

CM3120
HANDNOTES

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How to measure h ?

$$t < 0, T = T_0$$

$$t \geq 0$$



$$T_b = \text{hot}$$

to remove the
abs. value
bars, we
match
the signs
↙ left
↘ right

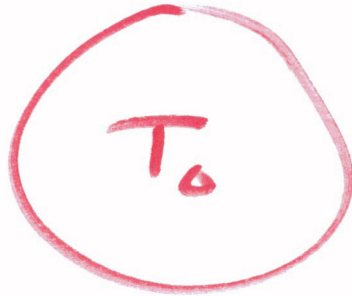
Newton's law of cooling at boundary $r=R$

$$-k \frac{dT}{dr} = \left| \frac{q_r}{A} \right| = h |T_w - T_b|$$

$$\frac{q_r}{A} < 0$$

$$\frac{q_r}{A} = h \underbrace{(T_w - T_b)}_{< 0}$$

$t < 0$



T_0

$$t < 0 \quad T = T_0 \quad \forall r$$

$t \geq 0$



T_b

$$r = R \quad -k \frac{dT}{dr} = h (T - T_b)$$

$$r = 0$$

$$\frac{\partial T}{\partial r} = 0$$

The **Equation of Energy** for systems with **constant k**

Microscopic energy balance, constant thermal conductivity; Gibbs notation

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + \underline{v} \cdot \nabla T \right) = k \nabla^2 T + S_e$$

Microscopic energy balance, constant thermal conductivity; Cartesian coordinates

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} + v_z \frac{\partial T}{\partial z} \right) = k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + S_e$$

Microscopic energy balance, constant thermal conductivity; cylindrical coordinates

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} + v_z \frac{\partial T}{\partial z} \right) = k \left(\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2} \right) + S_e$$

Microscopic energy balance, constant thermal conductivity; spherical coordinates

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} + \frac{v_\phi}{r \sin \theta} \frac{\partial T}{\partial \phi} \right) = k \left(\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial T}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 T}{\partial \phi^2} \right) + S_e$$

no elec current
no rxn

$\underline{v} = 0$

θ, ϕ symmetry

$$\frac{\partial T}{\partial t} = \frac{k}{\rho \hat{C}_p} \left(\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) \right)$$

Now, look up solution →

Reference: F. A. Morrison, "Web Appendix to An Introduction to Fluid Mechanics," Cambridge University Press, New York, 2013 (pages.mtu.edu/~fmorriso/cm310/IFMWebAppendixDMicroEBalanceMorrison.pdf). This worksheet is on the web at pages.mtu.edu/~fmorriso/cm310/energy.pdf.