26 Feb 2021 F. MORRISON CM3120 0

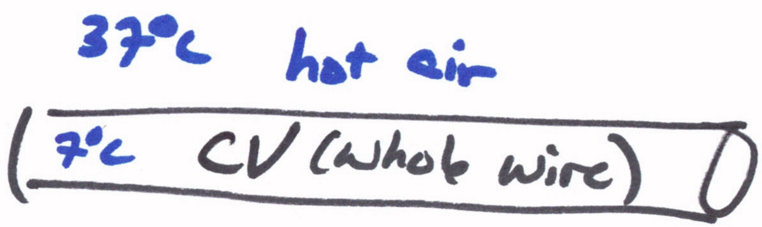
Michigan Tech

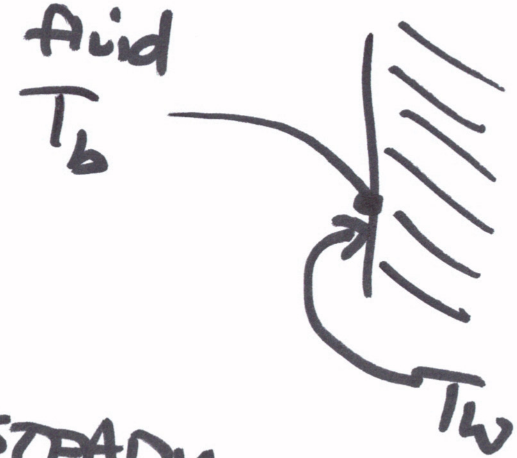
Requests for today?

- HW 2.7
- HW 2.3
- HW 2.14 (separate video)

2.7

$$\left|\frac{q_x}{A}\right| = h\left|T_b - T_w\right|$$





SOLN: MACRO E BAL UNSTEADY

$$\frac{d}{dt}\left(\underline{U_{sys}} + \cancel{E_k} + \cancel{E_p}\right) = \cancel{-\Delta H}\ \cancel{-\Delta E_k}\ \cancel{-\Delta E_p} + \underline{Q_{in}} + \cancel{W_{s,on}}$$

neglect $\qquad \dot{m} = 0 \qquad$ no shaft

Internal energy:
- T-change
- phase-change
- chemical composition change

$$\frac{dU_{sys}}{dt} = Q_{in}$$

$$\underbrace{\rho V}_{\text{mass of wire}} c_p \frac{dT}{dt} = hA(T_b - T_{wire})$$

$$A = \pi D L$$

$$\frac{\pi D^2}{4} \qquad V = \frac{\pi D^2 L}{4}$$

$$\rho \frac{\cancel{\pi} \cancel{D}^{\cancel{2}} \cancel{L}}{4} c_p \frac{dT}{dt} = h\, \cancel{\pi} \cancel{D} \cancel{L} (T_b - T_{wire})$$

$$\frac{dT}{dt} = \overbrace{\frac{h\,4}{\rho\, c_p\, D}}^{\equiv \tilde{\delta}} (T_b - T_{wire})$$

$$\frac{dT}{dt} = \tilde{\delta}\,(T_b - T)$$

$$(-1)\int \frac{-dT}{(T_b - T)} = \int \tilde{\delta}\, t$$

$$-\ln(T_b - T) = \tilde{\delta}\, t + C_1$$

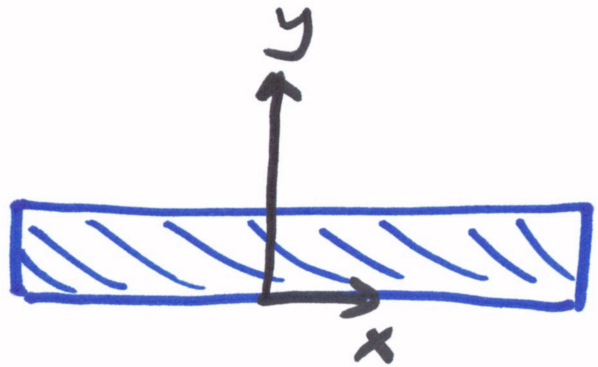
IC: $t = 0 \quad T = 7°C \implies C_1$

ANSWER QUESTION:

$t = 30 \quad T = 24°C \quad (t, T) = (30s, 24°C)$

$\implies h.$ //

2.3 Develop model:



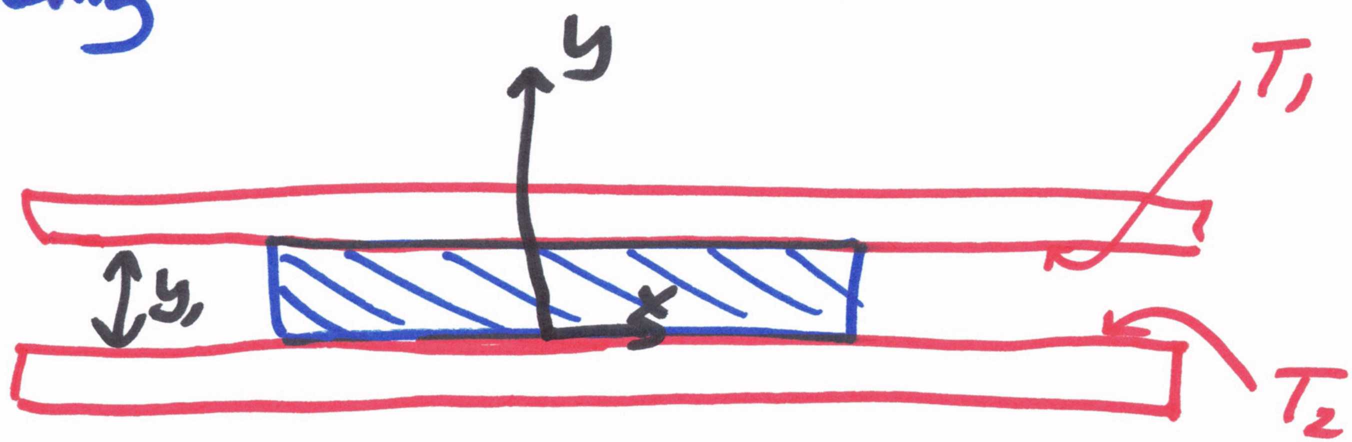
1D heat xfer in y dir

$T(y,$

$t = 0$

$T = T_0$

"Suddenly"



$y = 0 \qquad T = T_2$

$y = y_1 \qquad T = T_1$

5

The **Equation of Energy** for systems with **constant k**

Microscopic energy balance, constant thermal conductivity; Gibbs notation

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t}+\underline{v}\cdot\nabla T\right)=k\nabla^2 T+S_e$$

Microscopic energy balance, constant thermal conductivity; Cartesian coordinates

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t}+\cancel{v_x\frac{\partial T}{\partial x}}+\cancel{v_y\frac{\partial T}{\partial y}}+\cancel{v_z\frac{\partial T}{\partial z}}\right)=k\left(\cancel{\frac{\partial^2 T}{\partial x^2}}+\frac{\partial^2 T}{\partial y^2}+\cancel{\frac{\partial^2 T}{\partial z^2}}\right)+\cancel{S_e}$$

$\underline{v}=0$ long wide

no current
no rxn

Microscopic energy balance, constant thermal conductivity; cylindrical coordinates

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t}+v_r\frac{\partial T}{\partial r}+\frac{v_\theta}{r}\frac{\partial T}{\partial\theta}+v_z\frac{\partial T}{\partial z}\right)=k\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 T}{\partial\theta^2}+\frac{\partial^2 T}{\partial z^2}\right)+S_e$$

Microscopic energy balance, constant thermal conductivity; spherical coordinates

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t}+v_r\frac{\partial T}{\partial r}+\frac{v_\theta}{r}\frac{\partial T}{\partial\theta}+\frac{v_\phi}{r\sin\theta}\frac{\partial T}{\partial\phi}\right)$$
$$=k\left(\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial T}{\partial\theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 T}{\partial\phi^2}\right)+S_e$$

$$\frac{\partial T}{\partial t}=\underbrace{\frac{k}{\rho C_p}}_{\alpha}\frac{\partial^2 T}{\partial y^2}$$

BC: $y=0 \quad T=T_2$
$y=y_1 \quad T=T_1$
IC: $t=0 \quad T=T_0$

determining this + the sketch is what is meant by "modeling" the pbm.

Reference: F. A. Morrison, "Web Appendix to *An Introduction to Fluid Mechanics*," Cambridge University Press, New York, 2013 (*pages.mtu.edu/~fmorriso/cm310/IFMWebAppendixDMicroEBalanceMorrison.pdf*). This worksheet is on the web at *pages.mtu.edu/~fmorriso/cm310/energy.pdf*.

(separate video)

14. A brass sphere (diameter $= 2.54\ cm$) initially at 24^oC is suddenly immersed in vigorously stirred water (85^oC). After 5.0 seconds the center of the sphere is observed to be (72^oC). How long will it take the center of the sphere to reach $T = 83.8^oC$?

$$T(r,t) = T(0,t)$$

Since it's a question about the temp profile, it's a microscopic energy bal plm.

(slash + burn)

(Check if it's Heisler-compatible)

7

The **Equation of Energy** for systems with **constant k**

Microscopic energy balance, constant thermal conductivity; Gibbs notation

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t}+\underline{v}\cdot\nabla T\right)=k\nabla^2 T+S_e$$

Microscopic energy balance, constant thermal conductivity; Cartesian coordinates

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t}+v_x\frac{\partial T}{\partial x}+v_y\frac{\partial T}{\partial y}+v_z\frac{\partial T}{\partial z}\right)=k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right)+S_e$$

Microscopic energy balance, constant thermal conductivity; cylindrical coordinates

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t}+v_r\frac{\partial T}{\partial r}+\frac{v_\theta}{r}\frac{\partial T}{\partial \theta}+v_z\frac{\partial T}{\partial z}\right)=k\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}+\frac{\partial^2 T}{\partial z^2}\right)+S_e$$

Microscopic energy balance, constant thermal conductivity; spherical coordinates

no elec current
no rxn

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t}+\cancel{v_r\frac{\partial T}{\partial r}}+\cancel{\frac{v_\theta}{r}\frac{\partial T}{\partial \theta}}+\cancel{\frac{v_\phi}{r\sin\theta}\frac{\partial T}{\partial \phi}}\right)$$
$$=k\left(\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)+\cancel{\frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial T}{\partial \theta}\right)}+\cancel{\frac{1}{r^2\sin^2\theta}\frac{\partial^2 T}{\partial \phi^2}}\right)+\cancel{S_e}$$

$\underline{v}=0$

θ,ϕ symmetry

$$\frac{\partial T}{\partial t}=\underbrace{\frac{k}{\rho\hat{C}_p}}_{\alpha}\left(\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)\right)$$

IC: $t<0$ $T=24°C$ $\forall$ r
$=T_0$

BC: $r=R$ Newton's Law of cooling

Reference: F. A. Morrison, "Web Appendix to *An Introduction to Fluid Mechanics*," Cambridge University Press, New York, 2013 (*pages.mtu.edu/~fmorriso/cm310/IFMWebAppendixDMicroEBalanceMorrison.pdf*). This worksheet is on the web at *pages.mtu.edu/~fmorriso/cm310/energy.pdf*.

$r=0$ $\frac{\partial T}{\partial t}=0$ — $T_b=T_1=85°$

yes, it's the Heisler plm

RULES FOR HEISLER, Gurney-Lurie: (8)

Finite 1D Unsteady Heat Transfer, $T = T(t, x)$ or $T = T(t, r)$

Uniform initial temperature T_0; exposed to bulk temperature T_1; h known

- Flat plate long, wide, thickness $= 2x_1$ $T = T(t, x)$ $Y = Y(X, n)$
- Cylinder long, radius $= x_1$ $T = T(t, r)$ $Y = Y(X, n)$
- Sphere radius $= x_1$ $= \frac{2.54}{2}$ cm $T = T(t, r)$ $Y = Y(X, n)$

$$\text{Bi} = \frac{hD_{char}}{k} = \frac{hx_1}{k} = \frac{1}{m}$$

$$\text{Fo} = \frac{\alpha t}{x_1^2} = X$$

$$\frac{x}{x_1} = \frac{r}{x_1} = n$$

$$\frac{T_1 - T}{T_1 - T_0} = Y \quad \left(\frac{T - T_0}{T_1 - T_0} = 1 - Y\right)$$

Note:
$D_{char} = x_1$,
NOT V/A

$\alpha = 3.28 \times 10^{-5} \frac{m^2}{s}$

$x_1 = 1.27 \times 10^{-2}$ m

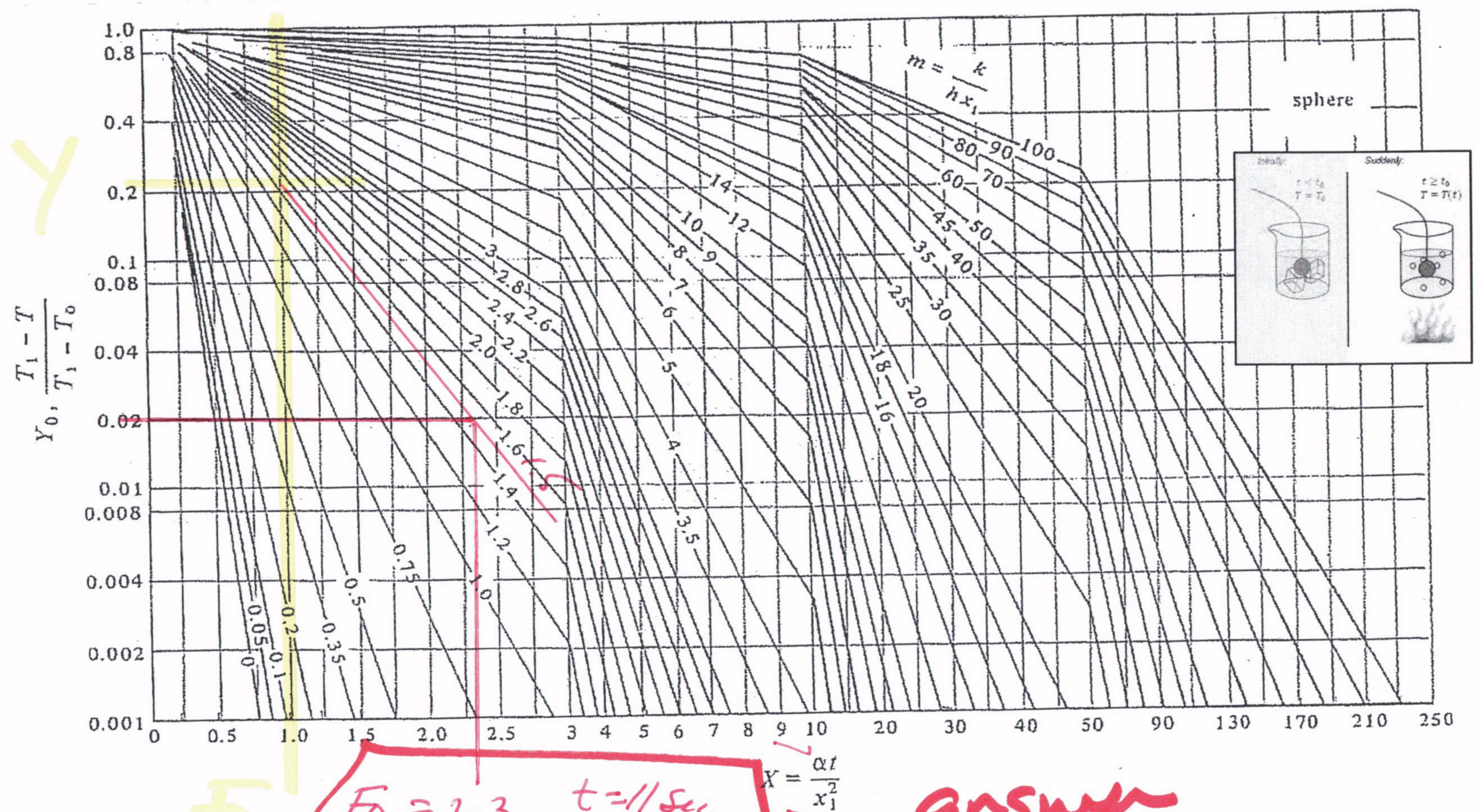
$Fo(5s)$
$= 1.02 \sim 1$

© Faith A. Morrison, Michigan Tech U.

9

Heisler Chart for Unsteady State Heat Transfer to a Sphere

(Geankoplis; see also Wikipedia)



Y

Fo

$\boxed{Fo = 2.3 \quad t = 11 \text{ sec}}$ answer

FIGURE 5.3-10. *Chart for determining the temperature at the center of a sphere for unsteady-state heat conduction. [From H. P. Heisler, Trans. A.S.M.E., 69, 227 (1947). With permission.]*

5

From Geankpolis, 4th edition, page 374

$\alpha = 3.28 \times 10^{-5}\ m^2/s$

$R = x_1 = 1.27 \times 10^{-2}\ m$

© Faith A. Morrison, Michigan Tech U.

FIRST POINT: at $Fo = 1 = \frac{\alpha t}{R^2}$

$$\frac{T_1 - T}{T_1 - T_0} = \frac{72}{} = 0.21$$

$85^oC = T_1$, $T_1 = 85^oC$, $T_0 = 24^oC$

From Heisler: $m = 1.5 = \frac{k}{hR}$

(we now know the m-line for the 2nd pt)

2nd pt: after t seconds $T = 83.8$

$$Y = \frac{85 - 83.8}{85 - 24} = 0.02 \leftarrow$$

What is $Fo = \frac{\alpha t}{x_1^2}$ for $m = 1.5$?