CM3120 8Feb2021 F. Morrison

1

Prm 2 unsteady "short-cut" solns

Example: A long, wide rectangular slab of butter ($46\ mm$ thick) at 4.4^oC is removed from refrigeration and placed on a table at room temperature. After five hours, what are the butter temperatures at the middle of the slab and at the bottom of the slab (in contact with the table)?

Properties of butter:

$$k_{butter} = 0.197\frac{W}{mK}$$

$$\hat{C}_{p\,butter} = 2.30\frac{kJ}{kg\ K}$$

$$\rho_{butter} = 998\frac{kg}{m^3}$$

Conditions of the room:

$$T_{bulk} = 24^oC$$

$$h_{conv} = 8.5\frac{W}{m^2K}$$

Handnotes: https://pages.mtu.edu/~fmorriso/cm3120/2021_solve_Gurney-Lurie_problem_butter.pdf butter warming: ③

The **Equation of Energy** for systems with **constant** $\boldsymbol{k}$

Microscopic energy balance, constant thermal conductivity; Gibbs notation

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t} + \underline{v}\cdot\nabla T\right) = k\nabla^2 T + S_e$$

Microscopic energy balance, constant thermal conductivity; Cartesian coordinates

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t} + \underbrace{\cancel{v_x\frac{\partial T}{\partial x}} + \cancel{v_y\frac{\partial T}{\partial y}} + \cancel{v_z\frac{\partial T}{\partial z}}}_{v=0}\right) = k\left(\frac{\partial^2 T}{\partial x^2} + \cancel{\frac{\partial^2 T}{\partial y^2}} + \cancel{\frac{\partial^2 T}{\partial z^2}}\right) + \cancel{S_e}$$

no rxn
no elec current

Microscopic energy balance, constant thermal conductivity; cylindrical coordinates

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t} + v_r\frac{\partial T}{\partial r} + \frac{v_\theta}{r}\frac{\partial T}{\partial \theta} + v_z\frac{\partial T}{\partial z}\right) = k\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2}\right) + S_e$$

long, wide

Microscopic energy balance, constant thermal conductivity; spherical coordinates

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t} + v_r\frac{\partial T}{\partial r} + \frac{v_\theta}{r}\frac{\partial T}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial T}{\partial \phi}\right)$$
$$= k\left(\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial T}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 T}{\partial \phi^2}\right) + S_e$$

$$\frac{\partial T}{\partial t} = \frac{k}{\rho C_p}\frac{\partial^2 T}{\partial x^2}$$ (familiar)

IC: $t<0 \quad T=T_0 \quad \forall x$

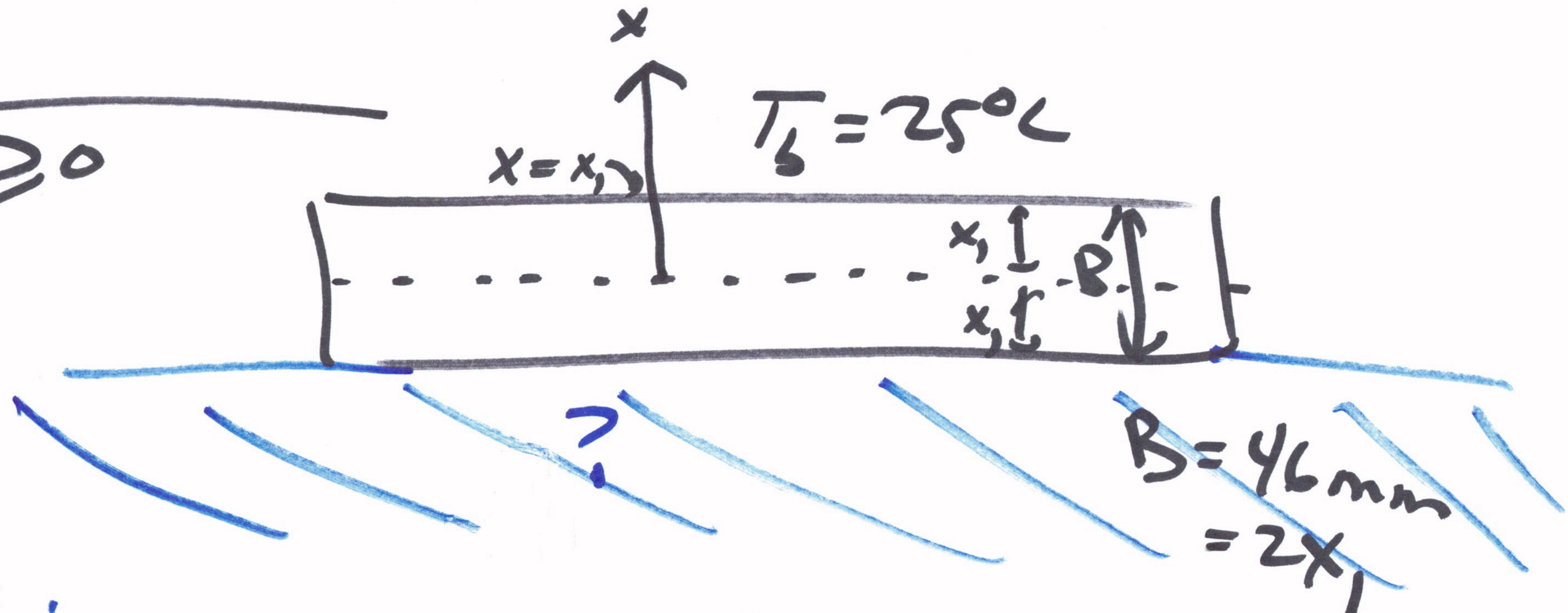
BC: top – Newton's law of cooling
bottom – ??

Reference: F. A. Morrison, "Web Appendix to *An Introduction to Fluid Mechanics*," Cambridge University Press, New York, 2013 (*pages.mtu.edu/~fmorriso/cm310/IFMWebAppendixDMicroEBalanceMorrison.pdf*). This worksheet is on the web at *pages.mtu.edu/~fmorriso/cm310/energy.pdf*.

$t < 0$

$T_0 = 4.4\,^{\circ}C$

$t \geq 0$



$x = x_1 \qquad \left|\frac{q_x}{A}\right| = h\left|T_b - T\right|$

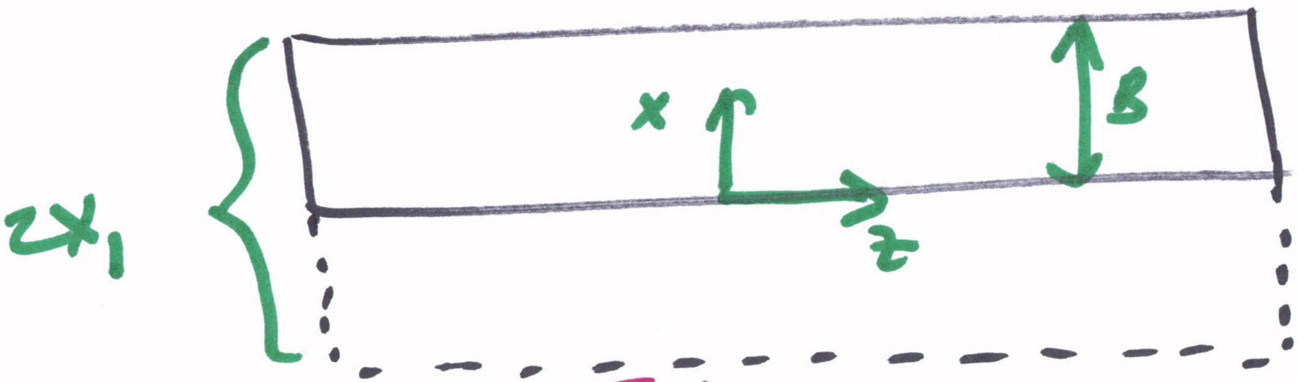
This is a "new" (to us) std BC

$x = 0 \qquad \frac{q_x}{A} = 0$

To match w/ Gurney-Lurie:

* Model a slab twice as thick

T_b, h



$B = x_1$

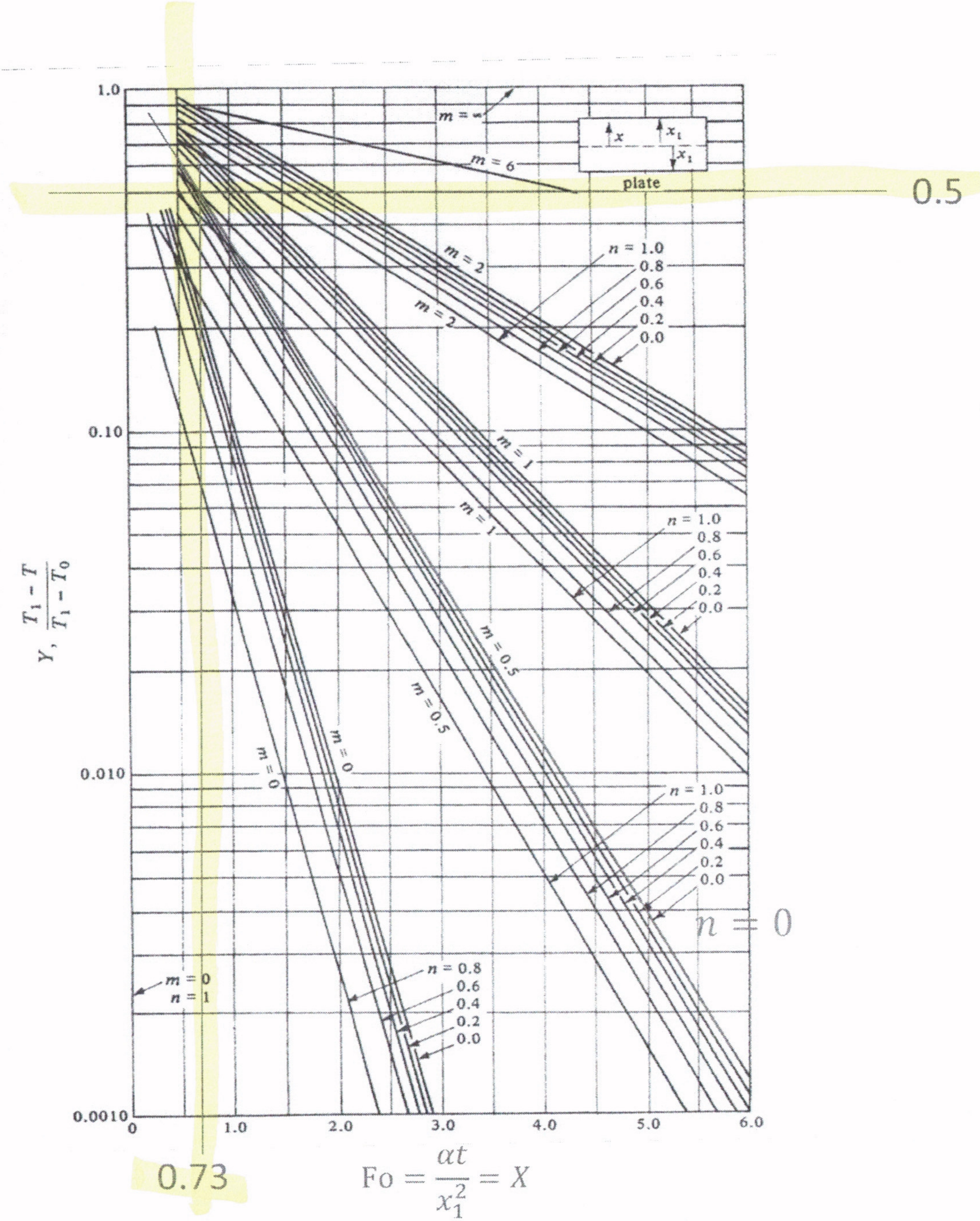
T_b, h

$x = B \qquad -k\frac{dT}{dx} = h(T - T_b)$

$x = 0 \qquad \frac{q_x}{A} = 0$

(equivalent to $x = -B$, Newton's law of cooling)

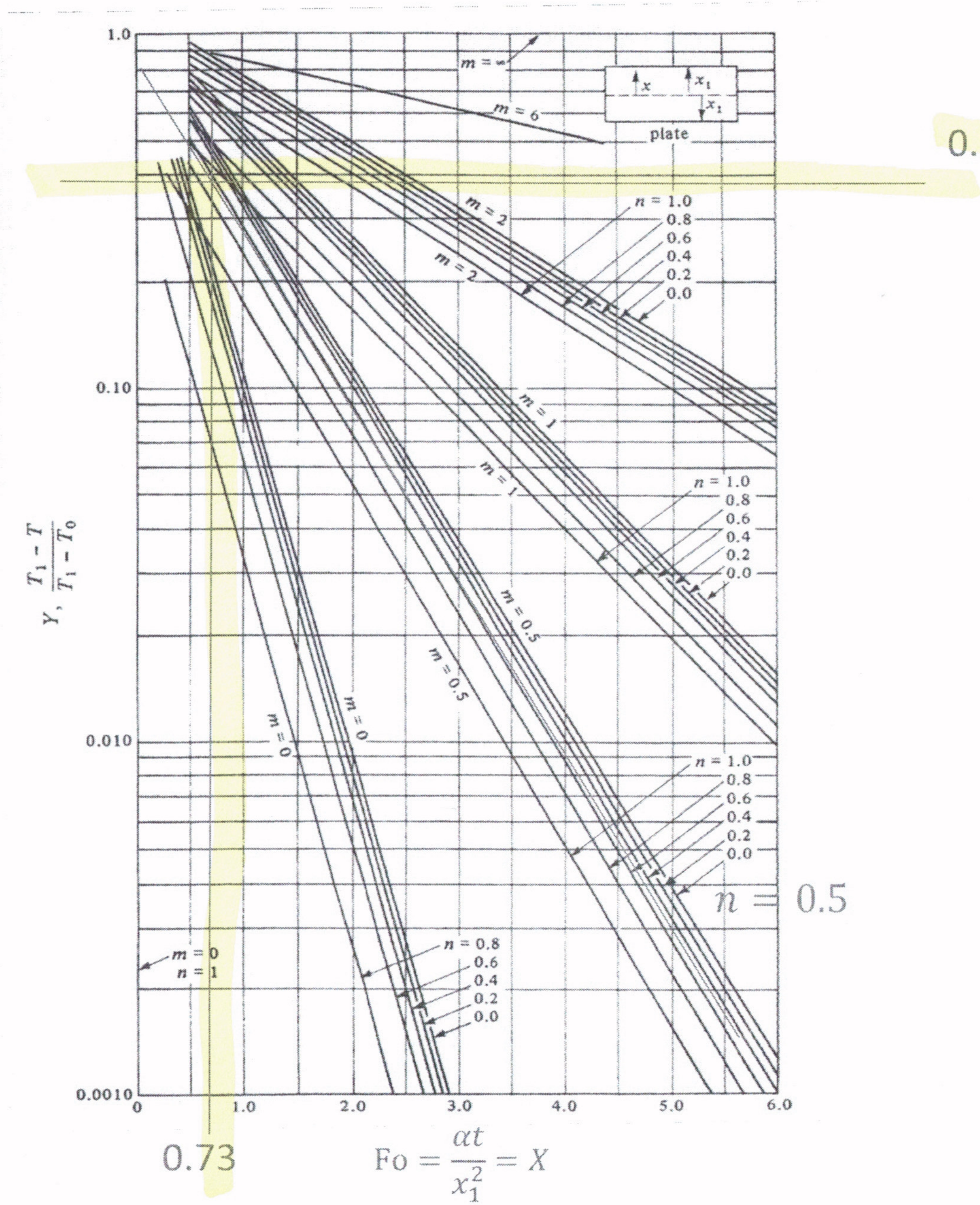
This set up forces the correct BC at the bottom of the real slab.



At the bottom, $x = 0$

$$\frac{T_1 - T}{T_1 - T_0} = 0.5$$
$$T = 14^o C$$

Ref: Geankoplis, 4th Ed, 2003



At the middle,
$x = 23mm$

$$\frac{T_1 - T}{T_1 - T_0} = 0.38$$
$$T = 17^o C$$

Ref: Geankoplis, 4th Ed, 2003