1 Feb 2021 MODULE 2 F. MORRISON ①
CM3120

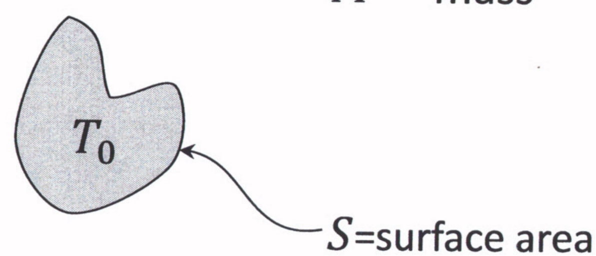
Unsteady macro Energy Bal

CM3120 Module 2—Cooling of a recently manufactured part

Example: Brass parts (oddly shaped, mass M with surface area S) are ejected at regular intervals from a machine that fabricates them. When ejected, the very hot parts at temperature T_0 enter a moving air stream where the air temperature is T_{bulk}. Create a model that will allow us to calculate the temperature of the part as a function of time. Using Excel, calculate $T(t)$ for the parts.

$t < 0$

$M =$ mass



You try.

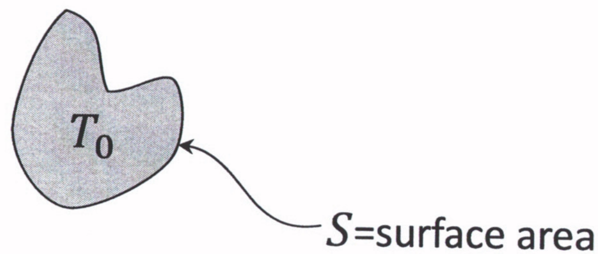
Exam 2 2019, problem 5

2

CM3120 Module 2—Cooling of a recently manufactured part

$t < 0$

M = mass

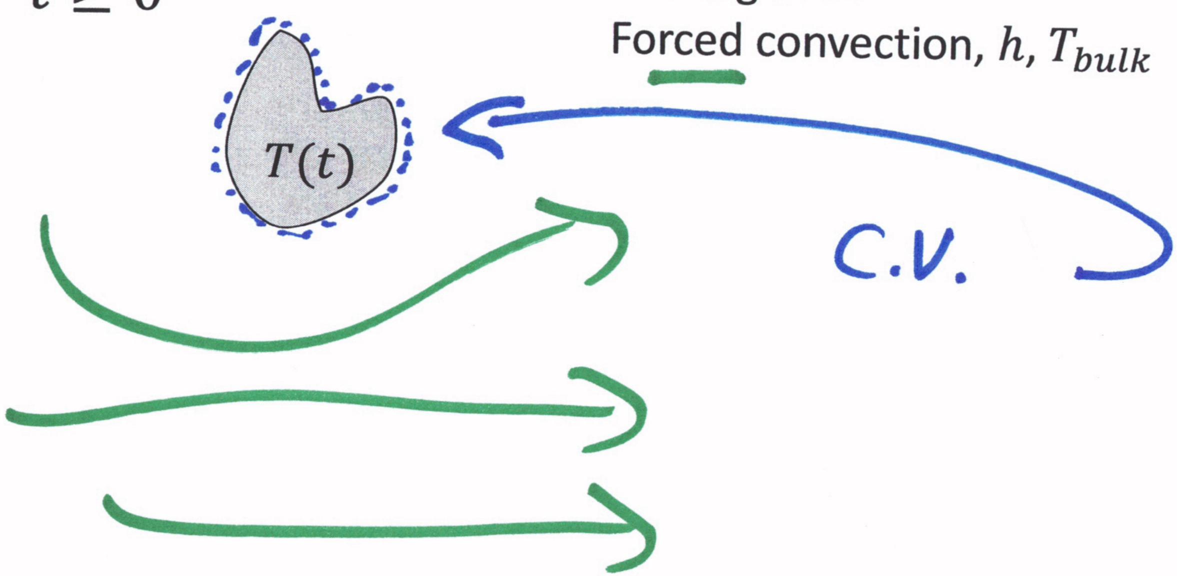


Suddenly,

$t \geq 0$

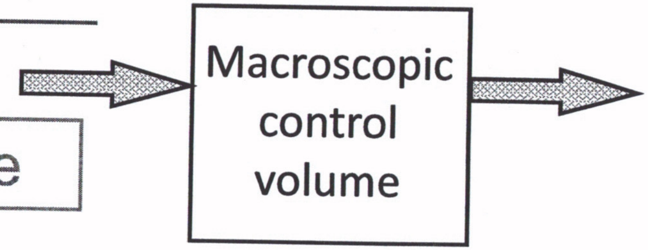
Cooling in air

Forced convection, h, T_{bulk}



Unsteady Macroscopic Energy Balance

Unsteady Macroscopic Energy Balance



$$accumulation = input - output$$

?

$$\frac{d}{dt}\left(U_{sys} + \cancel{E_{k,sys}} + \cancel{E_{p,sys}}\right) = -\cancel{\Delta H} - \cancel{\Delta E_k} - \cancel{\Delta E_p} + \dot{Q}_{in} + \cancel{W_{s,on}}$$

not important

$$\sum_{outs} \dot{m}(\,) - \sum_{ins} \dot{m}(\,)$$

no shafts

$$\frac{dU_{sys}}{dt} = \rho V C_v \frac{dT}{dt}$$

$\dot{m} = 0$
(no flow thru surface C.V.)

4

Unsteady Macroscopic Energy Balance

$$\frac{d}{dt}\left(U_{sys} + E_{k,sys} + E_{p,sys}\right) = -\Delta H - \Delta E_k - \Delta E_p + \dot{Q}_{in} + W_{s,on}$$

$\dot{Q}_{in} = \sum_i q_{in,i}$ **comes from a variety of sources:**

- **Thermal conduction:** $q_{in} = -kA\frac{dT}{dx}$

  e.g. device held by bracket; a solid phase that extends through boundaries of control volume

  Conduction to air included in h data correlations

- **Convection heat xfer:** $|q_{in}| = |hA(T_b - T)|$

  e.g. device dropped in stirred liquid; forced air stream flows past, natural convection occurs outside system; phase change at boundary

- **Radiation:** $q_{in} = \varepsilon\sigma A\left(T^4_{surroundings} - T^4_{surface}\right)$

  e.g. device at high temp. exposed to a gas/vacuum; hot enough to produce nat. conv.=possibly hot enough for radiation

  S-B constant: $\sigma = 5.676 \times 10^{-8}\frac{W}{m^2K^4}$

- **Electric current:** $q_{in} = I^2 R_{elec} L$

  e.g. if electric current is flowing within the device/control volume/ system

  no electric potential

- **Chemical Reaction:** $q_{in} = S_{rxn} V_{sys}$

  e.g. if a homogeneous reaction is taking place throughout the device/ control volume/system

  no chem rxn

(For check at the end)

$$\underbrace{M\hat{C}_V\frac{dT}{dt}}_{LHS} = \underbrace{hS(T_{bulk} - T)}_{RHS-1} + \underbrace{\varepsilon\sigma S\left(T_{bulk}^4 - T^4\right)}_{RHS-2}$$

S=surface area

Solve for $T(t)$

(Excel)

⑥

S=surface area

SOLVE

$$\boxed{M\hat{C}_V \frac{dT}{dt} = hS(T_{bulk} - T) + \varepsilon\sigma S(T_{bulk}^4 - T^4)}$$

bring T's together

$$\frac{dT}{dt} = \frac{hS}{M\hat{C}_V}(T_{bulk} - T) + \frac{\varepsilon\sigma S}{M\hat{C}_V}(T_{bulk}^4 - T^4)$$

$$\frac{dT}{dt} = \underbrace{\left[\frac{hS}{M\hat{C}_V}(T_{bulk}) + \frac{\varepsilon\sigma S}{M\hat{C}_V}(T_{bulk}^4)\right]}_{\Phi_0 \equiv} - \left[T\left(\frac{hS}{M\hat{C}_V}\right) + T^4\left(\frac{\varepsilon\sigma S}{M\hat{C}_V}\right)\right]$$

$$\frac{dT}{dt} = \Phi_0 - \left[T\left(\frac{hS}{M\hat{C}_V}\right) + T^4\left(\frac{\varepsilon\sigma S}{M\hat{C}_V}\right)\right]$$

$$\boxed{\frac{dT'}{\Phi_0 - \left[T\left(\frac{hS}{M\hat{C}_V}\right) + T^4\left(\frac{\varepsilon\sigma S}{M\hat{C}_V}\right)\right]} = dt}$$

integrate both sides

$T_0 < T' < T, \quad 0 < t' < t$

(7)

S=surface area

$$\int_{T_0}^{T} \underbrace{\frac{dT'}{\Phi_0 - \left[T'\left(\frac{hS}{M\hat{C}_V}\right) + T'^4\left(\frac{\varepsilon\sigma S}{M\hat{C}_V}\right)\right]}}_{F(T')} = \int_0^t dt'$$

We can use trapezoidal rule to integrate $f(T)$ in Excel

$$\text{area} = \frac{1}{2}h(B_1 + B_2)$$

$$\int_{T_0}^{T} f(T')dT' = t$$

$$f(T') = \frac{1}{\Phi_0 - \left[T'\left(\frac{hS}{M\hat{C}_V}\right) + T'^4\left(\frac{\varepsilon\sigma S}{M\hat{C}_V}\right)\right]}$$

$\alpha \equiv$ $\qquad$ $\boldsymbol{\beta} \equiv$

Need to integrate numerically.

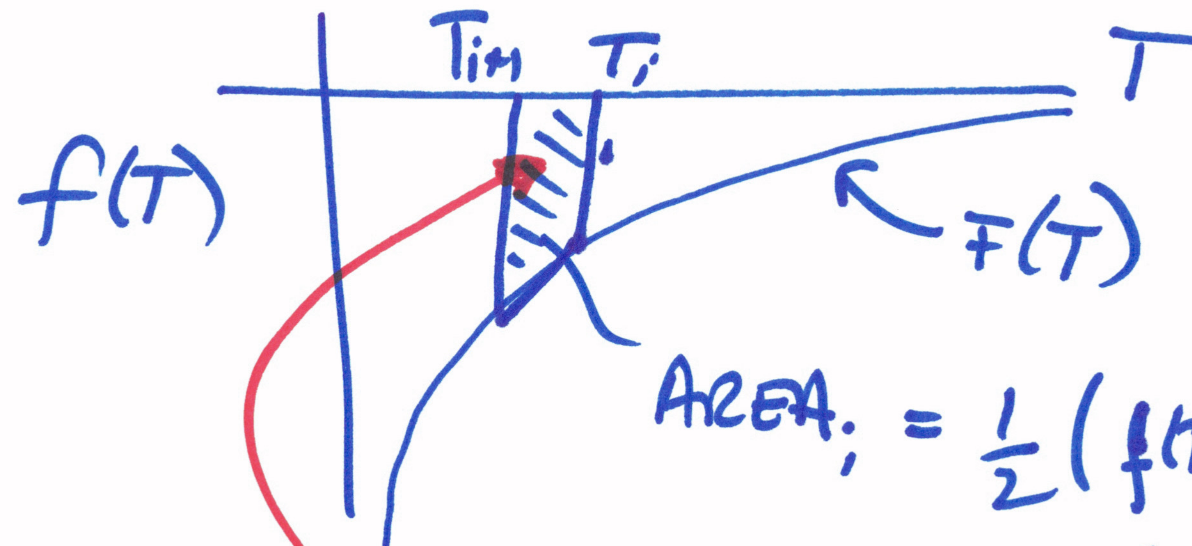
Numerical Integration

Process (Excel)

1. create a vector of Temps

$$T_0 < T_i < T_b \qquad (\text{intervals of arbitrary, but constant, size})$$
$$1 < i < n$$

2. Calculate $f(T_i)$ (a second vector)



$$\text{AREA}_i = \frac{1}{2}\left(f(T_i) + f(T_{i+1})\right) \cdot \Delta T$$

use trapazoidal rule

3. calc $(AREA)_i$ (a third vector)

Relate to integral

4. $$\int_{T_0}^{T} f(T')\,dT' = t$$

$$t_i = \sum_{j=1}^{i} (AREA)_j$$

5. create vector of t_i's – each one is one step forward in time

6) Plot T_i vs t_i

Excel spreadsheet posted on class website.