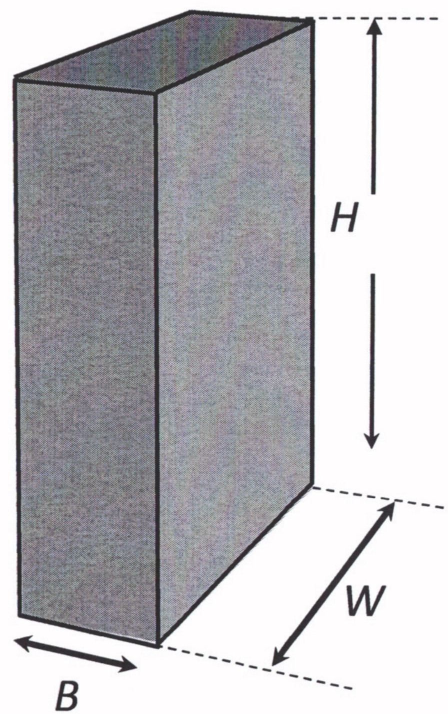
Example: A tall, wide rectangular ~~copper~~ 304 stainless steel slab, ~~five~~ ten centimeters thick, uniformly at a temperature of 17^oC, is suddenly exposed on all sides to ~~air~~ water ($h = 1380\ W/m^2K$) at 45^oC. After ~~30~~ 20 minutes, what is the temperature at the middle of the slab?

(numbers were changed in 2021 to improve the problem; the old numbers appear in the video)



2

$= 2X_1 \Rightarrow X_1 = 5cm$

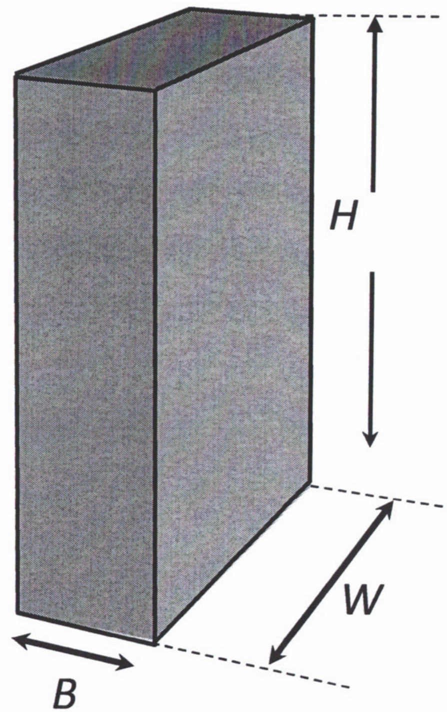
Example: A tall, wide rectangular 304 stainless steel slab, ten centimeters thick, uniformly at a temperature of 17^oC, is suddenly exposed on all sides to water ($h = 1380\ W/m^2K$) at 45^oC. After 20 minutes, what is the temperature at the middle of the slab?

$h = 1380 \frac{W}{m^2K}$

$T_0 = 17°C$

$T_1 = 45°C$

$t = 20\ min$



304 SS

$k = 13.8\ W/mK$

$\alpha = 4.2 \times 10^{-6} \frac{m^2}{s}$

Let's try

8Feb2021 CM3120 F. Morrison
Module 2

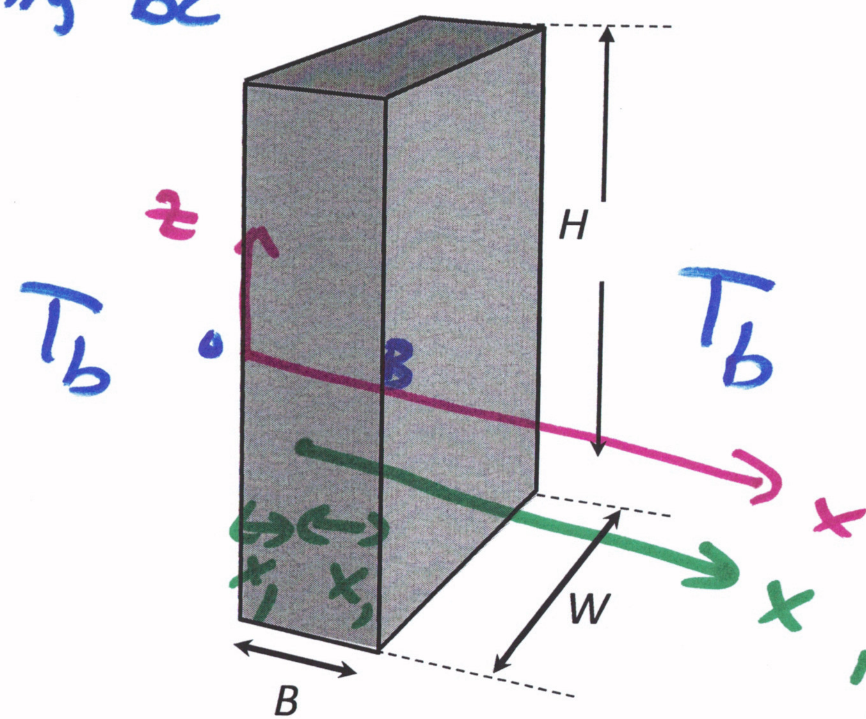
3

Example: A tall, wide rectangular 304 stainless steel slab, ten centimeters thick, uniformly at a temperature of 17^oC, is suddenly exposed on all sides to water ($h = 1380\ W/m^2K$) at 45^oC. After 20 minutes, what is the temperature at the middle of the slab?

$T_b = 45°C \Rightarrow$ Newton's law of cooling BC

$\Rightarrow$ microscopic energy bal

$t \geq 0$



pick a coord system

"temp in middle" $\Rightarrow$ Heisler

$B = 2x_1$

revise so that we can use short-cut solns

SS slab problem

The **Equation of Energy** for systems with **constant k**

Microscopic energy balance, constant thermal conductivity; Gibbs notation

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + \underline{v} \cdot \nabla T \right) = k \nabla^2 T + S_e$$

Microscopic energy balance, constant thermal conductivity; Cartesian coordinates

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + \cancel{v_x \frac{\partial T}{\partial x}} + \cancel{v_y \frac{\partial T}{\partial y}} + \cancel{v_z \frac{\partial T}{\partial z}} \right) = k \left(\frac{\partial^2 T}{\partial x^2} + \cancel{\frac{\partial^2 T}{\partial y^2}} + \cancel{\frac{\partial^2 T}{\partial z^2}} \right) + \cancel{S_e}$$

$v = 0$

thin wide

no rxn no electric current

Microscopic energy balance, constant thermal conductivity; cylindrical coordinates

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} + v_z \frac{\partial T}{\partial z} \right) = k \left(\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2} \right) + S_e$$

Microscopic energy balance, constant thermal conductivity; spherical coordinates

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} + \frac{v_\phi}{r \sin\theta} \frac{\partial T}{\partial \phi} \right)$$
$$= k \left(\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial \theta} \left(\sin\theta \frac{\partial T}{\partial \theta} \right) + \frac{1}{r^2 \sin^2\theta} \frac{\partial^2 T}{\partial \phi^2} \right) + S_e$$

$$\frac{\partial T}{\partial t} = \frac{k}{\rho C_p} \frac{\partial^2 T}{\partial x^2}$$

*BC: $x = B$, $x = 0$: $\left. -k \frac{dT}{dx} \right| = h \left(T_b - T \right|$

IC: $t = 0$ $T = T_0$ $\forall x$

Reference: F. A. Morrison, "Web Appendix to *An Introduction to Fluid Mechanics*," Cambridge University Press, New York, 2013 (*pages.mtu.edu/~fmorriso/cm310/IFMWebAppendixDMicroEBalanceMorrison.pdf*). This worksheet is on the web at *pages.mtu.edu/~fmorriso/cm310/energy.pdf*.

* Newton's law of cooling BC for both sides

Heisler Charts

Initial: Uniform initial temperature T_0
BC: Exposed to bulk temperature T_1
h known
Plots of temperature at the center

$$T\left(m = \frac{1}{\mathrm{Bi}}, x = 0, \mathrm{Fo}\right)$$

Flat plate

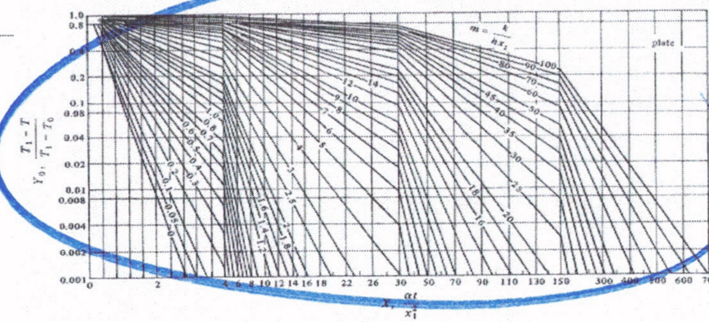


FIGURE 5.3-6. *Chart for determining temperature at the center of a large flat plate for unsteady-state heat conduction. [From H. P. Heisler, Trans. A.S.M.E., 69, 227 (1947). With permission.]*

Cylinder

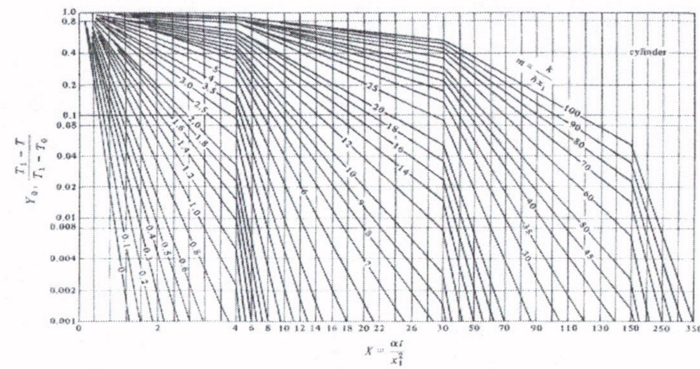


FIGURE 5.3-8. *Chart for determining temperature at the center of a long cylinder for unsteady-state heat conduction. [From H. P. Heisler, Trans. A.S.M.E., 69, 227 (1947). With permission.]*

Sphere

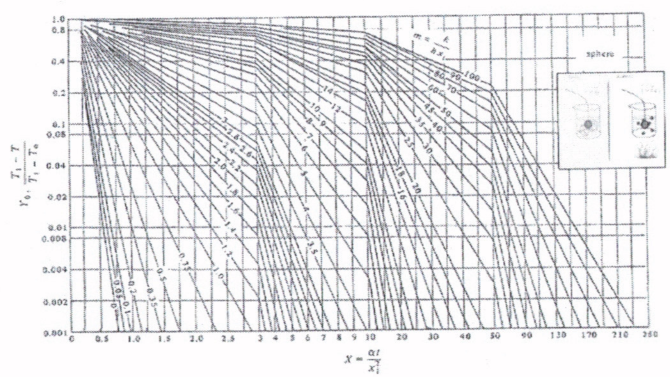


FIGURE 5.3-10. *Chart for determining the temperature at the center of a sphere for unsteady-state heat conduction. [From H. P. Heisler, Trans. A.S.M.E., 69, 227 (1947). With permission.]*

Ref: Geankoplis, 4th Ed, 2003

$$\mathrm{Fo} = \frac{\alpha t}{x_1^2} = X$$

5

Finite 1D Unsteady Heat Transfer, $T = T(t,x)$ or $T = T(t,r)$

Uniform initial temperature T_0; exposed to bulk temperature T_1; h known

- Flat plate long, wide, thickness $= 2x_1$ $T = T(t,x)$ $Y = Y(X,n)$
- Cylinder long, radius $= x_1$ $T = T(t,r)$ $Y = Y(X,n)$
- Sphere radius $= x_1$ $T = T(t,r)$ $Y = Y(X,n)$

$$\mathrm{Bi} = \frac{hD_{char}}{k} = \frac{hx_1}{k} = \frac{1}{m}$$

$$\mathrm{Fo} = \frac{\alpha t}{x_1^2} = X$$

$$\frac{x}{x_1} = \frac{r}{x_1} = n$$

$$\frac{T_1 - T}{T_1 - T_0} = Y \quad \left(\frac{T - T_0}{T_1 - T_0} = 1 - Y \right)$$

Note:
$D_{char} = x_1$,
NOT V/A

5t

$$Fo = \frac{\alpha t}{x_1^2} = \frac{\left(4.2\times10^{-6}\,\frac{m^2}{\cancel{s}}\right)\left(20\,\cancel{min}\right)\left(\frac{60\,\cancel{s}}{\cancel{min}}\right)}{\left(5\times10^{-2}\,\cancel{m}\right)^2}$$

$$Fo = 2.02 \sim \boxed{2}$$

$$Bi = \frac{h x_1}{k} = \frac{\left(380\,\frac{\cancel{W}}{\cancel{m^2 K}}\right)\left(5\times10^{-2}\,\cancel{m}\right)}{13.8\,\frac{\cancel{W}}{\cancel{mK}}}$$

$$= 5.00$$

$$m = \frac{1}{B_1} = \boxed{0.2}$$

Heisler Chart for Unsteady State Heat Transfer to a Flat Plate

(Geankoplis; see also Wikipedia)

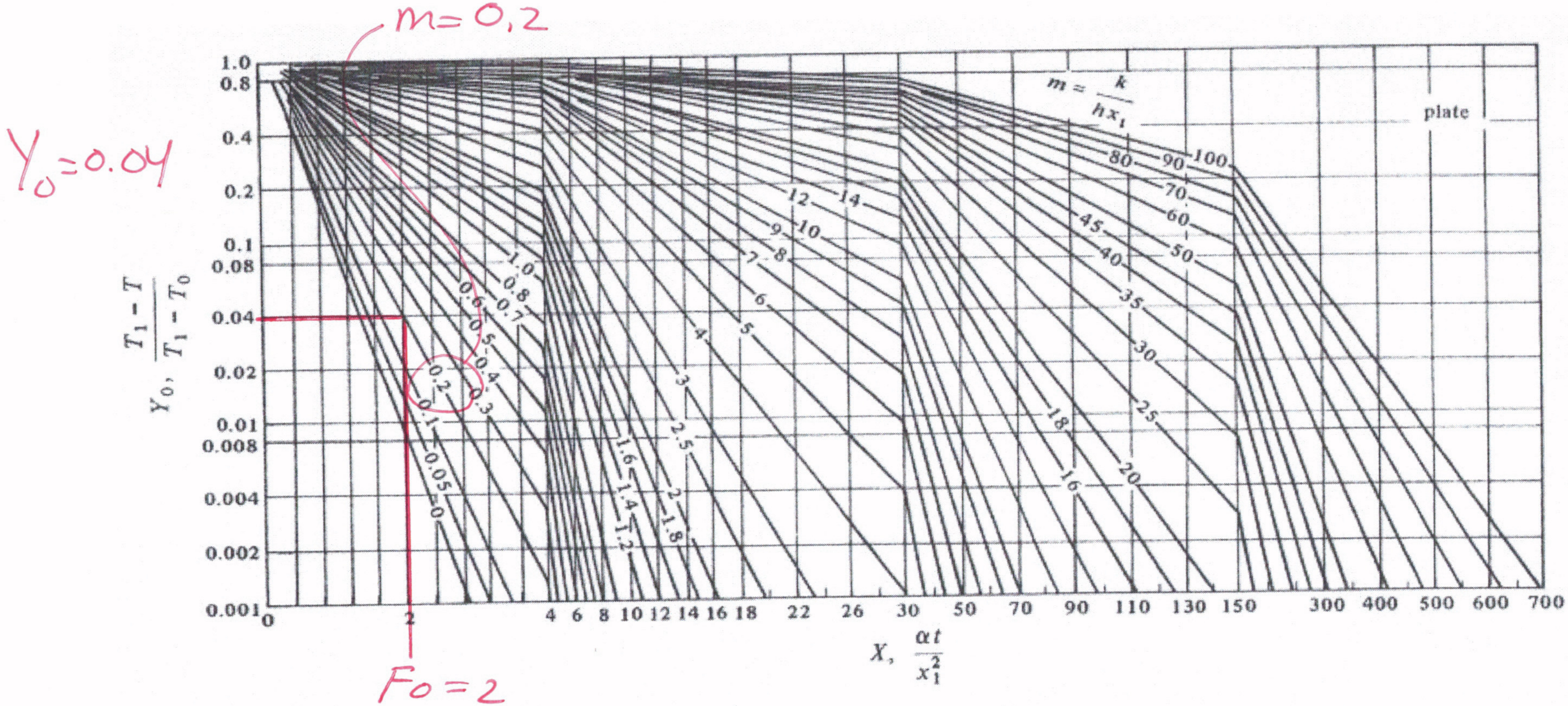


FIGURE 5.3-6. *Chart for determining temperature at the center of a large flat plate for unsteady-state heat conduction. [From H. P. Heisler, Trans. A.S.M.E.,* **69**, *227 (1947). With permission.]*

From Geankpolis, 4th edition

$$\frac{T_1 - T}{T_1 - T_0} = 0.04 = \frac{45 - T}{45 - 17} \quad (°C)$$

$$\boxed{T = 43.9°C}$$