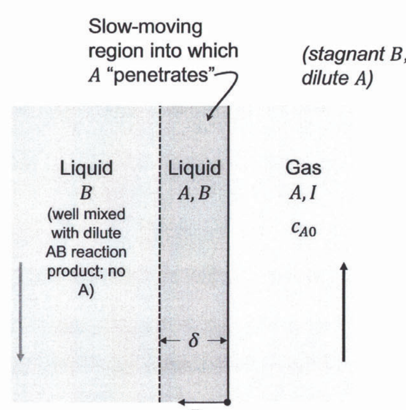
www.chem.mtu.edu/~fmorriso/cm3120/cm3120.html

In lecture II we developed a picture of how the mass moves around in an absorber (**penetration model**).

How can we design a packed bed gas absorber to achieve a desired separation?

In momentum and heat transfer we often sorted out "devices" with a macroscopic balance. Shall we try this?





III

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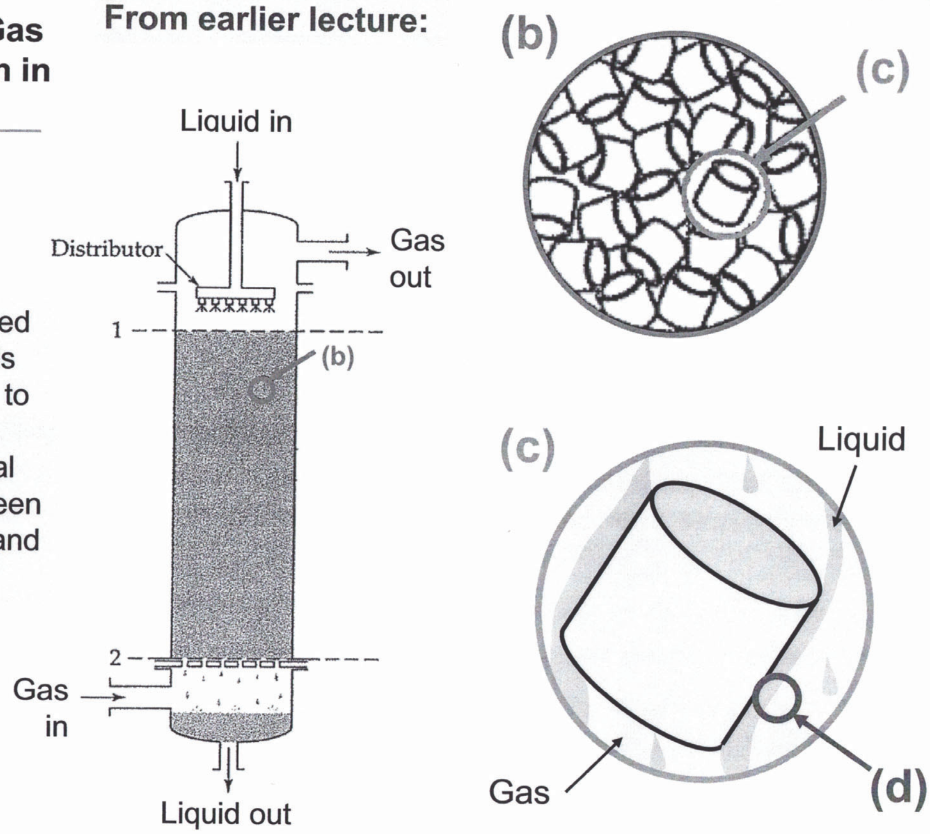
Model of Gas Absorption in a Column

From earlier lecture:

(a) Column

The packed column is designed to create interfacial area between the liquid and gas.



BSL2, p743

III

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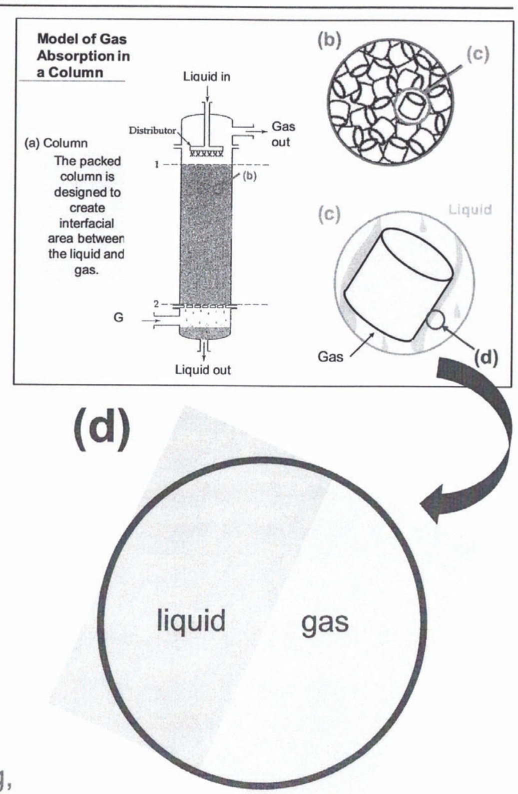
From earlier lecture:

The Mass-Transfer Model

We focus on the **liquid-gas interface** where the mass transfer takes place.

<u>Idealize</u> the **entire device** as comprising only the appropriate amount of this interface, with mass transfer taking place there.

Retain the role of the column, by forcing the appropriate concentrations of species A to enter and exit the column.



$$a \equiv \frac{\text{interfacial area}}{\text{volume}}$$

A property of the column and packing, in operation

III

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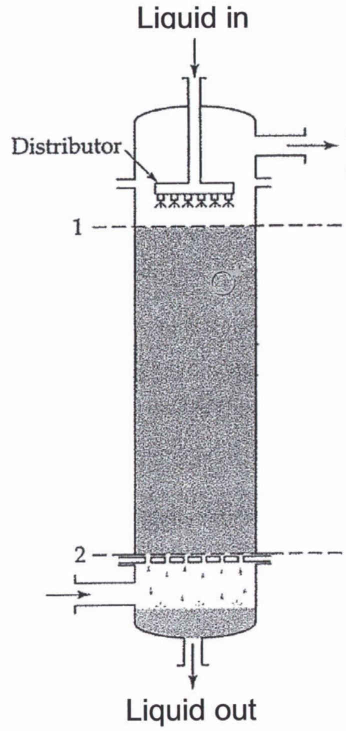
What is our model for the entire column??

(our assumptions)

1. Control volume =
2.



III

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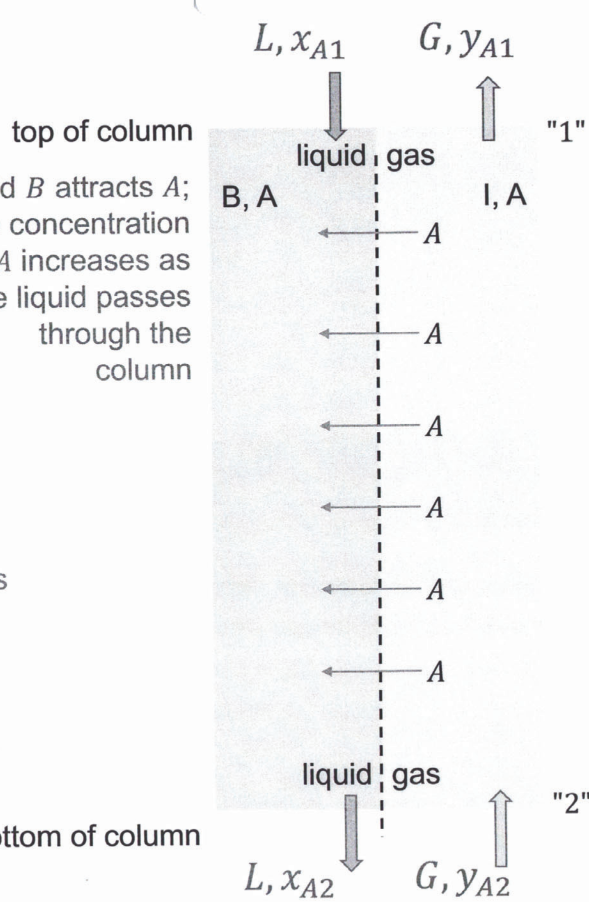
Model of Gas Absorption in a Column

Liquid B attracts A; the concentration of A increases as the liquid passes through the column



Gas contains A, which depletes as the gas passes through the column

The preference of species A for liquid B rather than inert gas I is due to a difference in *chemical potential*

III

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Model of Gas Absorption in a Column

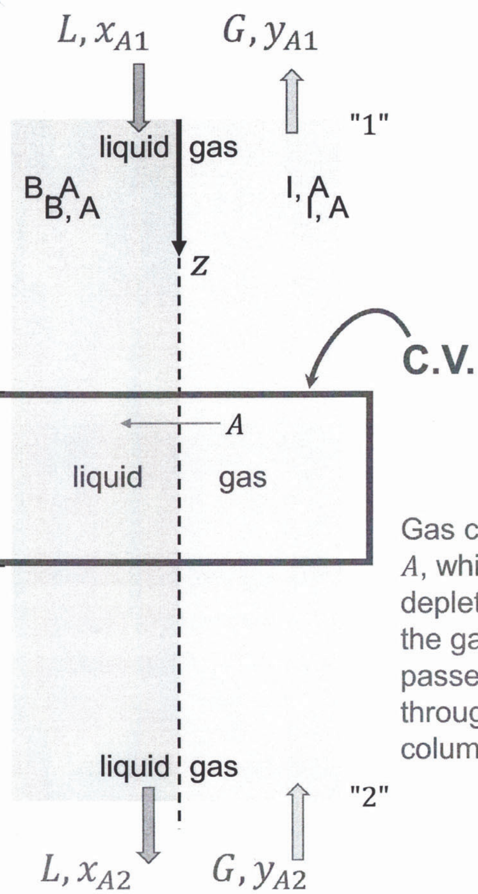
top of column

Liquid B attracts A; the concentration of A increases as the liquid passes through the column

$0 \le z \le L$

z

$z + \Delta z$



bottom of column

Gas contains A, which depletes as the gas passes through the column

III

MOLES

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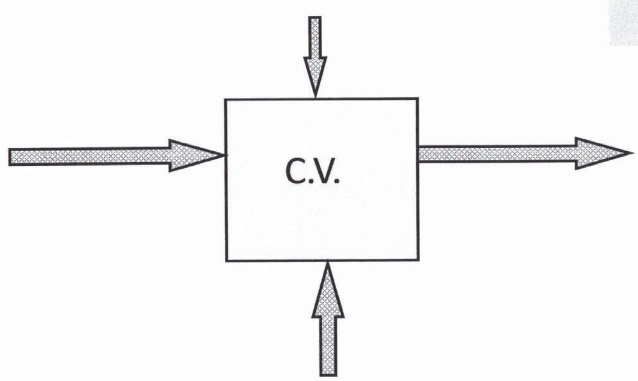
Unsteady, Macroscopic, Species A Mass Balance

balance over time interval Δt

Macroscopic control volume, C.V.



Keep track of:

- Bulk convection of species A into and out of the C.V.
- Mass transfer of species A across control surfaces
- Mass of species A created by chemical reaction

BSL2, p727

III

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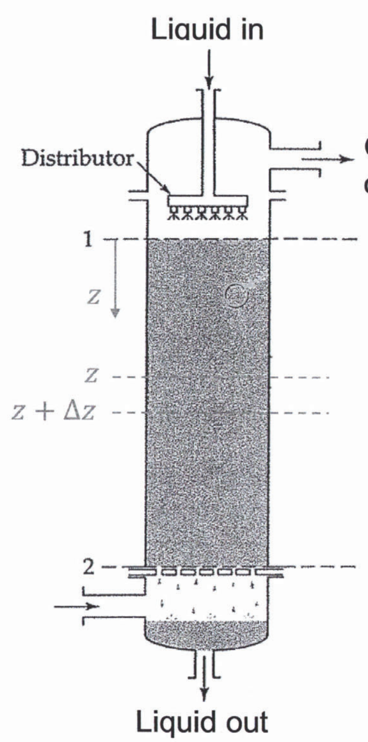
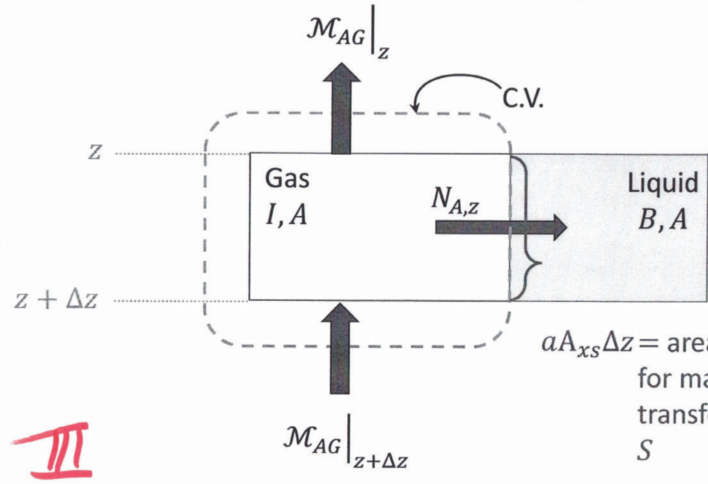
Unsteady Macroscopic Species A Mass Balance—Gas Absorption



$$a \equiv \frac{\text{interfacial area}}{\text{volume}}$$

A property of the column and packing, in operation

N_{Az} = (molar) mass flux 1 species A gas–liquid



$aA_{xs}\Delta z =$ area for mass transfer, S

III

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Unsteady Macroscopic Species A Mass Balance—Intro

MOLES

Unsteady, Macroscopic, Species A Mass Balance

balance over time interval Δt

Macroscopic control volume, C.V.

R_A = net rate of production of moles of A in the C.V. by reaction, per unit volume

$R_A V_{sys} \Delta t$

C.V.

$\underbrace{\dot{\mathcal{M}}_A \, \Delta t}$ moles of A that *flows into* the control volume between t and $t + \Delta t$

$\underbrace{\dot{\mathcal{M}}_A \, \Delta t}$ moles of A that *flows out of* the control volume between t and $t + \Delta t$

$-(N_A S)_j$

introduction of moles of A into the C.V. by mass transfer across the j^{th} bounding control surface S_j (C.S.)

C.S.

$$S_{sys} = \sum_j S_j$$

C.S. = control surface
C.V. = control volume

III

?

BSL2, p727

Unsteady Macroscopic Species A Mass Balance—Intro

MOLES

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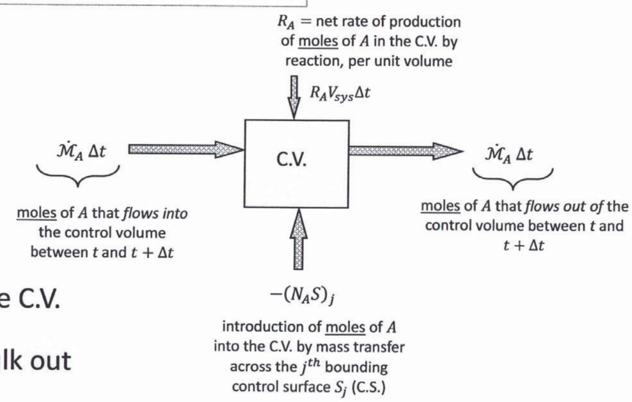
accumulation = net flow in + production + introduction

$$\boxed{\frac{d}{dt}\left(\mathcal{M}_{A,sys}\right) = -\Delta\dot{\mathcal{M}}_A + R_A V_{sys} - \sum_j (N_A S)_j}$$



$\mathcal{M}_{A,sys} = c_A V_{sys}$ = total moles of A in the C.V.

$\Delta\dot{\mathcal{M}}_A = \sum_{j,outs} \dot{\mathcal{M}}_{A,j} - \sum_{j,ins} \dot{\mathcal{M}}_{A,j}$ = bulk out

R_A = net rate of production of moles of A in the C.V. by reaction, per unit volume

V_{sys} = system volume

?

$N_{A_j} = K\Delta c_{df}$ =molar flux of A out through the j^{th} C.S.

$S_{sys} = \sum_j S_j$

Δ is "out"- "in"

C.S. = control surface

C.V. = control volume

III

Unsteady Macroscopic Species A Mass Balance--Intro

MOLES

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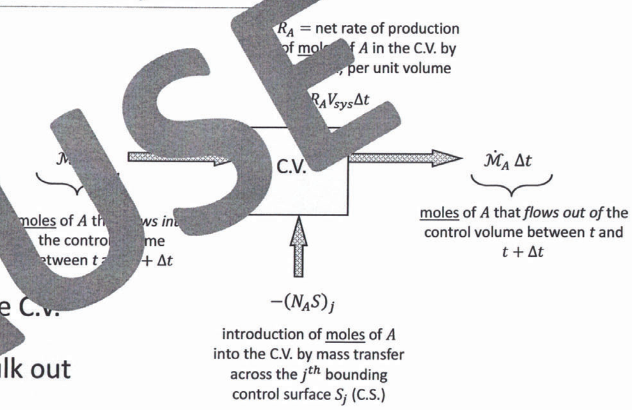
accumulation = net flow in + production + introduction

$$\boxed{\frac{d}{dt}\left(\mathcal{M}_{A,sys}\right) = -\Delta\dot{\mathcal{M}}_A + R_A V_{sys} - \sum_i (N_A S)_i}$$

PAUSE



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C.S. = control surface

C.V. = control volume

III

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Macroscopic Species A Balances Summary (so far)

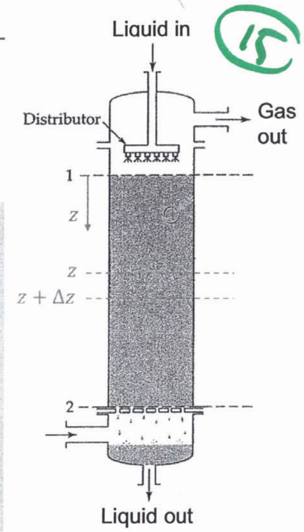
- To model entire devices, we need ***macroscopic*** control volumes
- The diffusion coefficient is not a convenient way to model mass transfer into a macroscopic control volume.
- In heat transfer we used Newton's law of cooling in the macroscopic balance as the source of convective heat flow:

$$\dot{q}_{\text{in}} = hA(T_b - T_w)$$

- Maybe someone has invented a "mass transfer coefficient" K analogous to the heat transfer coefficient h?

$$N_A = K\Delta c_{df}$$

Take a **PAUSE** to figure this out



III //

UN-PAUSE

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The **macroscopic species A mass balance** worked well for the these two problems.

Both problems involved transfer from the **interface conditions** to the **bulk**.

Let's now try a problem in which **the mass transfer is from one bulk condition to a second bulk condition:**

Example 6—Revisited

Example 6: Height of a packed bed absorber

How can we use mass transfer to design a packed bed gas absorber to achieve a desired separation?

(started in Module 4, Lecture III)

V

Example 6—Revisited

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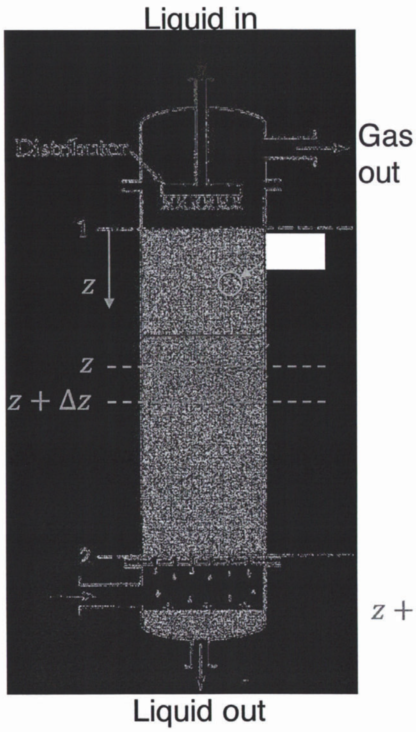
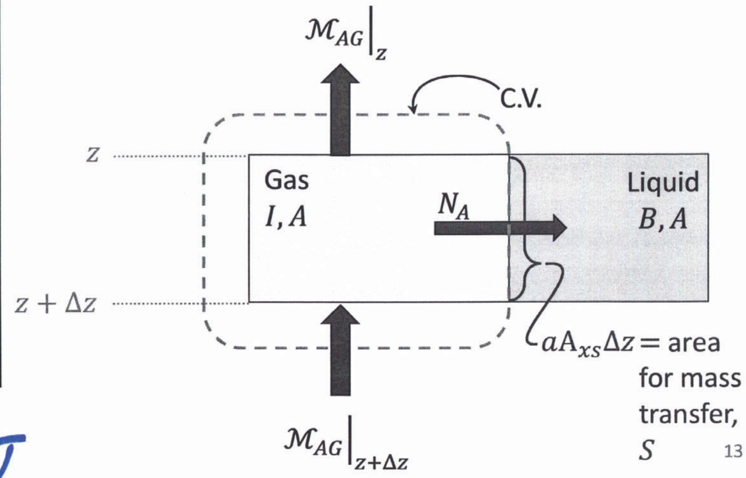


$$a \equiv \frac{\text{interfacial area}}{\text{volume}}$$

A property of the column and packing, in operation

Use film coefficients to obtain N_A?



V

Example 6—Revisited

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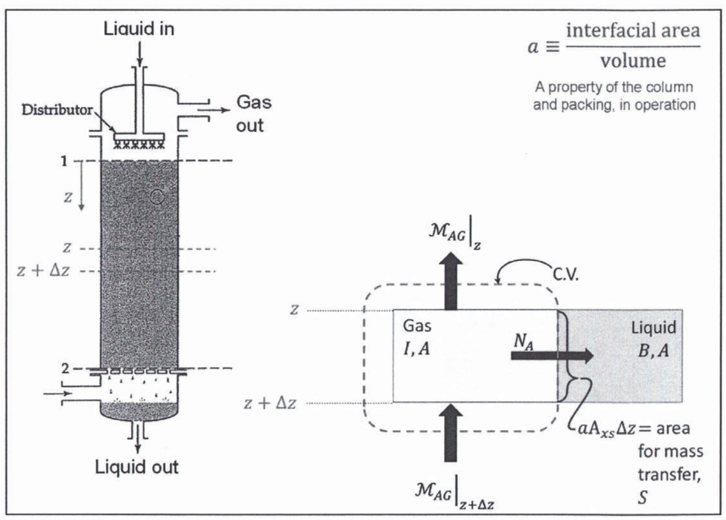
Difficulty:
The film coefficients do not allow us to include the status of the bulk liquid phase in the driving force for this problem.



Gas phase film LDF model:

$$N_A = k_p(p_A - p_{Ai})$$

→ We need a technique that uses a **bulk-to-bulk** driving force

V

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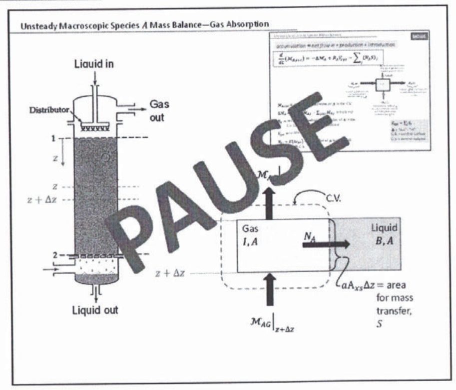
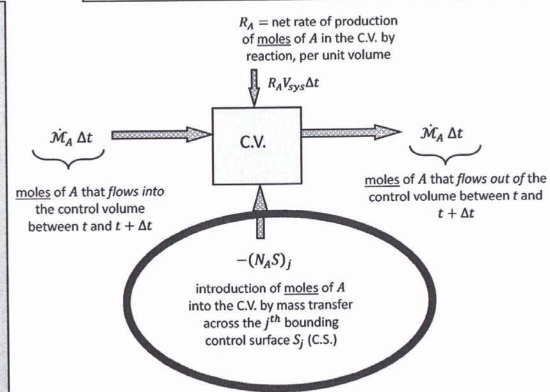
Next? Develop the macroscopic mass-transfer expression we need for bulk-to-bulk transfer $N_A S$.



Needed:

$$N_A = (?)$$
$$= (\text{expression})(\Delta c_{df})$$

Bulk-to-bulk, like
$\dot{Q} = UA\Delta T_{lm}$



IV //

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Example 6—concluded

Now, let's *finish* our practical problem

Example 6: Height of a packed bed absorber

How can we use mass transfer to design a packed bed gas absorber to achieve a desired separation? We assume dilute concentrations in both gas and liquid.

LECTURE VI

Example 6—concluded

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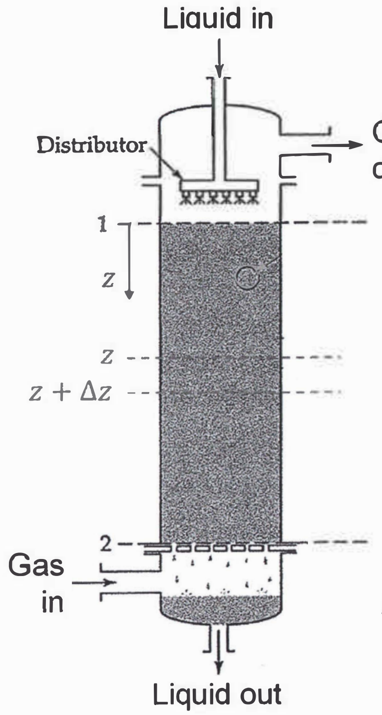
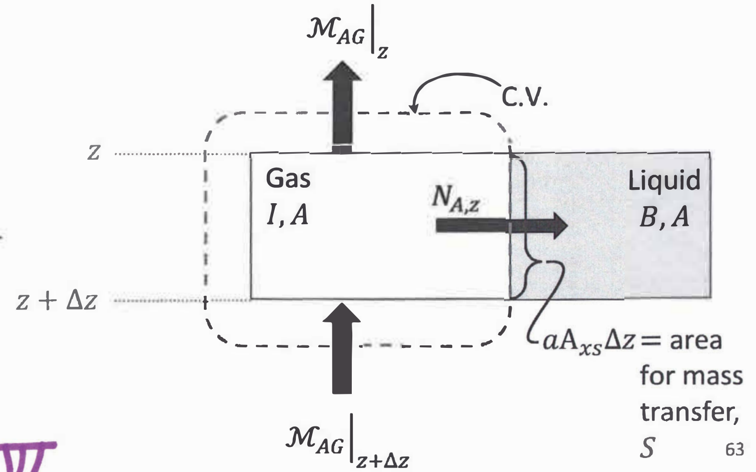
$$a \equiv \frac{\text{interfacial area}}{\text{volume}}$$

A property of the column and packing, in operation



Combined molar flux of species A **from gas to liquid** $\equiv N_{A,z}$

A_{xs} = column cross section



VI

MOLES

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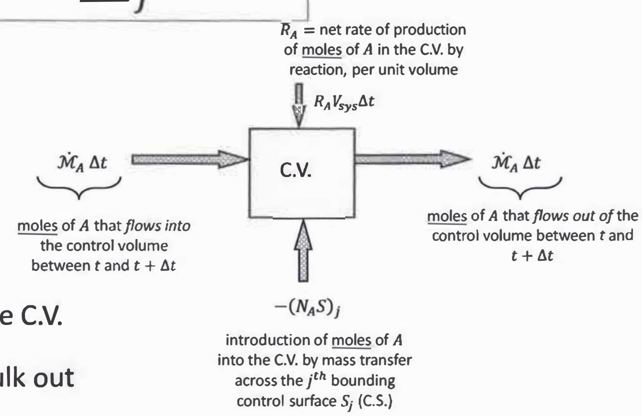
accumulation = net flow in + production + introduction

$$\boxed{\frac{d}{dt}\left(\mathcal{M}_{A,sys}\right) = -\Delta\dot{\mathcal{M}}_A + R_A V_{sys} - \sum_j (N_A S)_j}$$



$\mathcal{M}_{A,sys} = c_A V_{sys}$ = total moles of A in the C.V.

$\Delta\dot{\mathcal{M}}_A = \sum_{j,outs} \dot{\mathcal{M}}_{A,j} - \sum_{j,ins} \dot{\mathcal{M}}_{A,j}$ = bulk out

R_A = net rate of production of moles of A in the C.V. by reaction, per unit volume

V_{sys} = system volume

$N_{A_j} = K(\Delta c_{df})$ =molar flux of A out through the j^{th} C.S.

We now have expressions for K and Δc_{df}

$S_{sys} = \sum_j S_j$

Δ is "out"- "in"

C.S. = control surface

C.V. = control volume

VI

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Liquid-phase-units
Film Linear driving force model:

$$N_A \equiv k_x(x_{A,i} - x_{A,b})$$
$$N_A \equiv k_{cL}(c_{AL,i} - c_{AL,b})$$

Gas-phase-units:
Film Linear driving force model:

$$N_A \equiv k_p(p_{A,b} - p_{A,i})$$
$$N_A \equiv k_{cG}(c_{AG,b} - c_{A,i})$$
$$N_A \equiv k_y(y_{A,b} - y_{A,i})$$

Liquid-phase-units
Overall Linear driving force model:

$$N_A \equiv K_x(x_A^*(\ ) - x_{A,b})$$
$$N_A \equiv K_{cL}(c_{AL}^*(\ ) - c_{AL,b})$$

$(\ ) = p_{A,b}$ or $c_{A,b}$ or $y_{A,b}$

Gas-phase-units:
Overall Linear driving force model:

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$$N_A \equiv K_y\left(y_{A,b} - y_A^*(\ )\right)$$

$(\ ) = x_{A,b}$ or $c_{AL,b}$

Let's ~~take these tools out for a spin~~!

choose our mass transfer units (our tools)

$$\boxed{K_x = \frac{1}{\frac{1}{k_x} + \frac{1}{m''k_y}}},\quad \boxed{K_y = \frac{1}{\frac{1}{k_y} + \frac{m'}{k_x}}}, \text{ etc.}$$

VI

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Example 6—concluded

$$\boxed{\frac{d}{dt}(\mathcal{M}_{A,sys}) = -\Delta\dot{\mathcal{M}}_A + R_A V_{sys} - \sum_j (N_A S)_j}$$

steady

No homogenous reaction

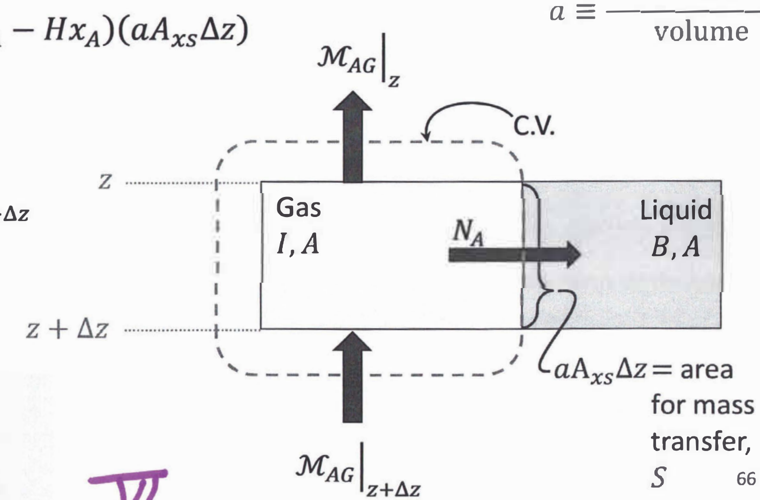
$$L, G = \frac{moles}{column\ area \cdot time}$$

A_{xs} = column cross section

$$a \equiv \frac{\text{interfacial area}}{\text{volume}}$$

$$N_A S = (K_y \Delta c_{df})(aA_{xs}\Delta z)$$
$$= \left(\frac{1}{\frac{H}{k_x} + \frac{1}{k_y}}\right)(y_A - Hx_A)(aA_{xs}\Delta z)$$

$$\mathcal{M}_{AG}\Big|_{z+\Delta z} = GA_{xs}\, y_A\Big|_{z+\Delta z}$$
$$\mathcal{M}_{AG}\Big|_{z} = GA_{xs}\, y_A\Big|_{z}$$



See Handnotes

VI

HANDNOTES

27 MAR 2021
F. MORRISON

1

Part 2

Example 6—~~Continued~~ concluded!

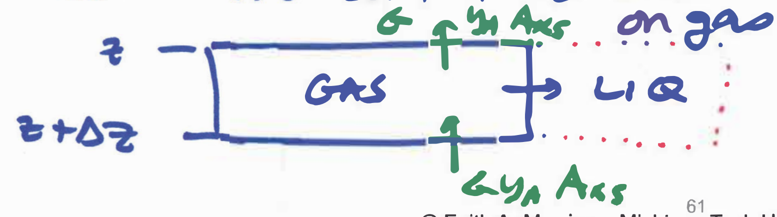
Once again, let's ~~work on~~ finish our practical problem

Example 6 (soln)

Example 6: Height of a packed bed absorber (assume dilute)

How can we use mass transfer to design a packed bed gas absorber to achieve a desired separation?

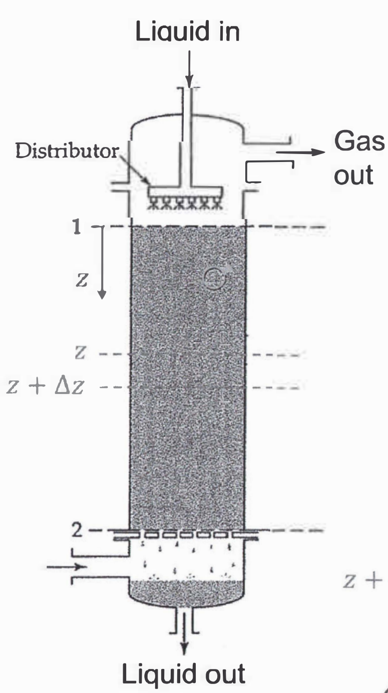
Choose macro control volume on gas



Cussler p314 section 10.3

2

Example 6—Continued



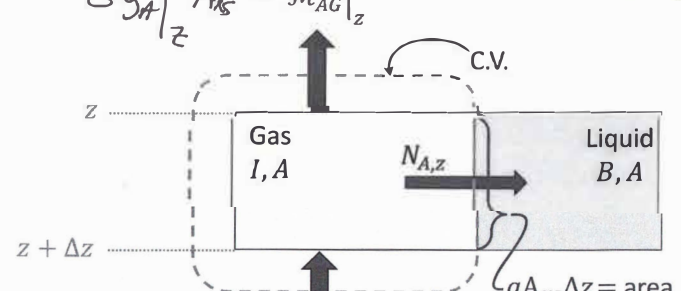
$$a \equiv \frac{\text{interfacial area}}{\text{volume}}$$

A property of the column and packing, in operation

Mass flux of species A **from gas to liquid** $\equiv N_{A,z}$

A_{xs} = column cross section

$G y_A\big|_z A_{xs} = \mathcal{M}_{AG}\big|_z$



$G y_A\big|_{z+\Delta z} A_{xs} = \mathcal{M}_{AG}\big|_{z+\Delta z}$

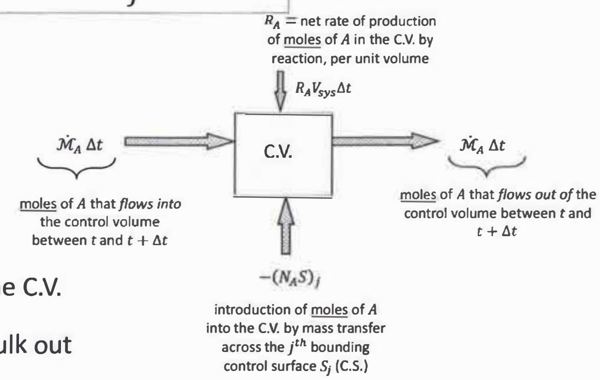
3

Unsteady Macroscopic Species A Mass Balance—Intro

MOLES

accumulation = net flow in + production + introduction

$$\boxed{\frac{d}{dt}\left(\mathcal{M}_{A,sys}\right) = -\Delta\dot{\mathcal{M}}_A + R_A V_{sys} - \sum\nolimits_j (N_A S)_j}$$



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We now have expressions for K and Δc_{df}

$S_{sys} = \sum_j S_j$
Δ is "out"- "in"
C.S. = control surface
C.V. = control volume

use gas-side overall mass xfer coef K_G

Choose units, gas side vs liquid-side +dilute

4

Overall Mass-Transfer Coefficient — concentrated regime

Liquid-phase-units
Film Linear driving force model:

$N_A \equiv k_x(x_{A,i} - x_{A,b})$
$N_A \equiv k_{cL}(c_{AL,i} - c_{AL,b})$

Gas-phase-units:
Film Linear driving force model:

$N_A \equiv k_p(p_{A,b} - p_{A,i})$
$N_A \equiv k_{cG}(c_{AG,b} - c_{A,i})$
$N_A \equiv k_y(y_{A,b} - y_{A,i})$

choose

Liquid-phase-units
Overall Linear driving force model:

$N_A \equiv K_x(x_A^*(\ ) - x_{A,b})$
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$(\ ) = p_{A,b}$ or $c_{A,b}$ or $y_{A,b}$

Gas-phase-units:
Overall Linear driving force model:

$N_A \equiv K_p\left(p_{A,b} - p_A^*(\ )\right)$
$N_A \equiv K_{cG}\left(c_{AG,b} - c_{AG}^*(\ )\right)$
$N_A \equiv K_y\left(y_{A,b} - y_A^*(\ )\right)$

choose

$(\ ) = x_{A,b}$ or $c_{AL,b}$

choose

Let's take these tools out for a spin!

$$\boxed{K_x = \frac{1}{\frac{1}{k_x} + \frac{1}{m''k_y}}},\quad \boxed{K_y = \frac{1}{\frac{1}{k_y} + \frac{m'}{k_x}}}, \text{ etc.}$$

(dilute equil)

dilute: $y_A^* = H x_A^*$

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At the interface: (gas-liq, equilibrium)

$$N_A = k_x \underset{\uparrow \text{solve}}{(x_{Ai} - x_A)} = k_y(y_A - \underset{= H x_{Ai}}{y_{Ai}})$$

$$k_x x_{Ai} + k_y H x_{Ai} = k_y y_A + k_x x_A$$

$$\boxed{x_{Ai} = \frac{k_y y_A + k_x x_A}{k_x + H k_y}}$$

$$N_A = k_x (x_{Ai} - x_A)$$

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$$N_A = \cancel{k_x}\left(\left(\frac{k_y y_A + k_x x_A}{k_x + H k_y}\right) - x_A\right)\frac{\frac{1}{\cancel{k_x}}}{\frac{1}{k_x}}$$

$$= \frac{\cancel{k_y} y_A + \cancel{k_x x_A} - \cancel{k_x x_A} - H \cancel{k_y} x_A}{1 + H\frac{k_y}{k_x}} \quad \frac{\frac{1}{\cancel{k_y}}}{\frac{1}{k_y}}$$

$$\boxed{N_A = \frac{(y_A - \overbrace{H x_A}^{y_A^*(x_A)})}{\left(\frac{1}{k_y} + \frac{H}{k_x}\right)}}$$

$$N_A = K_y (\overbrace{y_A}^{\text{bulk}} - y_A^*(\overbrace{x_A}^{\text{bulk}}))$$

$$\boxed{K_y = \left(\frac{1}{\frac{1}{k_y} + \frac{H}{k_x}}\right)}$$

Gas-side-units, overall, bulk to bulk

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Unsteady Macroscopic Species A Mass Balance—Gas Absorption

Example 6—concluded

$$\frac{d}{dt}\left(\mathcal{M}_{A,sys}\right) = -\Delta\dot{\mathcal{M}}_A + R_A V_{sys} - \sum_j \left(N_A S\right)_j$$

steady

No homogenous reaction

$$L, G = \frac{moles}{column\ area \cdot time}$$

liquid gas

A_{xs} = column cross section

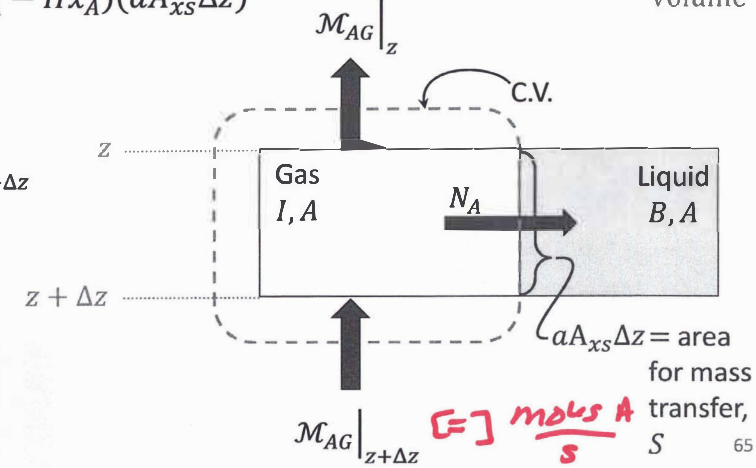
$$a \equiv \frac{\text{interfacial area}}{\text{volume}}$$

with the chosen units:

$$N_A S = \left(K_y \Delta c_{df}\right)\left(a A_{xs} \Delta z\right)$$
$$= \left(\frac{1}{\frac{H}{k_x} + \frac{1}{k_y}}\right)\left(y_A - H x_A\right)\left(a A_{xs} \Delta z\right)$$

$$\mathcal{M}_{AG}\Big|_{z+\Delta z} = G A_{xs}\, y_A\Big|_{z+\Delta z}$$

$$\mathcal{M}_{AG}\Big|_{z} = G A_{xs}\, y_A\Big|_{z}$$



[=] $\frac{\text{moles } A}{s}$

See Handnotes

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MACRO SPECIES A MASS BAL:
(on the gas side only)

$$0 = G\cancel{A_{xs}}\, y_A\Big|_{z+\Delta z} - G\cancel{A_{xs}}\, y_A\Big|_z - K_y \Delta c_{df}\, a \cancel{A_{xs}}\, \Delta z$$

$$G\,\frac{y_A\big|_{z+\Delta z} - y_A\big|_z}{\Delta z} = K_y a \overbrace{\left(y_A - H x_A\right)}$$

$$G\,\frac{dy_A}{dz} = K_y a\left(y_A - H x_A\right)$$

$$\boxed{\frac{dy_A}{dz} = \left(\frac{K_y a}{G}\right)\left(y_A - x_A H\right)}$$

"rate eqn"
(gas-side, species A mass bal)

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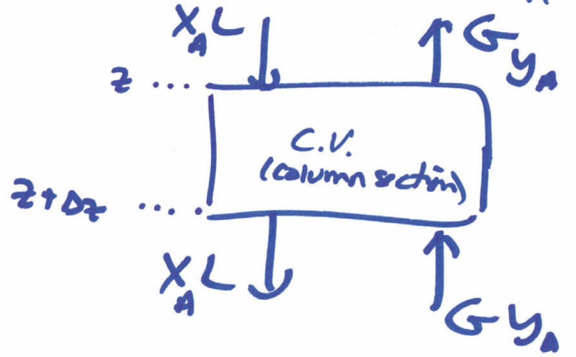
MACROSCOPIC OVERALL species A MASS BAL:

(over both gas + liquid streams, on species A only)



$$L, G = \frac{\text{moles}}{\underbrace{\text{column area}}_{A_{xs}} \cdot \text{time}}$$

$$\cancel{A_{xs}} L\, x_A\Big|_z + \cancel{A_{xs}} G\, y_A\Big|_{z+\Delta z} = \cancel{A_{xs}} L\, x_A\Big|_{z+\Delta z} + \cancel{A_{xs}} G\, y_A\Big|_z$$

$$G\, \frac{\left(y_A\big|_{z+\Delta z} - y_A\big|_z\right)}{\Delta z} = L\, \frac{\left(x_A\big|_{z+\Delta z} - x_A\big|_z\right)}{\Delta z}$$

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$$G\frac{dy}{dz} = L\frac{dx}{dz}$$

$$\frac{dy}{dx} = \frac{L}{G}$$

$$y = \left(\frac{L}{G}\right)x + c_1$$

$$\boxed{\text{BC: } \begin{array}{l} x = x_1 \\ y = y_1 \end{array}}$$

entering conditions

$$y_1 = \frac{L}{G}x_1 + c_1$$

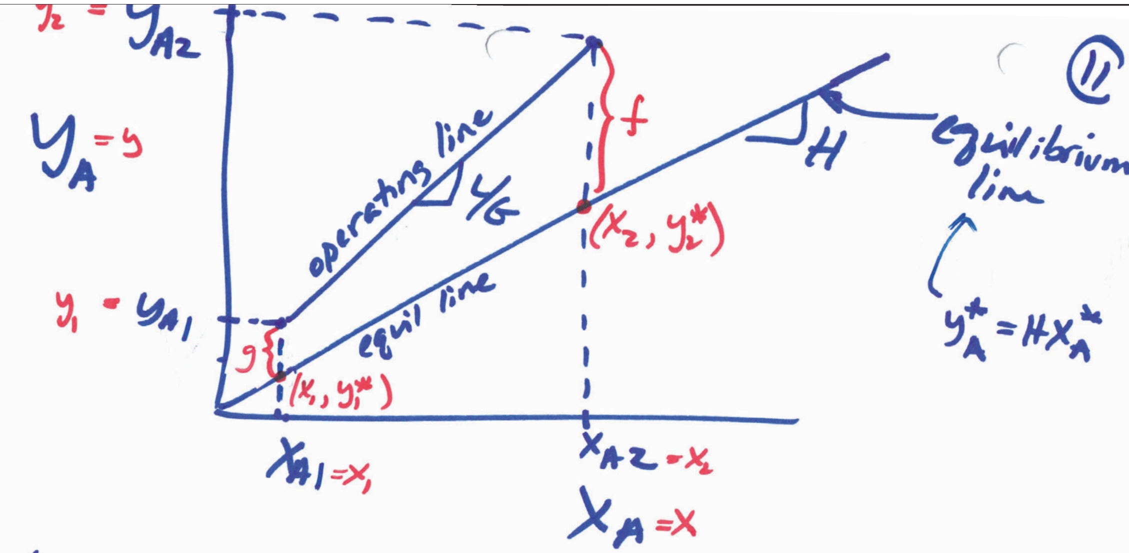
$$\boxed{c_1 = y_1 - \frac{L}{G}x_1}$$

operating line

$$\boxed{y = \frac{L}{G}(x - x_1) + y_1}$$

(overall, species A mass bal)

$y_A(x_A) = y(x)$ along the column (operating line)



to simplify:

$$y_A = y$$
$$x_A = x$$

$$f = y_2 - y_2^*$$
$$g = y_1 - y_1^*$$

(*f* and *g* are defined on page 14; they turn out to be the segments indicated, as established in the derivation that follows)

Combine the "rate eqn" with the "operating line," and integrate:

rate eqn:

$$\frac{dy}{dz} = \frac{K_y a}{G}(y - xH)$$

operating line:

$$y = \frac{L}{G}(x - x_1) + y_1$$

$$\frac{(y - y_1)}{L/G} = x - x_1$$

$$x = x_1 + \left(\frac{y - y_1}{L/G}\right)$$

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$$\frac{dy}{dz} = \frac{K_y a}{G}\left[y - H\left(x_1 + \frac{y - y_1}{L/G}\right)\right]$$

gather the y terms:

$$\frac{G}{K_y a}\frac{dy}{dz} = y - H\left[x_1 + \frac{y}{L/G} - \frac{y_1}{L/G}\right]$$

$$= y - \frac{H}{L/G}y - Hx_1 + \frac{Hy_1 G}{L}$$

$$= y\underbrace{\left(1 - \frac{GH}{L}\right)}_{\equiv \tilde{c}} + \underbrace{\left(\frac{Hy_1 G}{L} - Hx_1\right)}_{\equiv b}$$

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$$\frac{G}{K_y a}\frac{dy}{dz} = \tilde{c}y + b$$

$$\frac{1}{\tilde{c}}\int_{y_1}^{y_2}\left(\frac{dy\,(\tilde{c})}{\tilde{c}y + b}\right) = \int_0^B \frac{K_y a}{G}\,dz$$

B = column height

$$\frac{1}{\tilde{c}}\ln(\tilde{c}y + b)\Big|_{y_1}^{y_2} = \frac{K_y a}{G}z\Big|_0^B = \frac{K_y a}{G}B$$

$$\boxed{B = \frac{G}{K_y a}\left(\frac{1}{\tilde{c}}\right)\ln\underbrace{\left(\frac{\tilde{c}y_2 + b \;\}\equiv f}{\tilde{c}y_1 + b \;\}\equiv g}\right)}_{= \frac{f}{g}}}$$

f, g defined here

(Cussler eqn 10.3-2)

Substituting back redefined variables (15)

denominator:

$$g = \tilde{c}\, y_1 + b = \left(1 - \cancel{\frac{HG}{L}}\right) y_1 + \cancel{\frac{HG}{L} y_1} - Hx_1$$

$$= y_1 - Hx_1 = \boxed{y_1 - \underbrace{y^*(x_1)}_{\text{the gas composition in equilibrium w/ liquid of composition } x_{1,\text{bulk}}}} = g$$

numerator:

$$f = \tilde{c}\, y_2 + b = ? \longrightarrow$$

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$$\tilde{c}\, y_2 + b = \left(1 - \frac{HG}{L}\right) y_2 + b$$

$$b = H\left(\frac{G}{L} y_1 - x_1\right)$$

Use operating line to write b in terms of x_2, y_2

operating line:

$$y = \frac{L}{G}(x - x_1) + y_1$$

at x_2, y_2:

$$y_2 = \frac{L}{G}(x_2 - x_1) + y_1$$

$$\frac{G}{L} y_2 = x_2 \underbrace{- x_1 + \frac{G}{L} y_1}$$

$$\frac{G}{L} y_1 - x_1 = \frac{G}{L} y_2 - x_2 = \frac{b}{H}$$

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Pull it together:

$$\tilde{c} y_2 + b = \left(1 - \cancel{\frac{HG}{L}}\right) y_2 + H\left(\cancel{\frac{G}{L} y_2} - x_2\right)$$

$$= y_2 - H x_2 = \boxed{y_2 - y^*(x_2)} = f$$

HEIGHT OF COLUMN, B:

$$\boxed{B = \frac{G}{K_y a}\left(\frac{1}{1-\frac{G}{L}H}\right)\ln\left(\frac{y_2 - y_2^*}{y_1 - y_1^*}\right)}$$

note: $y_2^* = y(x_2) = H x_2$

$y_1^* = y(x_1) = H x_1$

(Cussler 10.3-12) p316

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Example 6—concluded

Example 6 Solution:

"rate" equation (gas-side, species A mass balance)	$\frac{dy}{dz} = \frac{K_y a}{G}(y - Hx)$
"operating line" equation (overall, species A mass balance)	$y = \frac{L}{G}(x - x_1) + y_1$
"equilibrium line" equation (thermodynamic equilibrium, dilute mixtures)	$y^* = Hx^*$
Column height, ***B*** (result)	$\boldsymbol{B} = \frac{G}{K_y a}\left(\frac{1}{1-\frac{G}{L}H}\right)\ln\left(\frac{y_2 - y_2^*}{y_1 - y_1^*}\right)$

$y_2^* = y^*(x_2)$
$y_1^* = y^*(x_1)$

See Handnotes VI HW 4.16