CM3120 LEC IV & LEC V 7 April 2021 F. Morrison ①

Part 2

Example 6: Height of a packed bed absorber

How can we use mass transfer to design a packed bed gas absorber to achieve a desired separation?

Example 6 is presented as a series of **linked examples** that navigate around apparent "dead ends" in modeling mass-transfer units

Previously:

III

LAST Time

LECTURES

Identify a question	Invent something	Try to use it
1. How can we model a large, practical device dependent on mass transfer? III	1. Apply the species A mass balance to a macroscopic C.V. III	1. Lack a system to account for A going between phases PAUSE III
2. How an we account for A going between phases? IV	2. Invent k_x through linear driving force (LDF) model IV	2. Gets A *to* the boundary, but not *across* PAUSE V
3. How can we improve LDF model to cross the boundary (bulk-to-bulk transfer)? VI	3. Write LDF in both phases and combine to create overall effect of multiple resistances VI	3. Working, but can we devise a convenient shorthand? PAUSE VI
4. Can we model a large, practical device, incorporating K_L, K_G to account for mass xfer between phases? VI	4. Yes VI	

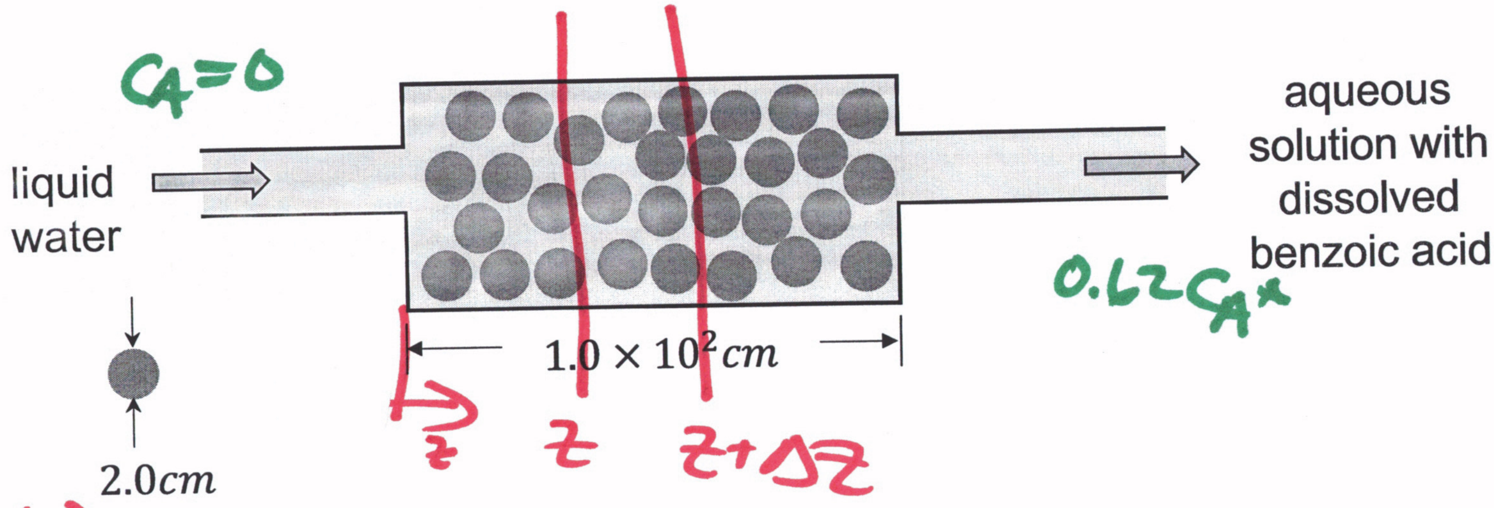
Today: Exampl 8 + brief into Ex 9, 10

Then, Lec V return to Ex 6

LECTURE IV (continued) ②

Unsteady Macroscopic Species A Mass Balance

Example 8: Flow through a packed bed of soluble spherical pellets.



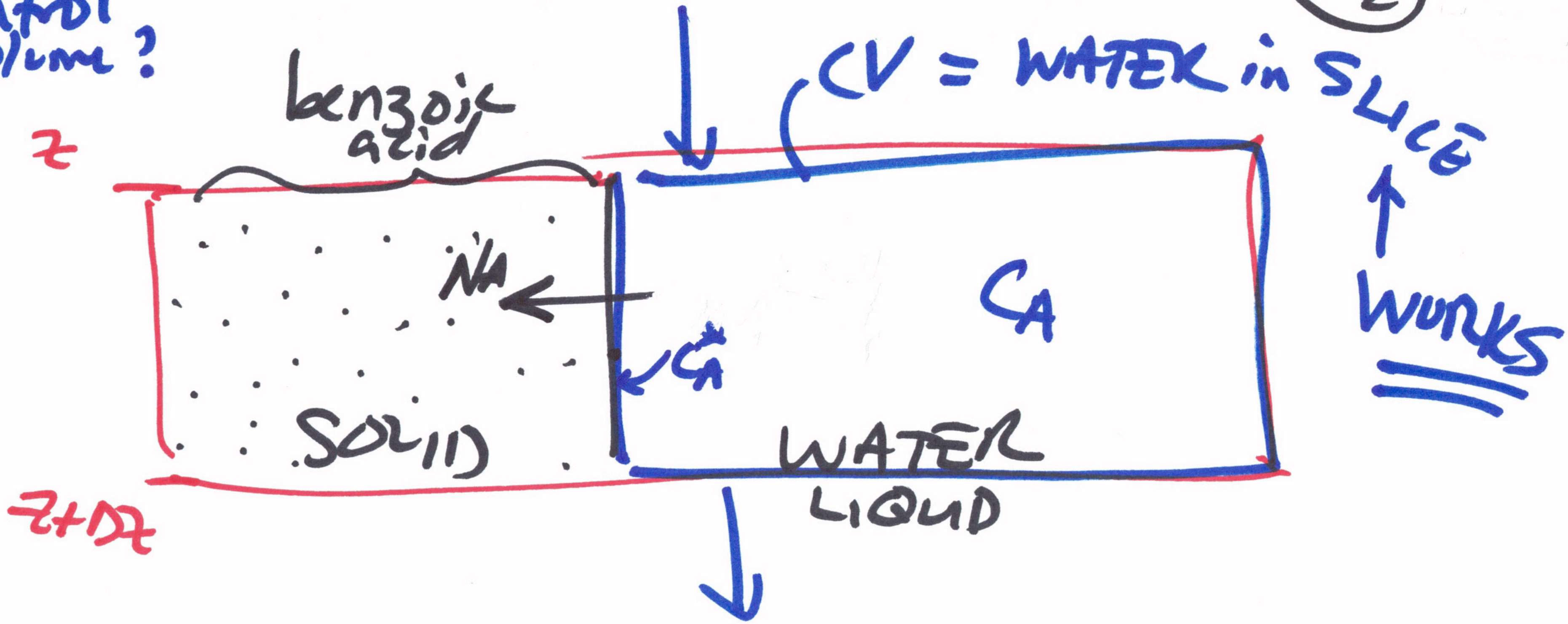
$C_A=C_A(z)$

Two-centimeter diameter spheres of benzoic acid (soluble in water) are packed into a bed as shown. The spheres have $23\ cm^2$ of surface area per cm^3 volume of bed. What is the mass transfer coefficient when pure water flowing in ("superficial velocity"= $5.0\ cm/s$) exits 62% saturated with benzoic acid?

Lets do a pseudo steady state solution

$$a = 23\ \frac{cm^2}{cm^3}$$

Control volume?



$A_{xs} \Delta z$ = VOLUME of RED

cannot be c.v. since species A mass xfer between phases is not captured

$S = a \left(A_{xs} \Delta z \right)$ = surface area of the red total volume of the slice

☆ N_A = flux "out" according to our macro species A mass bal eqn

Unsteady Macroscopic Species A Mass Balance

MOLES

accumulation = net flow in + production + introduction

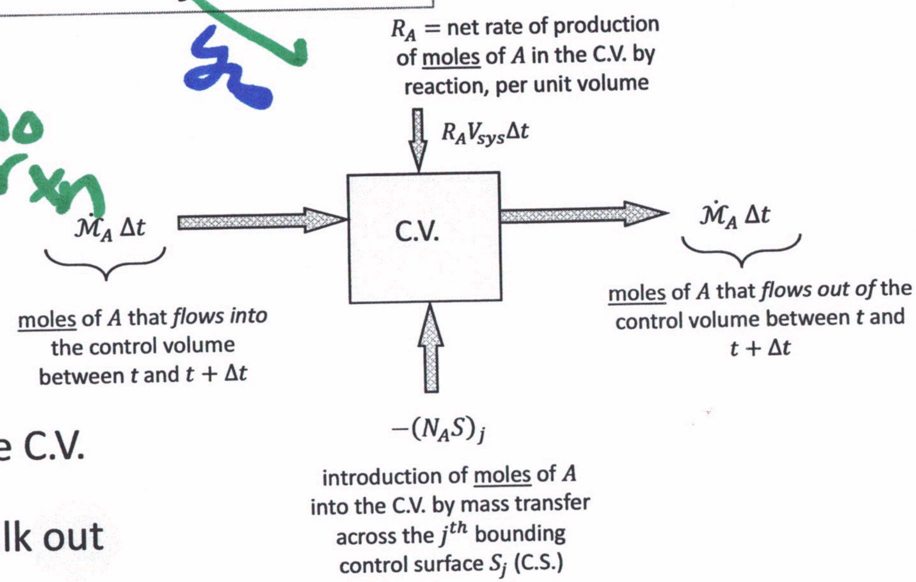
$$\frac{d}{dt}\left(\mathcal{M}_{A,sys}\right) = -\Delta\dot{\mathcal{M}}_A + R_A V_{sys} - \sum_j (N_A S)_j$$

pseudo steady state

ss

no rxn

ss



Species A conc grows over length of the bed

$\mathcal{M}_{A,sys} = c_A V_{sys}$ = total moles of A in the C.V.

$\Delta\dot{\mathcal{M}}_A = \sum_{j,outs} \dot{\mathcal{M}}_{A,j} - \sum_{j,ins} \dot{\mathcal{M}}_{A,j}$ = bulk out

R_A = net rate of production of moles of A in the C.V. by reaction, per unit volume

V_{sys} = system volume

$N_{A_j} = K\Delta c_{df}$ = molar flux of A out through the j^{th} C.S. ☆

out of C.V.

$S_{sys} = \sum_j S_j$

Δ is "out" - "in"

C.S. = control surface

C.V. = control volume

How to characterize the mass xfr between phases? (3½)

Liquid-phase-units
Film Linear driving force model:

$$N_A \equiv k_x\left(x_{A,i} - x_{A,b}\right)$$
$$N_A \equiv k_{cL}\left(c_{AL,i} - c_{AL,b}\right)$$

Gas-phase-units:
Film Linear driving force model:

$$N_A \equiv k_p\left(p_{A,b} - p_{A,i}\right)$$
$$N_A \equiv k_{cG}\left(c_{AG,b} - c_{A,i}\right)$$
$$N_A \equiv k_y\left(y_{A,b} - y_{A,i}\right)$$

$\tilde{N}_A$ = flux in
$N_A = -\tilde{N}_A$ = flux out

Let's take these tools out for a spin!

Liquid-phase-units
Overall Linear driving force model:

$$N_A \equiv K_x\left(x_A^*(\ ) - x_{A,b}\right)$$
$$N_A \equiv K_{cL}\left(c_{AL}^*(\ ) - c_{AL,b}\right)$$

$$(\ ) = p_{A,b} \text{ or } c_{A,b} \text{ or } y_{A,b}$$

Gas-phase-units:
Overall Linear driving force model:

$$N_A \equiv K_p\left(p_{A,b} - p_A^*(\ )\right)$$
$$N_A \equiv K_{cG}\left(c_{AG,b} - c_{AG}^*(\ )\right)$$
$$N_A \equiv K_y\left(y_{A,b} - y_A^*(\ )\right)$$

$$(\ ) = x_{A,b} \text{ or } c_{AL,b}$$

Warning: There's even more complexity coming

"in"

$$K_x = \frac{1}{\frac{1}{k_x} + \frac{1}{m'' k_y}},\quad K_y = \frac{1}{\frac{1}{k_y} + \frac{m'}{k_x}},\ \text{etc.}$$

$$N_A = \tilde{N}_A = k_c\left(C_{Ai} - C_{Ab}\right)$$
$$= k_c\left(C_A^* - C_A\right)$$

"out"

$NAS = ?$

$$\boxed{-\underbrace{N_A}_{\substack{\text{moles A} \\ \overline{\text{area time}} \\ \text{(out of LIQUID)}}} = k_c \left(C_A^* - C_A\right)}$$

C_A^*: saturated at surface of solid benzoic acid

k_c: mass transfer coef

$$N_A S = -\underbrace{a A_{xs} \Delta z}_{S} k_c \left(C_A^* - C_A\right)$$

S: interfacial area between solid and liquid

ΔM_A?

moles in: $C_A\big|_z \underbrace{V^\circ A_{xs}}_{\dot{V}\ \frac{\text{volume}}{\text{time}}}$

$\frac{\text{moles A}}{\cancel{\text{volume}}}$ $\dot{V}\ \frac{\cancel{\text{volume}}}{\text{time}}$

volumetric flow rate

$\frac{\dot{V}}{A_{xs}} = V^\circ$

bed cross section

"superficial velocity"

moles out: $C_A\big|_{z+\Delta z} V^\circ A_{xs}$

combine + integrate ...

(see HW)

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$$0 = -\left(C_A\big|_{z+\Delta z} - C_A\big|_z\right) V^\circ \cancel{A_{cs}} - \underbrace{\overset{\text{flux out}}{N_A} S}_{a\cancel{A_{cs}}\,\Delta z}$$

"outs" – "ins"

$$0 = -\left(C_A\big|_{z+\Delta z} - C_A\big|_z\right) V^\circ + a\,\Delta z\, k_c \left(C_A^* - C_A\right)$$

$$\lim_{\Delta z \to 0} \frac{C_A\big|_{z+\Delta z} - C_A\big|_z}{\Delta z} = \frac{a k_c}{V^\circ}\left(C_A^* - C_A\right)$$

$$\frac{dC_A}{dz} = \left(\frac{a k_c}{V^\circ}\right)\left(C_A^* - C_A\right)$$

solve w/ $t = 0$, $C_A = 0$ //