CM3120 – Module 1

15 Jan 2021
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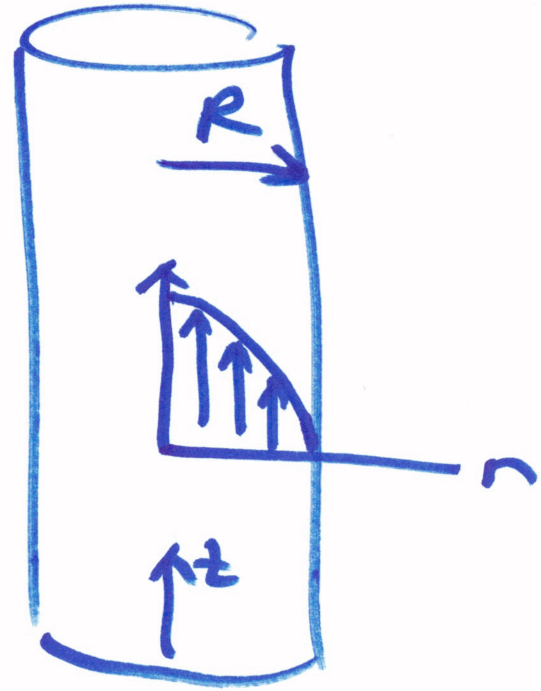
Requests for today?

HW Prob	
7	sketches – k vs h
12	micro E bal
14	fluids data correlations
15b	heat " "
21	nat'l conv + radiation
Ex 2019 #4	MEB
17	Q from $\underline{v}$
18	$\underline{F}$ from $\underline{v}$
11	Temp micro E bal
13	given T(r) $q_r = ?$

HW1 prob 17 book: UFM 6.21 15Jan2021 (1)

$$Q = \iint_S \overset{(\hat{n}\cdot\underline{V})}{v_z}\; \underbrace{dS}_{r\,dr\,d\theta}$$



$S = \pi R^2$

cross sectional area

$$Q = \int_0^R \int_0^{2\pi} v_z \, r\,dr\,d\theta$$

$$\int_0^{2\pi} d\theta = \theta \Big|_0^{2\pi} = 2\pi$$

$$\frac{Q}{2\pi} = \int_0^R 12.0\left(1 - \frac{r^2}{\underbrace{0.012}_{R}{}^2}\right) r\,dr$$

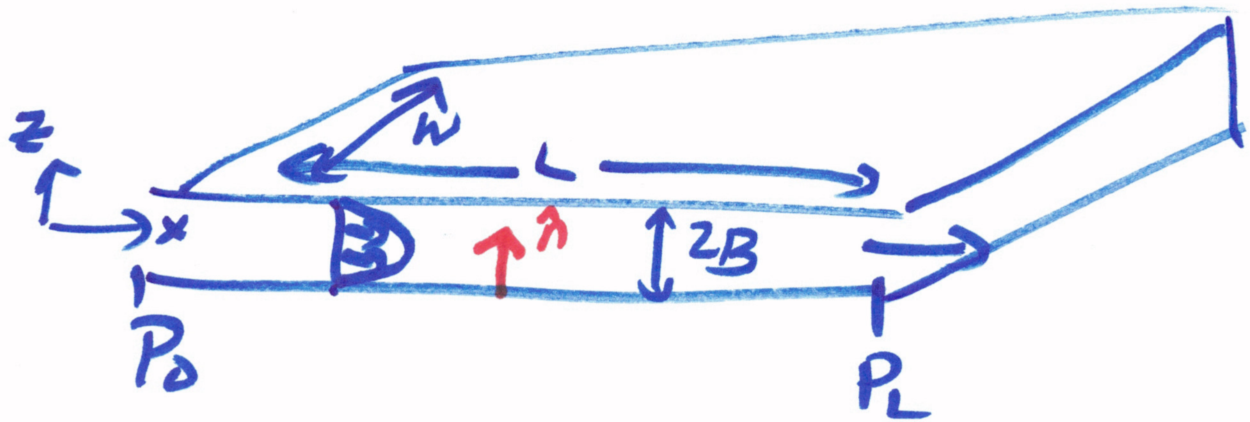
$$\frac{Q}{(2\pi)(12.0)} = \int_0^R \left(r - \frac{r^3}{R^2}\right) dr$$

$$\left(\frac{r^2}{2} - \left(\frac{1}{R^2}\right)\frac{r^4}{4}\right)\Big|_0^R$$

$$\frac{R^2}{2} - \frac{R^2}{4} = \frac{R^2}{4}$$

$$Q = \frac{R^2}{4}(2\pi)(12.0) \quad //$$

1.18



$P = c\,x$ pressure distribution

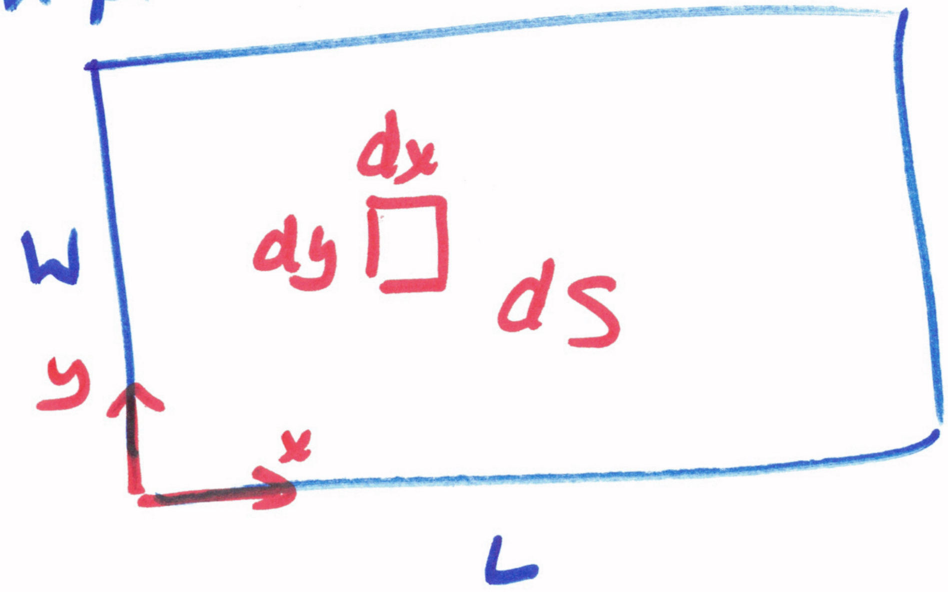
$-B < z < B$

$$\underline{F} = \iint_{S} \int_0^W \int_0^L \left(\hat{n} \cdot \underline{\underline{\widetilde{\Pi}}}\right)_{\substack{\text{at the surface} \\ z=-B}} \overbrace{dS}^{dx\,dy}$$

normal to bottom plate, "wet" side

$$\hat{n} = \begin{pmatrix} n_x \\ n_y \\ n_z \end{pmatrix}_{xyz} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}_{xyz}$$

bottom plate $(z = B)$



$-B$

$$-\overbrace{\left(\frac{2\mu A}{B^2}\right)}^{\alpha} z$$

$0 \le x \le L$

$0 \le y \le W$

$$\hat{n} \cdot \underline{\underline{\tilde{\Pi}}} = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -P & 0 & B\alpha \\ 0 & -P & 0 \\ B\alpha & 0 & -P \end{pmatrix}$$

$$= \begin{pmatrix} B\alpha & 0 & -P \end{pmatrix}_{xyz}$$

$$\underline{F} = \int_0^W \int_0^L \begin{pmatrix} B\alpha \\ 0 \\ -p \end{pmatrix}_{xyz} dx\,dy$$

$$F_x = \int_0^W \int_0^L \overbrace{B\alpha}^{\text{constant}} \, dx\,dy$$

$$B\alpha \, x \Big|_0^L = B\alpha L$$

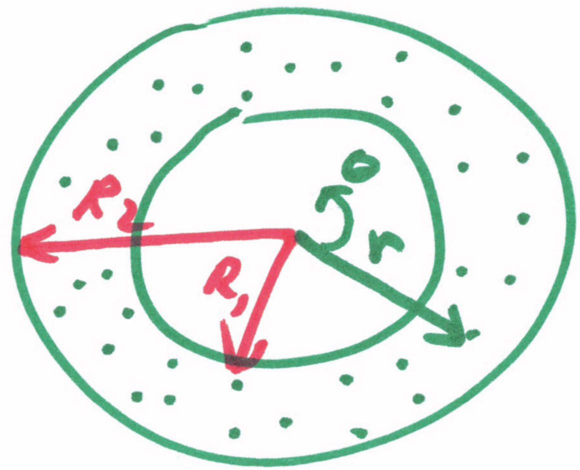
$$\int_0^W B\alpha L \, dy = B\alpha L \cdot y \Big|_0^W$$

$$= B\alpha L W$$

$$F_x = \cancel{B} L W \left(\frac{2\mu A}{B\cancel{x}} \right)$$

$$\boxed{F_x = \frac{2\mu A L W}{B}}$$

1.11 (HW1)



$R_1 \leq r \leq R_2$

$0 \leq \theta \leq 2\pi$

$r = R_1 \quad T = T_1$

$r = R_2 \quad T = T_2$

What is $T(r)$?

+1/2

The **Equation of Energy** for systems with **constant k**

Microscopic energy balance, constant thermal conductivity; Gibbs notation

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + \underline{v} \cdot \nabla T \right) = k \nabla^2 T + S_e$$

Microscopic energy balance, constant thermal conductivity; Cartesian coordinates

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} + v_z \frac{\partial T}{\partial z} \right) = k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + S_e$$

⊖ symmetry

Microscopic energy balance, constant thermal conductivity; cylindrical coordinates

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} + v_z \frac{\partial T}{\partial z} \right) = k \left(\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2} \right) + S_e$$

Microscopic energy balance, constant thermal conductivity; spherical coordinates

steady

long

no rxn

no elec current

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} + \frac{v_\phi}{r \sin \theta} \frac{\partial T}{\partial \phi} \right)$$
$$= k \left(\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial T}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 T}{\partial \phi^2} \right) + S_e$$

$\underline{V} = 0$ pipe is solid

$$r \cdot \frac{0}{k} = \frac{\cancel{k}}{\cancel{k}} \frac{1}{\cancel{r}} \frac{d}{dr} \left(r \frac{dT}{dr} \right)$$

$$0 = \frac{d}{dr} \underbrace{\left(r \frac{dT}{dr} \right)}_{\equiv \Phi}$$

Reference: F. A. Morrison, "Web Appendix to *An Introduction to Fluid Mechanics*," Cambridge University Press, New York, 2013 (*pages.mtu.edu/~fmorriso/cm310/IFMWebAppendixDMicroEBalanceMorrison.pdf*). This worksheet is on the web at *pages.mtu.edu/~fmorriso/cm310/energy.pdf*.

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$$\frac{d\Phi}{dr} = 0$$

$$r\frac{dT}{dr} = \Phi = 0 + C_1 = C_1$$

$$\frac{dT}{dr} = C_1\left(\frac{1}{r}\right)$$

$$\boxed{T = C_1 \ln r + C_2}$$

BC: $r = R_1 \quad T = T_1$

$r = R_2 \quad T = T_2$

$$\begin{cases} T_1 = C_1 \ln R_1 + C_2 \\ T_2 = C_1 \ln R_2 + C_2 \end{cases}$$

2 eqns, 2 unknowns

subtract: $T_1 - T_2 = c_1 (\overbrace{\ln R_1 - \ln R_2}^{\ln R_1/R_2})$

$$\boxed{c_1 = \frac{(T_1 - T_2)}{\ln \frac{R_1}{R_2}}}$$

substitute:

(or could substitute into the other eqn) BC

$$T_1 = \left(\frac{T_1 - T_2}{\ln \frac{R_1}{R_2}} \right) \ln R_1 + c_2$$

Answer: $$T = \left(\frac{T_1 - T_2}{\ln \frac{R_1}{R_2}} \right) \ln r + \overbrace{T_1 - \left(\frac{T_1 - T_2}{\ln \frac{R_1}{R_2}} \right) \ln R_1}^{c_2}$$

$$T - T_1 = \left(\frac{T_1 - T_2}{\ln \frac{R_1}{R_2}} \right) (\ln r - \ln R_1)$$

$$\boxed{\left(\frac{T - T_1}{T_1 - T_2} \right) = \frac{\ln \frac{r}{R_1}}{\ln \frac{R_1}{R_2}}}$$

1.13 (HW1) Calc radial flux

Expand $T(r)$ algebraically:

$$T = T_{b2} + \underbrace{\left(\frac{T_{b1} - T_{b2}}{\frac{k}{h_2 R_2} + \ln\frac{R_2}{R_1} + \frac{k}{h_1 R_1}} \right)}_{\alpha} \ln\frac{R_2}{r} + \underbrace{\left(\quad \right) \frac{k}{h_2 R_2}}_{\beta}$$

$$T = T_{b2} + \alpha \ln\frac{R_2}{r} + \beta$$

$$= T_{b2} + \alpha\left(\ln R_2 - \ln r\right) + \beta$$

$$= \underbrace{\left(T_{b2} + \alpha \ln R_2 + \beta\right)}_{\delta} - \alpha \ln r$$

$$T = \delta - \alpha \ln r$$

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$$\frac{q_r}{A} = -k \frac{dT}{dr}$$ from above

Fourier's Law

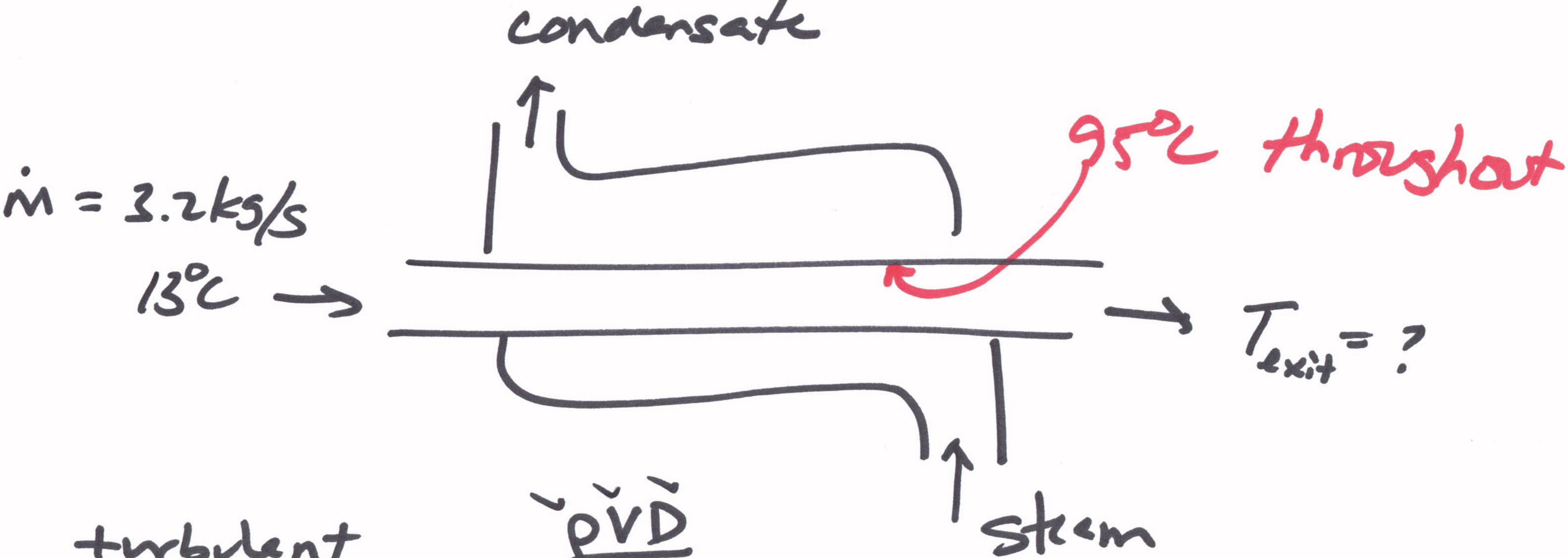
$$= -k\left(-\alpha \frac{1}{r}\right)$$

$$= \frac{k\alpha}{r} = \boxed{k\left(\frac{T_{b_1} - T_{b_2}}{\frac{k}{h_2 R_2} + \ln\frac{R_2}{R_1} + \frac{k}{h_1 R_1}}\right)\frac{1}{r} = \frac{q_r}{A}}$$

STUDENT HOURS
(following class)

Discussion HW1
prob 15

1.15 (HW1) OH 15 Jan 2021 ⑯



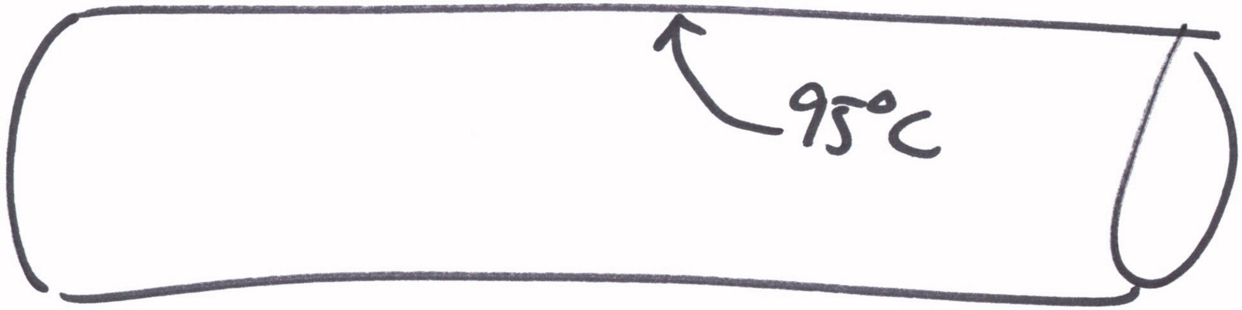
turbulent Sieder-Tate

$$\frac{h_{lm} D}{k} = 0.027\, Re^{0.8}\, Pr^{\frac{1}{3}} \left(\frac{\mu_b}{\mu_w}\right)^{0.14}$$

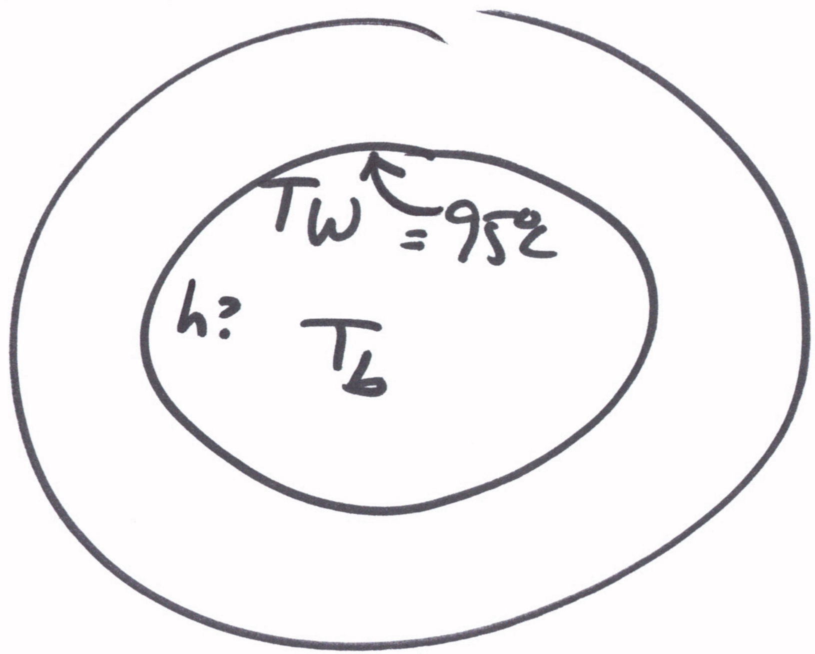
$$Re = \frac{\tilde{\rho} \tilde{V} \tilde{D}}{\tilde{\mu}}$$

see steam handout packet

$$q_{lm} = h_{lm}\, \overbrace{A}^{\pi D L}\, \Delta T_{lm}$$



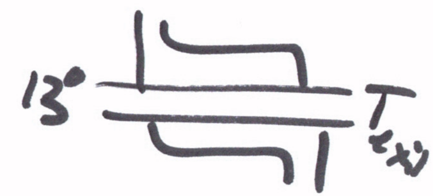
obtain h
from
data
correlations



... a

$$h = 5300 \ \frac{W}{m^2 K}$$

b) What is exit temperature?



PART OF SEIDER-TATE:

$$q_{lm} = h_{lm} (\pi D L) \Delta T_{lm}$$

$$\Delta T_{lm} = \frac{(95-13)-(95-T_{exit})}{\ln\left(\frac{95-13}{95-T_{exit}}\right)}$$

equate; solve for T_{exit}

MACRO E BAL INSIDE:

$$\cancel{\Delta E_p} + \cancel{\Delta E_k} + \Delta H = Q_{in} + \cancel{W_{s,on}}$$

$$q_{lm} = Q_{in} = \Delta H = \dot{m} C_p (T_{exit} - 13°C)$$