CM3120 3Feb2021 F. Morrison

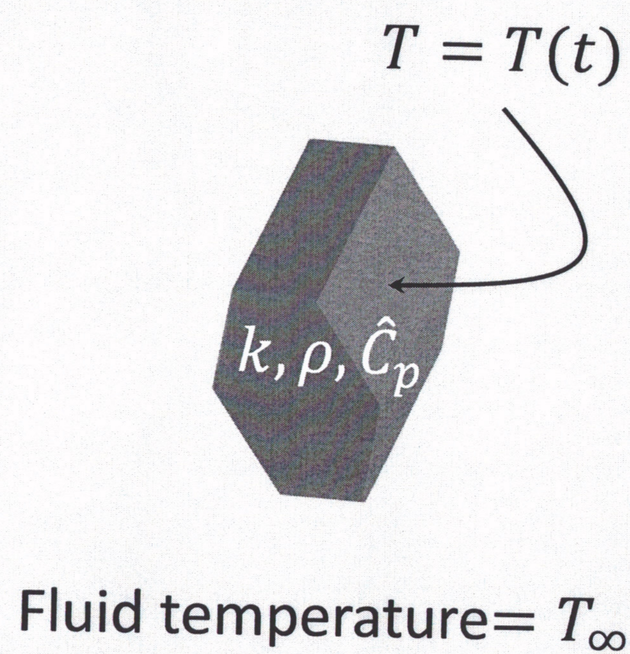
1

$$\text{Bi}_{LP} = \frac{hV_{sys}}{kA_{sys}}$$

Example: Quench cooling of a manufactured part.

If a piece of steel with $T = T_0$ is dropped into a large, well stirred reservoir of fluid at bulk temperature T_∞, what is the temperature of the steel as a function of time?

- k = large, which means that there is no internal resistance to heat transfer in the part
- Therefore, we are NOT calculating a temperature profile (internal T is uniform)
- **⇒ Use Unsteady, Macroscopic Energy Balance**



2

Unsteady State Heat Transfer: Low Biot Number

Unsteady Macroscopic Energy Balance

$$accumulation = input - output$$

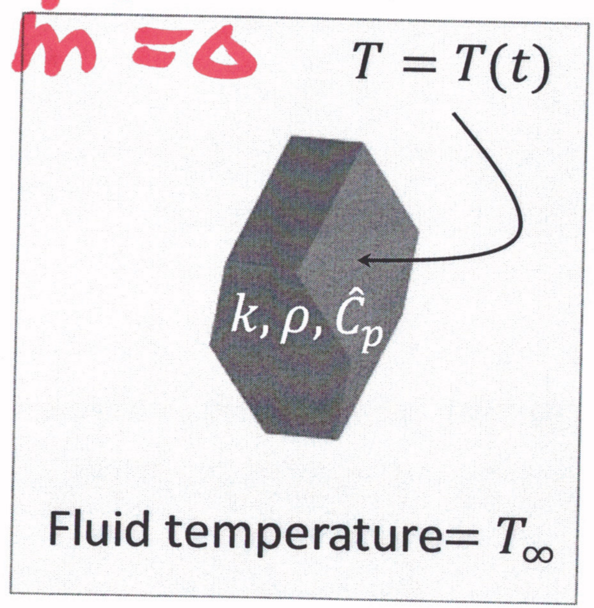
$$\frac{d}{dt}\left(U_{sys} + \cancel{E_{k,sys}} + \cancel{E_{p,sys}}\right) = \cancel{-\Delta H} - \cancel{\Delta E_k} - \cancel{\Delta E_p} + \dot{Q}_{in} + \cancel{W_{s,on}}$$

keep

neglect

$\dot{m} = 0$

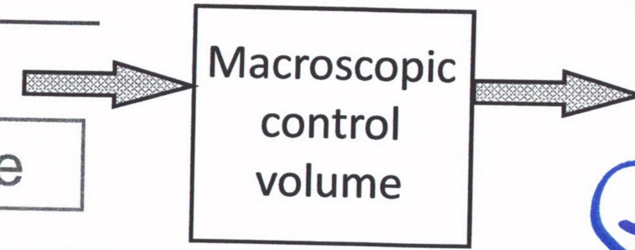
no shaft



You try.

Typical Modeling Assumptions (MACRO UNSTEADY)

Unsteady Macroscopic Energy Balance



3

Unsteady Macroscopic Energy Balance

How do we quantify the heat in $\dot{Q}_{in}$?

(single phase)

?

$$\frac{d}{dt}\left(U_{sys} + \cancel{E_{k,sys}} + \cancel{E_{p,sys}}\right) = \cancel{-\Delta H} \cancel{- \Delta E_k} \cancel{- \Delta E_p} + \dot{Q}_{in} + \cancel{W_{s,on}}$$

often negligible

no flow

no shafts

(no pump, turbine, mixing shaft)

$$\frac{dU_{sys}}{dt} = \rho V_{sys}\hat{C}_v\frac{dT}{dt} = M_{sys}\hat{C}_v\frac{dT}{dt}$$

In heat-transfer problems, there is often heat-in, $\dot{Q}_{in}$

$\hat{C}_v \approx \hat{C}_p$ for liquids, solids

④

$$\frac{d}{dt}\left(U_{sys}+E_{k,sys}+E_{p,sys}\right) = -\Delta H - \Delta E_k - \Delta E_p + \dot{Q}_{in} + W_{s,on}$$

$\dot{Q}_{in} = \sum_i q_{in,i}$ comes from a variety of sources:

- **Thermal conduction:** $q_{in} = -kA\frac{dT}{dx}$

  e.g. device held by bracket; a solid phase that extends through boundaries of control volume

- **Convection heat xfer:** $|q_{in}| = |hA(T_b - T)|$

  e.g. device dropped in stirred liquid; forced air stream flows past, natural convection occurs outside system; phase change at boundary

- **Radiation:** $q_{in} = \varepsilon\sigma A\left(T^4_{surroundings} - T^4_{surface}\right)$

  e.g. device at high temp. exposed to a gas/vacuum; hot enough to produce nat. conv.=possibly hot enough for radiation

- **Electric current:** $q_{in} = I^2 R_{elec} L$

  e.g. if electric current is flowing within the device/control volume/ system

- **Chemical Reaction:** $q_{in} = S_{rxn} V_{sys}$

  e.g. if a homogeneous reaction is taking place throughout the device/ control volume/system

S-B constant: $\sigma = 5.676 \times 10^{-8}\,\frac{W}{m^2K^4}$

SOLVE

$$M C_p \frac{dT}{dt} = hA(T_\infty - T)$$

$$\frac{dT}{dt} = \underbrace{\left(\frac{hA}{mc_p}\right)}_{\equiv B}[T_\infty - T]$$

$$-\left(\frac{-dT}{(T_\infty - T)}\right) = B\,dt$$

$$\left.\begin{aligned} u &= T_\infty - T \\ du &= -dT \end{aligned}\right\}$$

Recall:

$$\int \frac{du}{u} = \ln u + c_1$$

$$-\int \frac{-dT}{T_\infty - T} = B \int dt$$

$$-\ln(T_\infty - T) = Bt + C_1$$

Initial condition:

$t = 0 \quad T = T_0 \Rightarrow C_1$

$$\boxed{-\ln(T_\infty - T_0) = C_1}$$

substitute back

Simplify:

7

$$-\ln(T_\infty - T) = Bt - \ln(T_\infty - T_0)$$

$$-Bt = \ln(T_\infty - T) - \ln(T_\infty - T_0)$$

recall

$$\ln\frac{a}{b} = \ln a - \ln b$$

$$-Bt = \ln\left(\frac{T_\infty - T}{T_\infty - T_0}\right)$$

recall

$$\ln x = y$$

$$e^y = x$$

$$\Rightarrow \left(\frac{T_\infty - T}{T_\infty - T_0}\right) = e^{-Bt}$$

$$\boxed{\left(\frac{T_\infty - T}{T_\infty - T_0}\right) = e^{-\frac{hA}{Mc_p}t}}$$

Initial conditions? check: $t = 0$

$$\cancel{T_\infty} - T = \cancel{T_\infty} - T_0$$

$$T = T_0 \quad ✓$$

check units:

$$\frac{\frac{\cancel{W}}{\cancel{m^2}\cancel{K}}\ \cancel{m^2}}{\cancel{kg}\ \frac{kJ}{\cancel{kg}\,\cancel{K}}} \quad \frac{\cancel{kJ}}{W\,s}$$

✓

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