Requests for today?

② • 2.11 Blocks of snow - troubleshooting

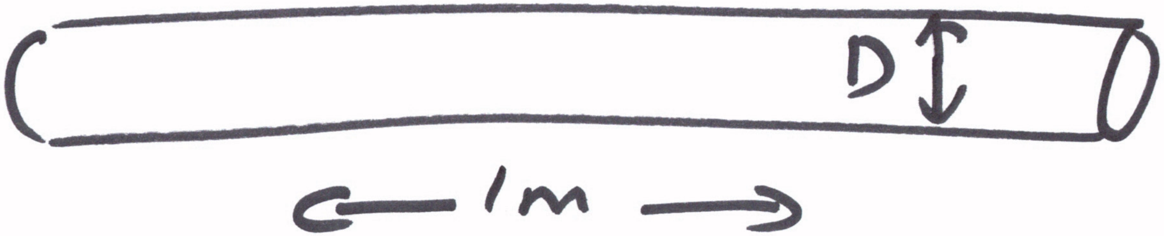
① • 2.9 Determine longest shaft diameter

- try Gurney Lurie
- turn to semi-infinite slab

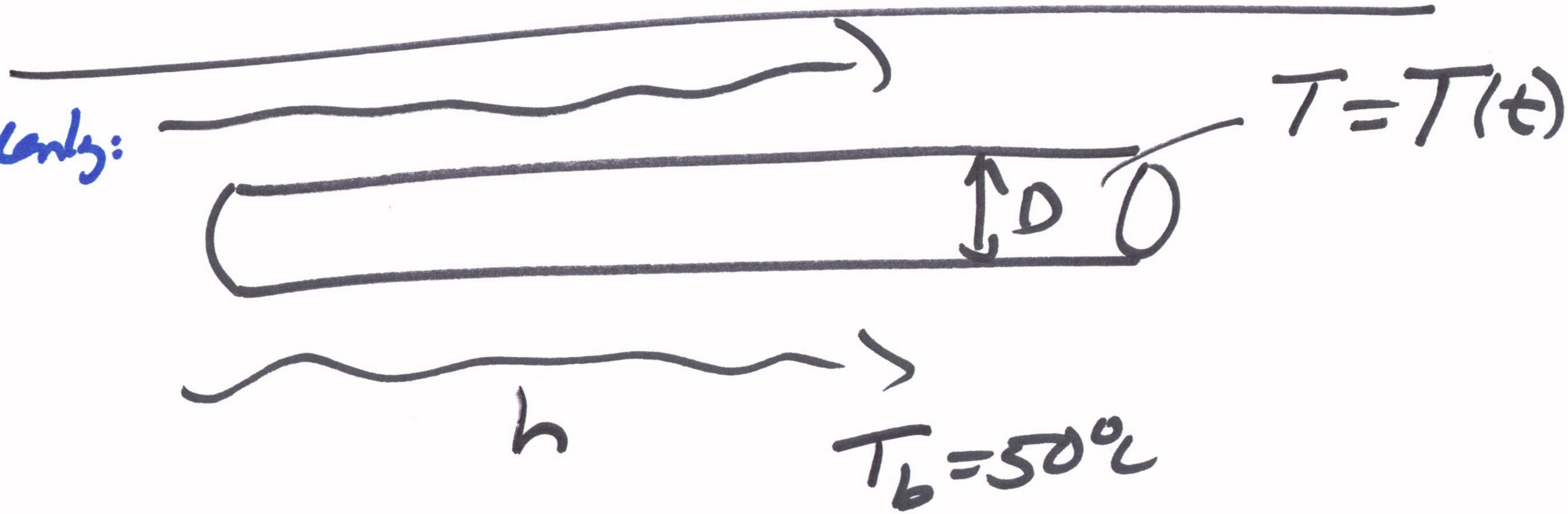
2.9

$t<0$

$T = T_0 = 400°C$



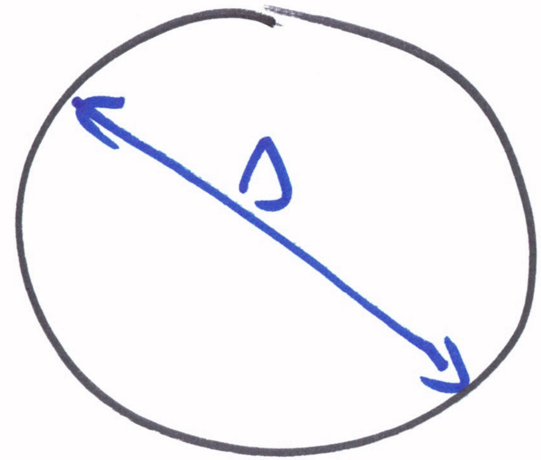
Suddenly:



Biot # organizes what we see:

(ratio of resistances)

$$Bi = \frac{h D_{char}}{k_{solid}} = \frac{\frac{D}{k}}{\frac{1}{h}}$$



☆ lumped parameter case

Bi small
T is uniform w/in object

Bi medium

Bi large
$T_w = T_b$

criterion for lumped parameter case:

$$Bi = \frac{h D_{char}}{k} < 0.1$$

$$D_{char} = \frac{V}{SA}$$

surface area

know

$$0.1 = \frac{h}{k}\,\frac{V}{A}$$

know

$V = \cancel{\pi} \cancel{R^2} \cancel{L}$

$A = 2 \cancel{\pi} \cancel{R} \cancel{L}$

Solve for $R = 2.5\,cm$

$$\boxed{D_{char} = 5\,cm}$$

Second problem

2.11

Blocks of compacted snow (dimensions 2 meters by 1.2 meters by 0.7 meters) are created to form part of a wintertime festive display (Winter Carnival). The weather has been such that the blocks are at a uniform temperature of -10.0^oC. If the weather turns unseasonably warm (7.0^oC) what will be the temperature near the surface of the block (5 and 10 centimeters from the surface) after 0.5 hours?

$t = 0.5\,h$

$x_1 = 5\,cm$

$x_2 = 10\,cm$

$T(x,y,z,t)$

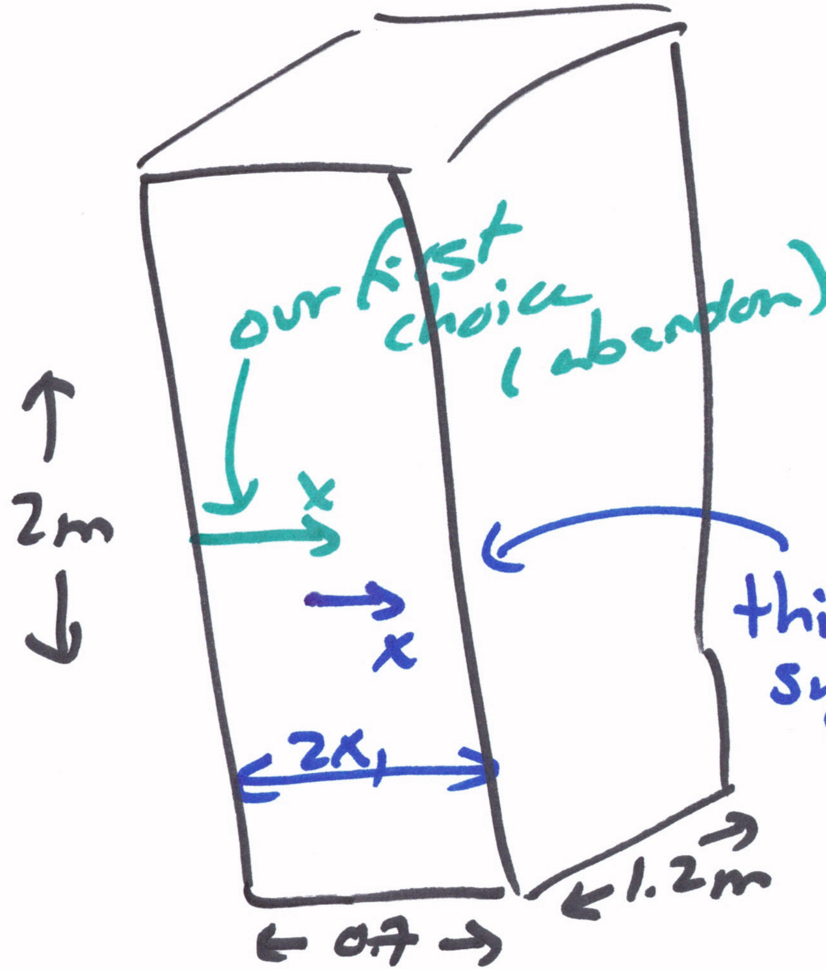
$t < 0$

$T = T_0 \quad \forall\, x,y,z$

Suddenly,

$t \geq 0$

T_b



this coord system is required by G-L chart

S

11. Blocks of compacted snow (dimensions 2 meters by 1.2 meters by 0.7 meters) are created to form part of a wintertime festive display (Winter Carnival). The weather has been such that the blocks are at a uniform temperature of -10.0^oC. If the weather turns unseasonably warm (7.0^oC) what will be the temperature near the surface of the block (5 and 10 centimeters from the surface) after 0.5 hours?

Physical property data for compacted snow: $k = 0.54\frac{W}{mK}, \rho = 68\frac{kg}{m^3}, \hat{C}_p = 2090\frac{J}{kgK}$. The heat transfer coefficient under the prevailing weather conditions may be estimated to be $12W/m^2K$. Answers: $5\ cm, -2.5^oC$; $10\ cm, -6^oC$

physical property data provided (but may be approximate

$\alpha = \frac{k}{\rho C_p} = 3.8 \times 10^{-6}\ m^2/s$

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The **Equation of Energy** for systems with **constant k**

Microscopic energy balance, constant thermal conductivity; Gibbs notation

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t}+\underline{v}\cdot\nabla T\right)=k\nabla^2 T+S_e$$

Microscopic energy balance, constant thermal conductivity; Cartesian coordinates

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t}+\cancel{v_x\frac{\partial T}{\partial x}}+\cancel{v_y\frac{\partial T}{\partial y}}+\cancel{v_z\frac{\partial T}{\partial z}}\right)=k\left(\frac{\partial^2 T}{\partial x^2}+\cancel{\frac{\partial^2 T}{\partial y^2}}+\cancel{\frac{\partial^2 T}{\partial z^2}}\right)+\cancel{S_e}$$

no rxn
no current

Microscopic energy balance, constant thermal conductivity; cylindrical coordinates

solid
$\underline{v}=0$

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t}+v_r\frac{\partial T}{\partial r}+\frac{v_\theta}{r}\frac{\partial T}{\partial\theta}+v_z\frac{\partial T}{\partial z}\right)=k\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 T}{\partial\theta^2}+\frac{\partial^2 T}{\partial z^2}\right)+S_e$$

1D
rect
heat
conduction

Microscopic energy balance, constant thermal conductivity; spherical coordinates

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t}+v_r\frac{\partial T}{\partial r}+\frac{v_\theta}{r}\frac{\partial T}{\partial\theta}+\frac{v_\phi}{r\sin\theta}\frac{\partial T}{\partial\phi}\right)$$
$$=k\left(\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial T}{\partial\theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 T}{\partial\phi^2}\right)+S_e$$

IC: $t\leq 0 \quad T=T_0 \quad \forall x$

BC: $t>0 \quad \frac{q_x}{A}=0 \quad x=0$

$x=x_1 \quad \left|\frac{q_x}{A}\right|=h\left|T_b-T_w\right|$

Reference: F. A. Morrison, "Web Appendix to *An Introduction to Fluid Mechanics*," Cambridge University Press, New York, 2013 (*pages.mtu.edu/~fmorriso/cm310/IFMWebAppendixDMicroEBalanceMorrison.pdf*). This worksheet is on the web at *pages.mtu.edu/~fmorriso/cm310/energy.pdf*.

try Gurney Lurie
or semi infinite slab

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Finite 1D Unsteady Heat Transfer, $T = T(t,x)$ or $T = T(t,r)$

Initial: Uniform initial temperature T_0; BC: exposed to bulk temperature T_1; h known

- Flat plate long, wide, thickness $= 2x_1$ $\quad T = T(t,x) \quad Y = Y(X,n)$
- Cylinder long, radius $= x_1$ $\quad T = T(t,r) \quad Y = Y(X,n)$
- Sphere radius $= x_1$ $\quad T = T(t,r) \quad Y = Y(X,n)$

The Gurney-Lurie charts may work for us:

$$\mathrm{Bi} = \frac{hD_{char}}{k} = \frac{hx_1}{k} = \frac{1}{m}$$

$$\mathrm{Fo} = \frac{\alpha t}{x_1^2} = X$$

$$\frac{x}{x_1} = \frac{r}{x_1} = n$$

$$\frac{T_1 - T}{T_1 - T_0} = Y \quad \left(\frac{T - T_0}{T_1 - T_0} = 1 - Y\right)$$

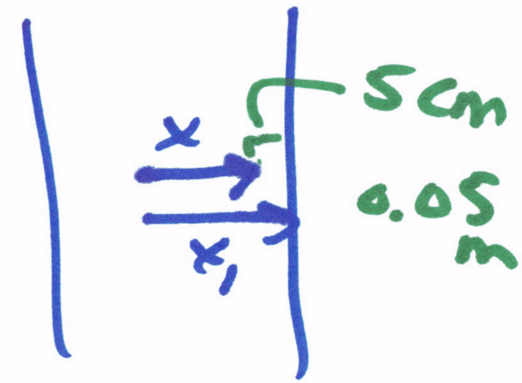
7°C

−10°C

Note:
$D_{char} = x_1$,
<u>NOT</u> V/A

TRY GURNEY–LURIE CHART

slab



$$2X_1 = 0.7$$

$$\boxed{X_1 = 0.35\,m}$$

$$m = \frac{1}{Bi} = \frac{k}{hD} = \frac{k}{hx_1} = \boxed{0.13 = m}$$

$$n = \frac{x}{x_1} = \frac{0.35 - 0.05}{0.35} = 0.86 \approx \overset{n}{\boxed{0.9}}$$

$$F_o = \frac{\alpha t}{x_1^2} = \frac{3.8\times10^{-6}\,\frac{\cancel{m^2}}{\cancel{s}}\,(1800\,\cancel{s})}{(.35\,\cancel{m})^{\cancel{2}}} = \underset{F_o}{\boxed{0.056}}$$

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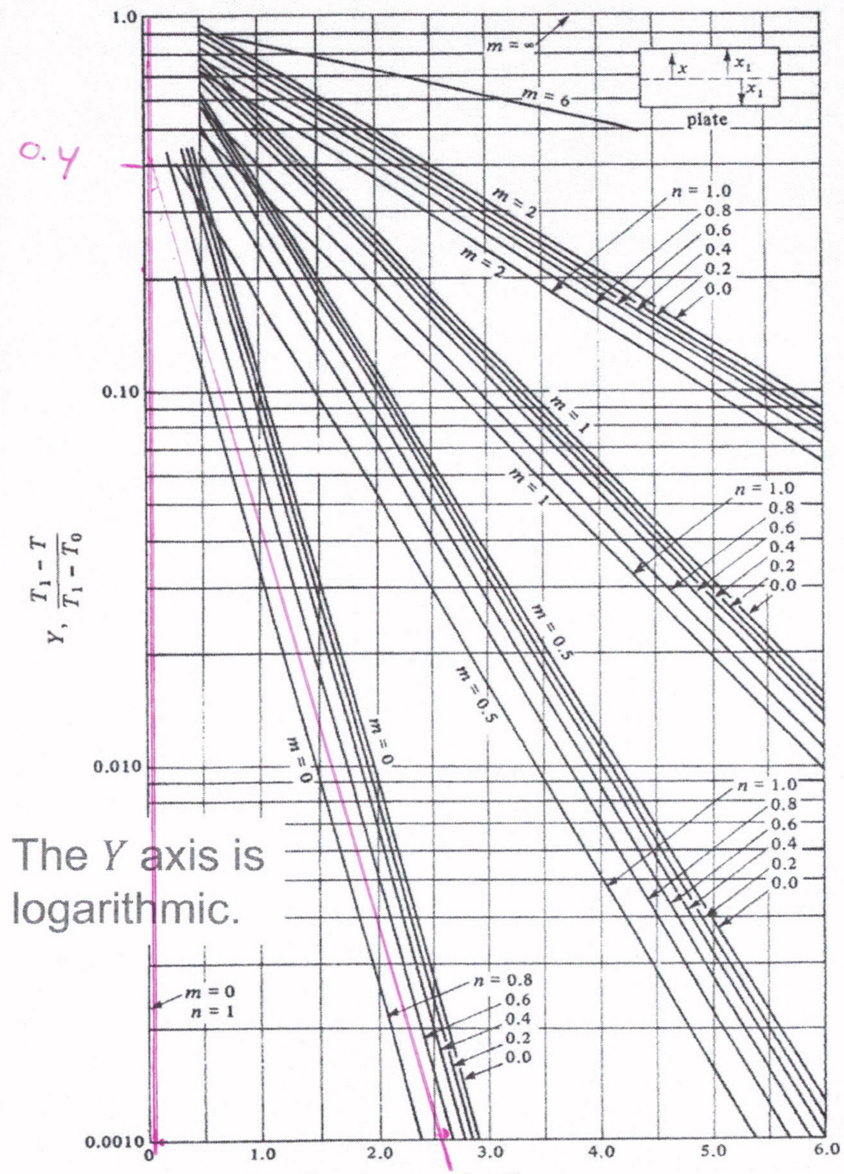

$$\mathrm{Fo} = \frac{\alpha t}{x_1^2} = X$$

Gurney and Lurie Chart
Finite 1D Unsteady Heat Transfer

Initial: Uniform initial temperature T_0

BC: Exposed to bulk temperature T_1

h known

- Flat plate, long, wide
- thickness $2x_1$
- $T = T(t, x)$
- $Y = Y\left(\mathrm{Fo}, n = \frac{x}{x_1}\right)$

$$T\left(m = \frac{1}{\mathrm{Bi}}, n = \frac{x}{x_1}, \mathrm{Fo}\right)$$

$m = 0.1 = 0$

$n = 0.9 \sim 1$

$\mathrm{Fo} = 0.06$

Not on the chart!

The question is about short times + locations near the surface — maybe the semi infinite slab is a better picture

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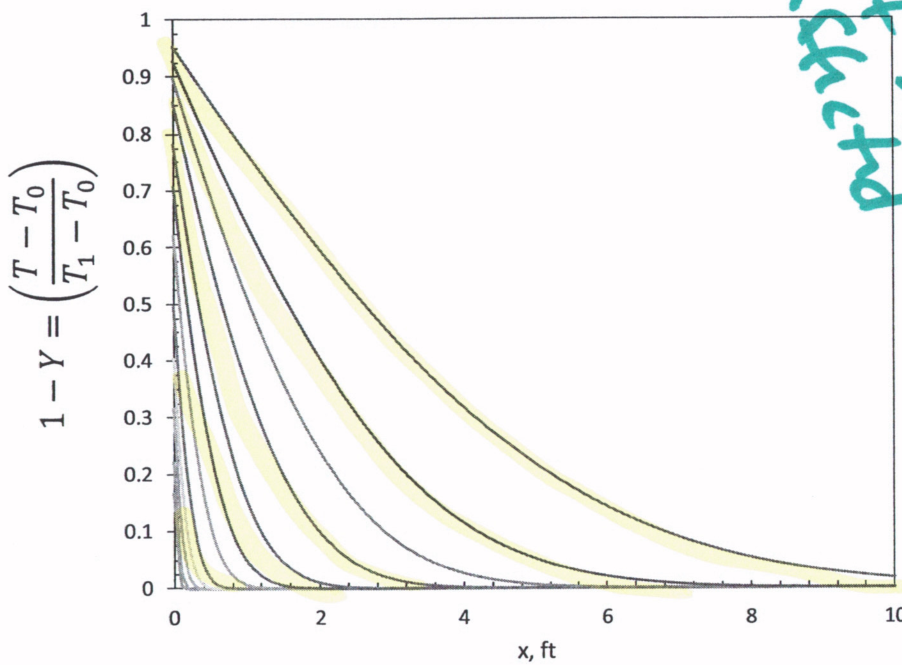
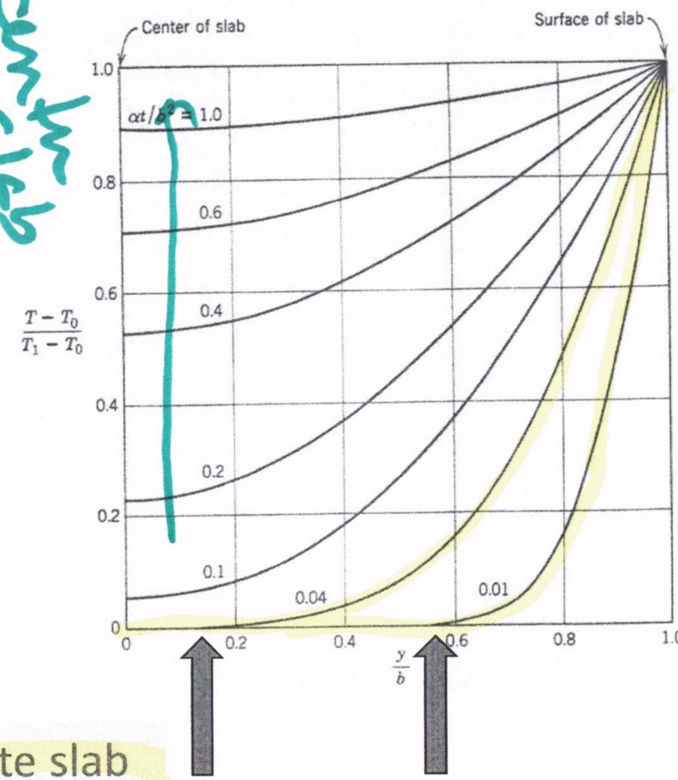
Note: We can use the semi-infinite slab solution for finite slabs, within limits

Semi-Infinite Slab



center of slab affected

Finite Slab



BSL1, p356, 1960

For some cases, the finite slab looks semi-infinite

- Short time
- Thicker slab

The temp change at most is by $T_1 - T_0$ degrees. When $1-y=0.01$, 99% of the max temp change has occurred

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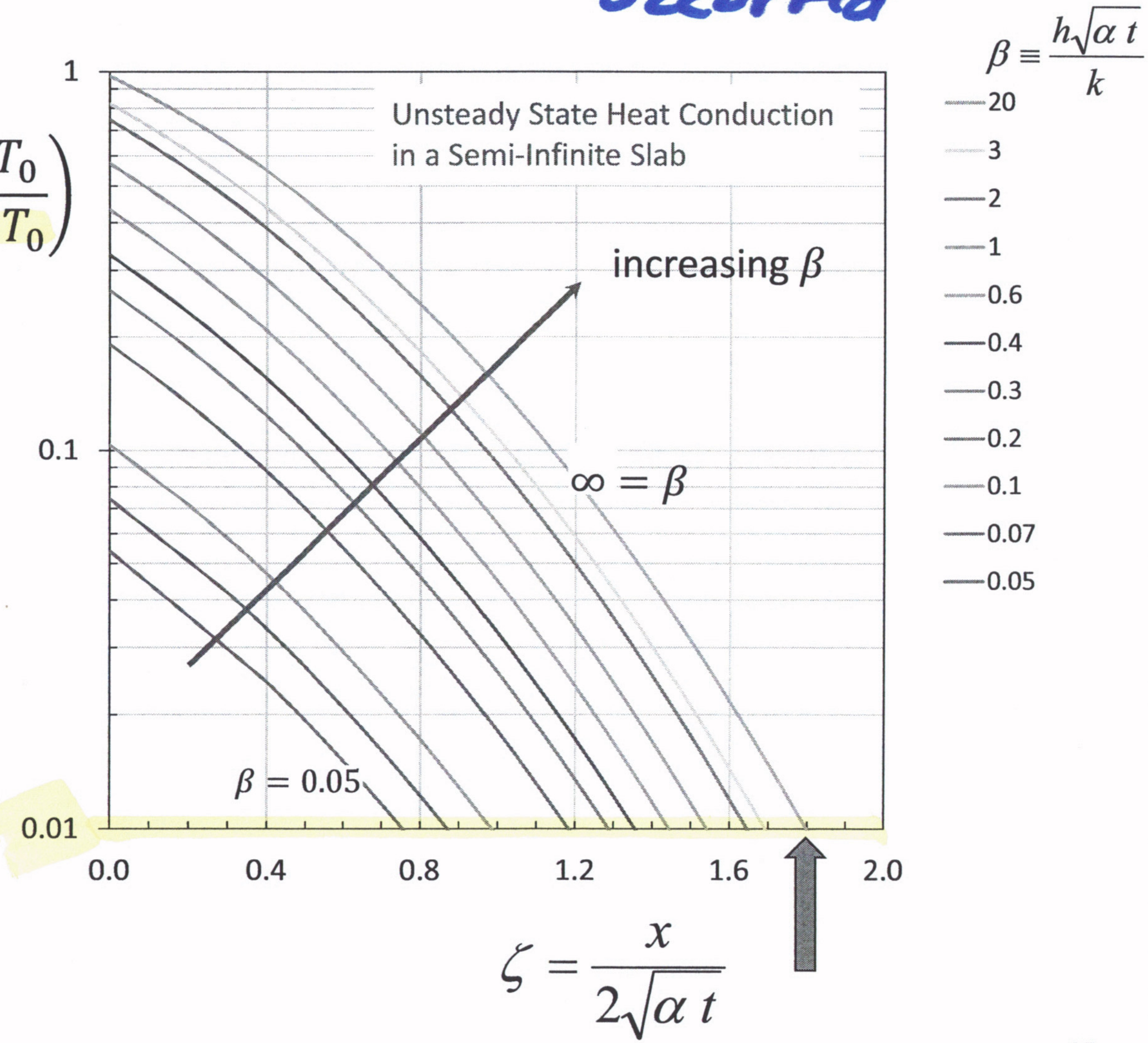
Note: We can use the semi-infinite slab solution for finite slabs, within limits

$$1 - Y = \left(\frac{T - T_0}{T_1 - T_0}\right)$$

ζ = ZETA



For values of $\zeta(t,x)$ that predict $T \approx T_0$ or $(1-Y) = 0.01$ at a distance equal to the half-slab thickness b, the semi-infinite slab solution is equivalent to the finite slab; this occurs $\forall\ \beta$ when $\zeta_{half-slab} = \frac{b}{2\sqrt{\alpha t}} > 1.8$

When $\zeta_b > 1.8$, no matter what h is, the temp at $x=b$ is still T_0

Try semi-infinite slab:

$$\zeta_{x_1} > 1.8 \ ?$$

$$\frac{x_1}{2\sqrt{\alpha t}} = \frac{(0.35\,\cancel{m})}{2\sqrt{3.8\times10^{-6}\,\frac{\cancel{m^2}}{\cancel{s}}\,1800\,\cancel{s}}}$$

$$= 2.1 \searrow$$

To calc T(5 cm), T(10 cm) $\longrightarrow$

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$$\beta = \frac{h}{k}\sqrt{\alpha t} = 1.84 \sim 2$$

$$\zeta = \frac{x}{2\sqrt{\alpha t}}$$

5cm $\zeta = 0.3$ $T = -2.5°C$

10cm $\zeta = 0.6$ $T = -6°C$

(read off graph
1-sig-fig)