# MOD 4 STUDY GUIDE #9

14 Ap 2021
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Explain linear driving force model to an interface ($k_x$) + compare to Fick's LAW

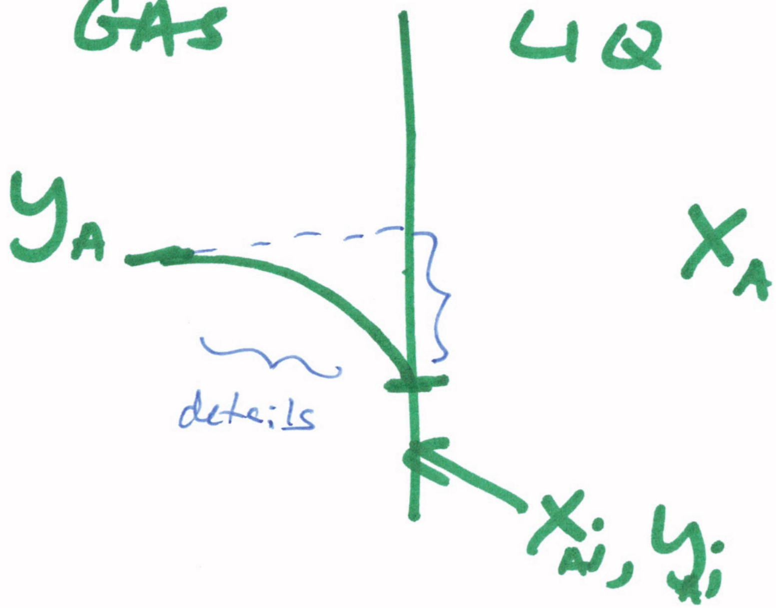
What is Fick's Law?
What is LDF model? } (p2)

How are they used in mass xfr } (p3)

Compare + contrast } (p4)

**Fick's Law** $N_{A_z} \propto \dfrac{dx_A}{dz}$ $N_{A_z} \propto \dfrac{dy_A}{dz}$

localized flux value or flux as function of position




**LDF** $(y_A - y_{Ai})\, k_y = N_A$

details are "LUMPED" together

get from data correlations (scenario property) microscopic C.V.

micro spec A mole Bal $\frac{\partial c_A}{\partial t} = -\nabla \cdot \underline{N}_A + R_A$

Fick's Law

$$N_A = x_A(\underline{N}_A + \underline{N}_B) - c D_{AB} \nabla x_A$$

MACROSCOPIC C.V.

MACRO spec A mole Bal

s.a. mass xfr

$$\frac{dM_A}{dt} = -\Delta \dot{M}_A - R_A V_{sys} - N_A S$$

LDF model: $N_A = K \Delta c_{df} = K_y(y_A - y_A^*(x_A))$

compare + contrast: at phase interface

| $D_{AB}$ (Fick's Law) | $k_y$ (LDF model) |
|---|---|
| gives detailed $N_A(z)$ as a func of position | lumped approach (simplified) $N_A \propto (y_A - y_{Ai})$ |
| $D_{AB}$ = material parameter (look up) | $k_y$ is a scenario property |
| complex to do in non-simple scenarios (usually) | much more accessible in non-simple scenarios (but need data correlations) $Sh(Re, Sc, \frac{L}{D})$ |
| MICRO C.V. | MACRO C.V. |

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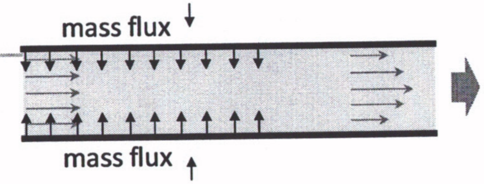


**Example 15**

According to our **dimensional analysis** calculations, the dimensionless heat transfer coefficient should be found to be a function of <u>three</u> dimensionless groups:

Peclet number

$$Pe_m = ReSc = \frac{VD}{\mathcal{D}_{AB}}$$

Schmidt number

$$Sc = \frac{\mu}{\rho \mathcal{D}_{AB}}$$

$$\boxed{Sh = Sh\left(Re, Sc, \frac{L}{D}\right)}$$

$$Sh = \frac{k_c D}{\mathcal{D}_{AB}}$$

dimensionless mass xfer coef

Now, do the experiments.





(compare w/ Nu)

# Dimensionless Numbers

Dimensionless numbers from the **Equations of Change**

$\mathrm{Re}$ – Reynolds $= \frac{\rho V D}{\mu} = \frac{VD}{\nu}$

$\mathrm{Fr}$ – Froude $= \frac{V^2}{gD}$

$\mathrm{Pe}$ – Péclet$_h$ $= \mathrm{RePr} = \frac{\hat{C}_p \rho V D}{k} = \frac{VD}{\alpha}$

$\mathrm{Pe}$ – Péclet$_m$ $= \mathrm{ReSc} = \frac{VD}{\mathcal{D}_{AB}}$

These numbers tell us about the **relative importance of the terms** they precede in the microscopic balances (*scenario properties*).

$\mathrm{Pr}$ – Prandtl $= \frac{\hat{C}_p \mu}{k} = \frac{\nu}{\alpha}$

$\mathrm{Sc}$ – Schmidt $= \mathrm{LePr} = \frac{\mu}{\rho \mathcal{D}_{AB}} = \frac{\nu}{\mathcal{D}_{AB}}$

$\mathrm{Le}$ – Lewis $= \frac{\alpha}{\mathcal{D}_{AB}}$

These numbers compare the **magnitudes of the diffusive transport coefficients** $\nu, \alpha, \mathcal{D}_{AB}$ (*material properties*).

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# Dimensional Analysis

These numbers are defined to help us build **transport data correlations** based on the fewest number of grouped (dimensionless) variables (*scenario property*).

Dimensionless numbers from the **Engineering Quantities of Interest**

momentum

Dimensionless Force on the Wall (Drag)

$$f = \frac{1}{\pi}\frac{D}{L}\frac{1}{\mathrm{Re}}\int_0^{\frac{L}{D}}\int_0^{2\pi}\left(\frac{\partial v_z^*}{\partial r^*}\right)\Bigg|_{r^*=\frac{1}{2}} d\theta dz^*$$

*(Fanning)*

$f$ – Friction Factor

$\frac{L}{D}$ – Aspect Ratio

$$f = \frac{\mathcal{F}_{drag}}{\left(\frac{1}{2}\rho V^2\right) A_c}$$

energy

Newton's Law of Cooling

$$Nu = \frac{1}{2\pi L/D}\int_0^{2\pi}\int_0^{L/D} -\frac{\partial T^*}{\partial r^*}\Bigg|_{r^*=1/2} dz^* d\theta$$

Nu – Nusselt

$\frac{L}{D}$ – Aspect Ratio

$$\mathrm{Nu} = \frac{hD}{k}$$

$$\mathrm{St}_h = \frac{h}{\rho V \hat{C}_p} = \frac{\mathrm{Nu}}{\mathrm{RePr}}$$

mass xfer

Dimensionless Mass Transfer Coefficient

$$Sh = \frac{1}{2\pi}\frac{D}{L}\int_0^{\frac{L}{D}}\int_0^{2\pi}\left(-\frac{\partial x_A^*}{\partial r^*}\right)\Bigg|_{r^*=\frac{1}{2}} d\theta dz^*$$

Sh – Sherwood

$\frac{L}{D}$ – Aspect Ratio

$$\mathrm{Sh} = \frac{k_c D}{\mathcal{D}_{AB}}$$

$$\mathrm{St}_m = \frac{k_c}{V} = \frac{\mathrm{Sh}}{\mathrm{ReSc}}$$

St – Stanton