MODULE 3

Homework 3 **CM3120 Transport/UO 2**

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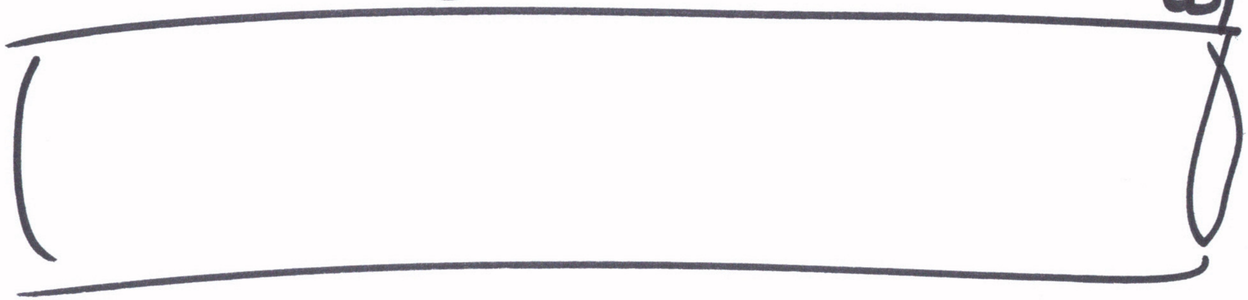
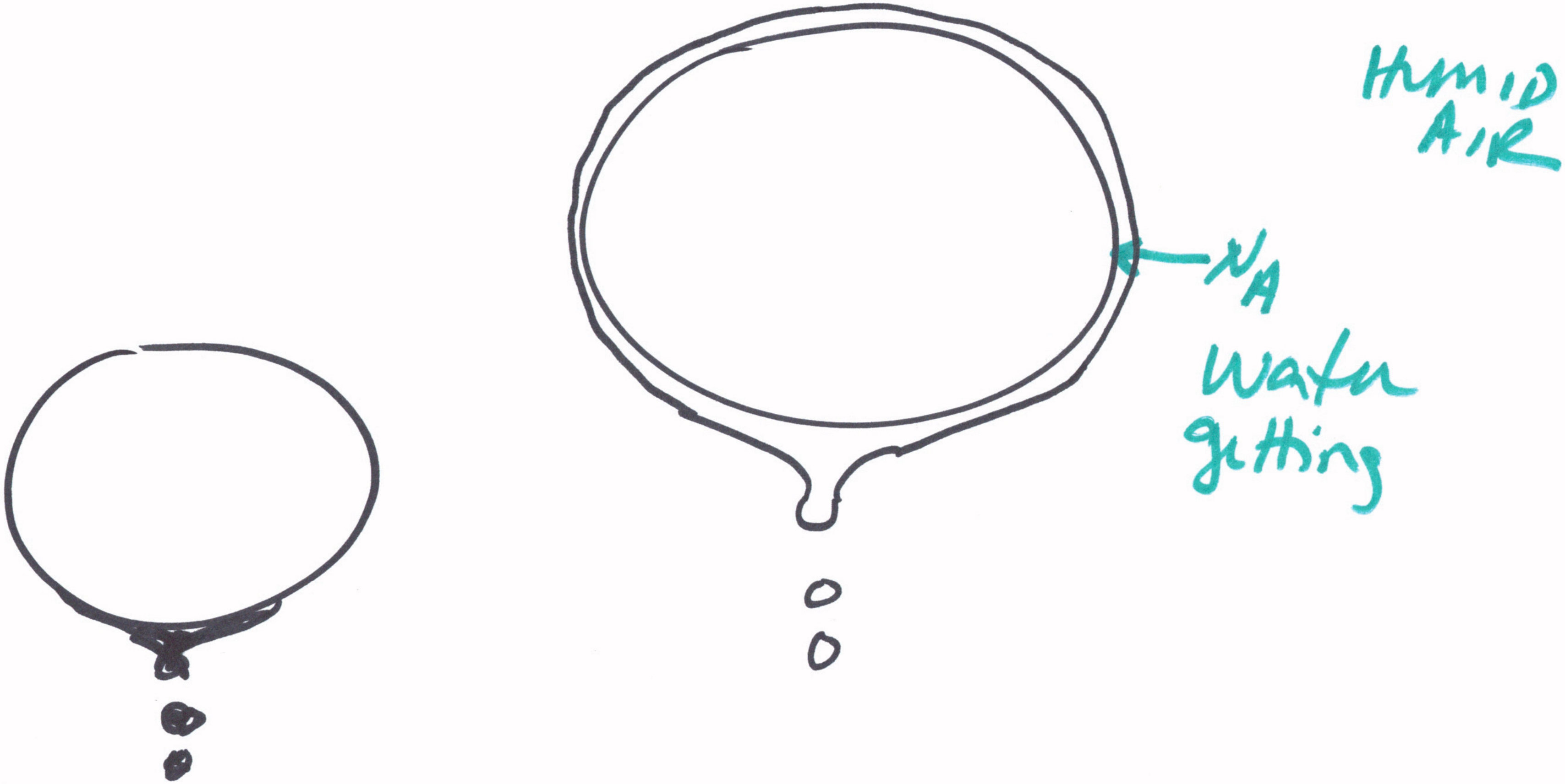
A cold-water pipeline runs through a hot, humid space in a processing plant. Water condenses onto the pipe and drips onto the ground. Model this problem in such a way that we can estimate the rate of water dripping from the pipe.

Notes: Cold pipe Sketch the problem and see where diffusion comes into the picture. The molar flux $\underline{N}_A$ can tell us the condensation rate at steady state. Near the water film, the air will be saturated with water (Raoult's law).

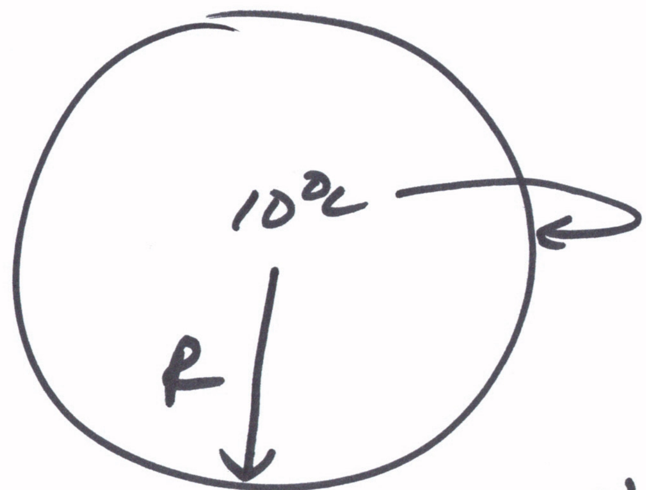
HW 3
PLM
6

Saturday 20 MAR 21 F. Morrison w/ study group



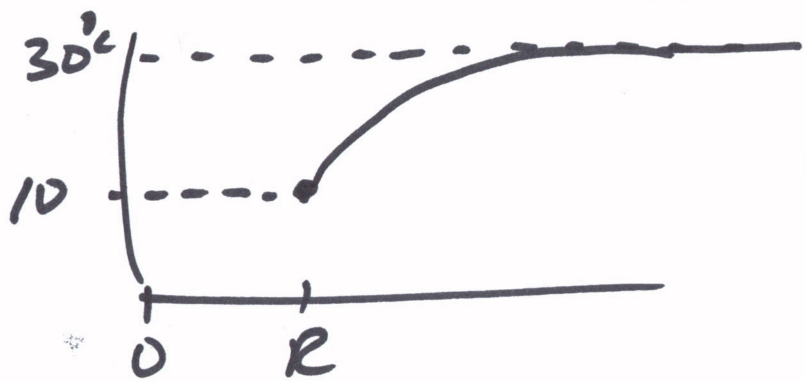


How would we model a temperature distribution plm?



$T_b = 30°C$

BDA

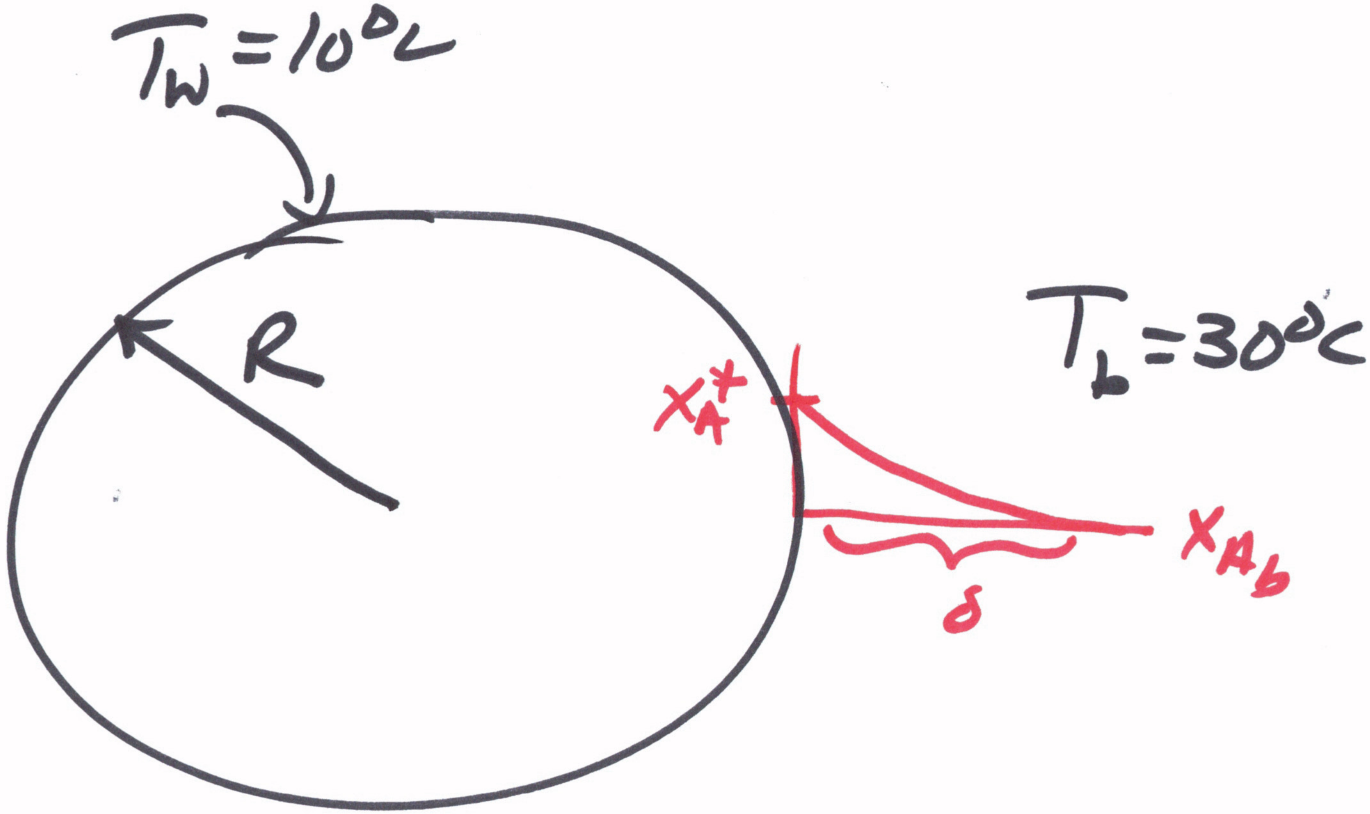


steady

$R \leq T < \infty$

$T(r)$

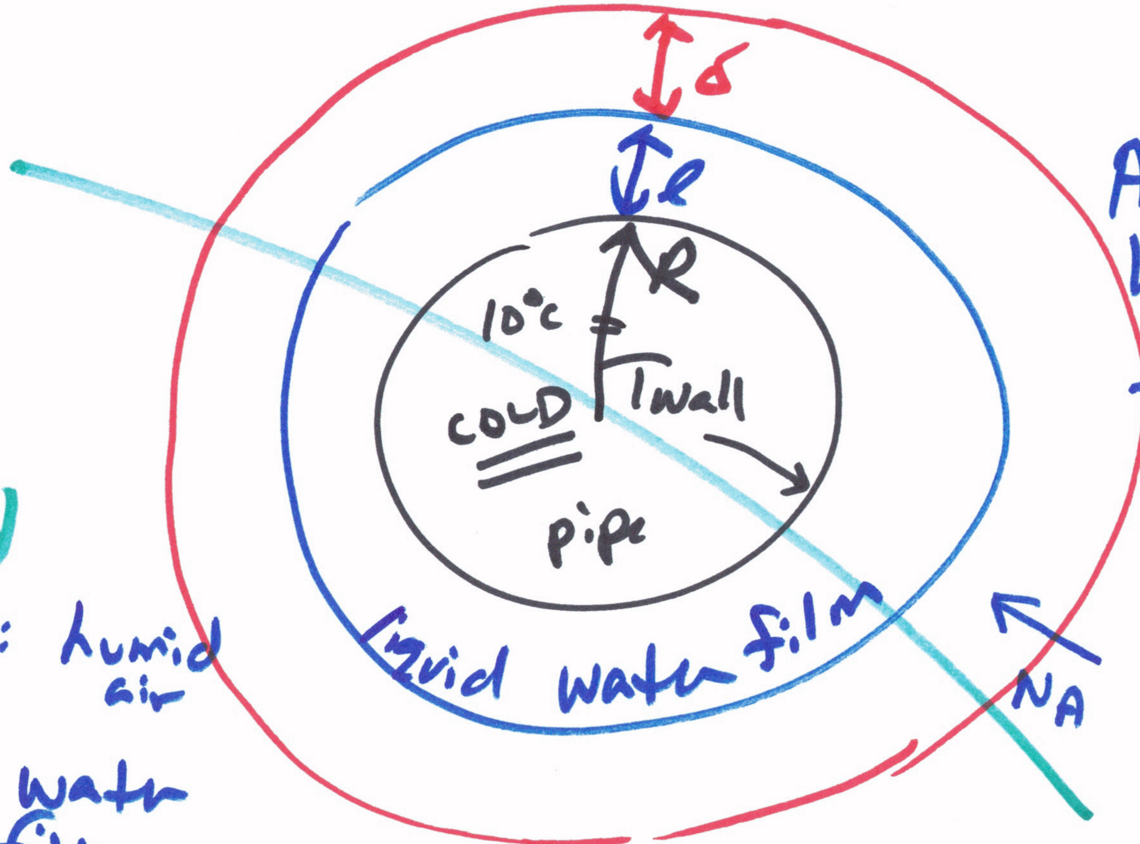
Mass xfr plm



What do we think is happening?

$T = 30°C$

too many films (confusing)



AIR
WATER

$P x_{A_b} = P^*(T)$

$x_{A_b} = \frac{P^*}{P}$

source: humid air

sink: water film

$x_A = \frac{P^*(T)}{P}$

sat'd at the outside location of our modeling domain

$R+l \leq r \leq R+l+\delta$

Source: Humid Air

Sink

RH = 80%

$T_b = 30°C$

$T_w = 10°C$

x_w

model liquid water as infinitely thin

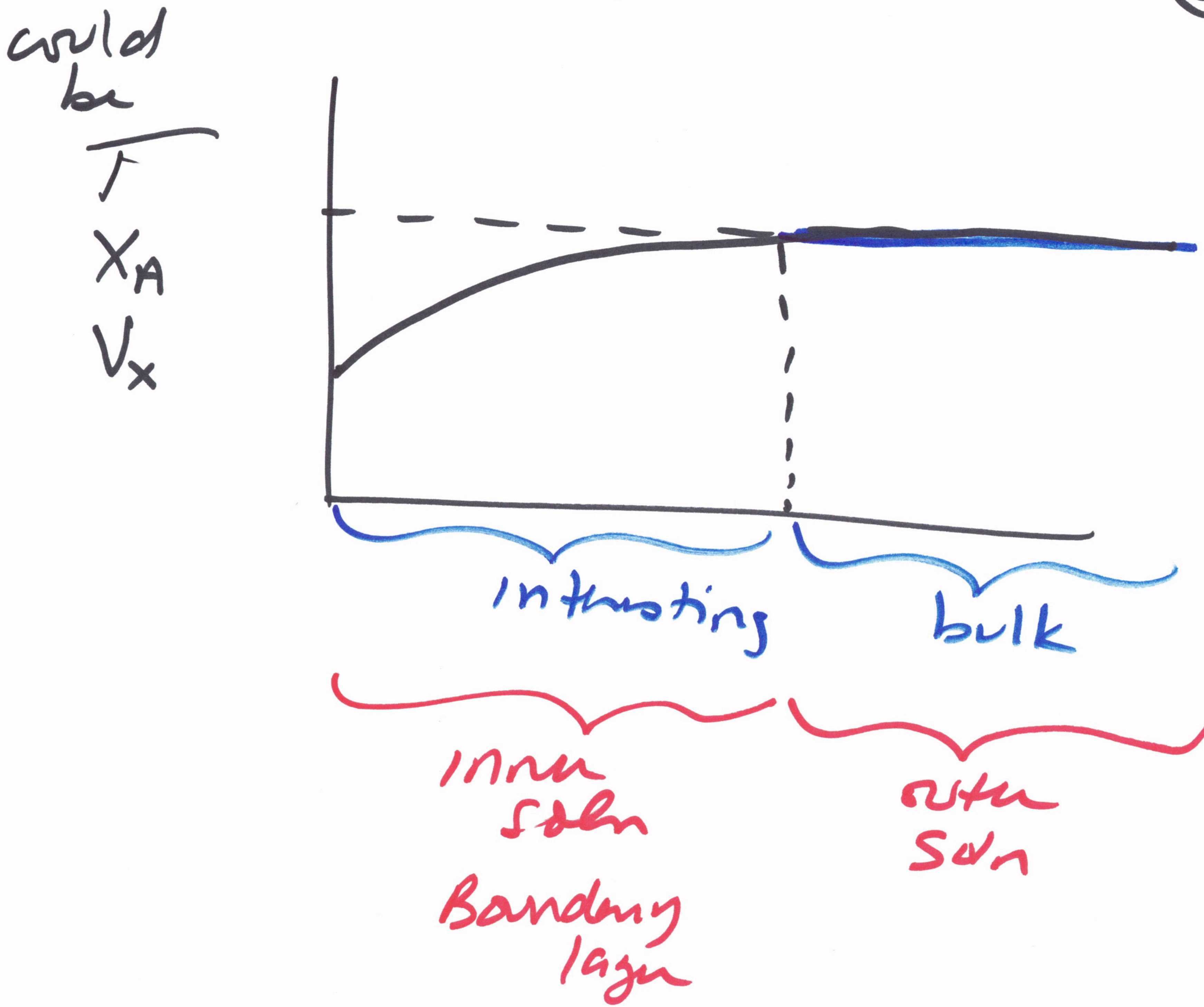
domain: choice

the top of the liquid film $\delta \le r \le \infty$ gas

$\le \delta$ film thickness

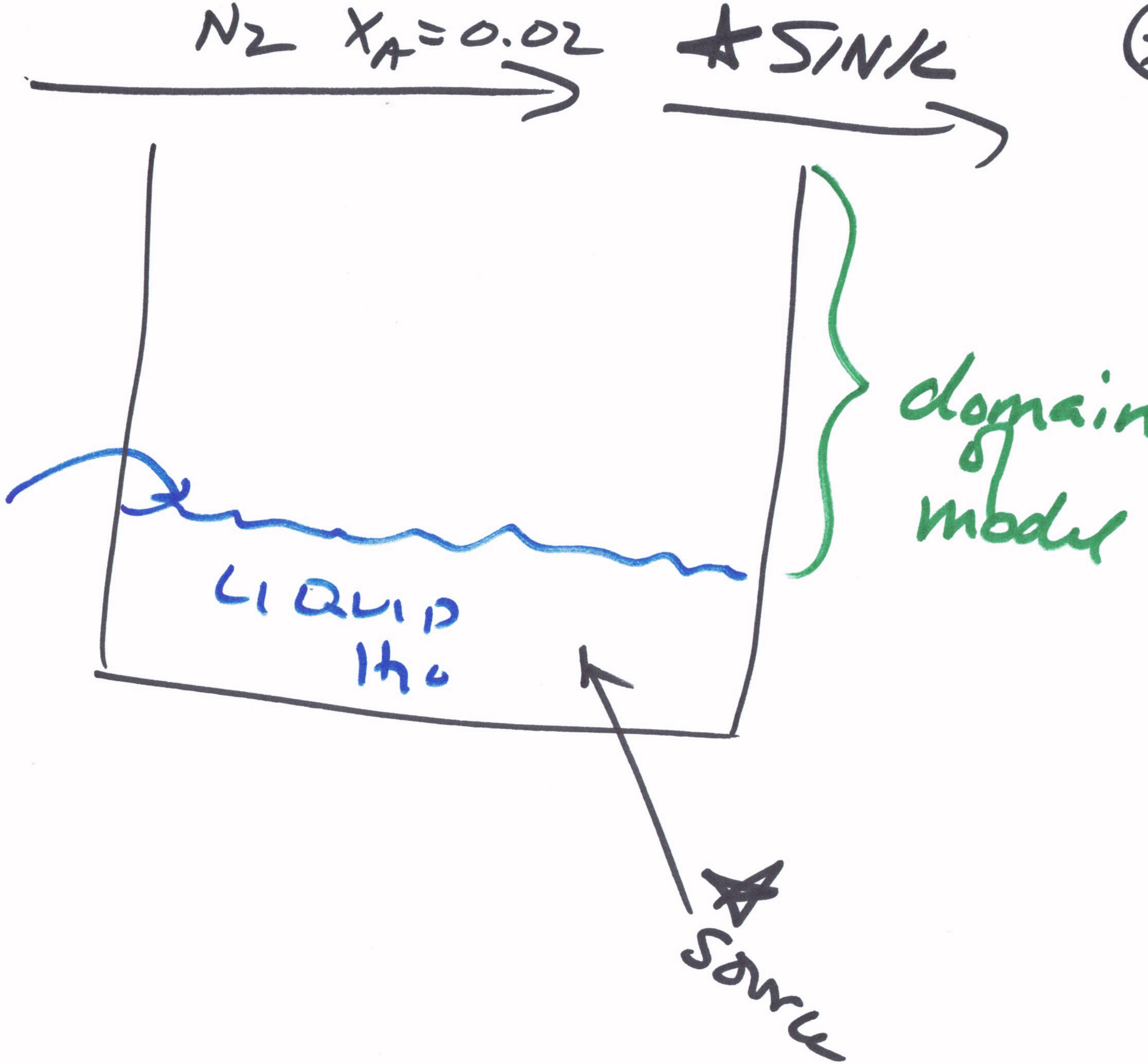
MACRO MASS ON green water replenished by humid air = mass drips

CLASSIC XPORT:



Recall EXAMPLE 1

$$\frac{P_A^*}{P} = x_A^*$$



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The **Equation of Species Mass Balance in Terms of Combined Molar quantities** in Cartesian, cylindrical, and spherical coordinates for binary mixtures of A and B.

The general case, where the combined molar flux with respect to molar velocity ($\underline{N}_A$), is given on page 1.

Spring 2019 Faith A. Morrison, Michigan Technological University

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In terms of total molar flux, $\underline{N}_A$

Microscopic species mass balance, in terms of molar flux; Gibbs notation

$$\frac{\partial c_A}{\partial t} = -\nabla \cdot \underline{N}_A + R_A \qquad \text{WRF 25-11}$$

Microscopic species mass balance, in terms of combined molar flux; Cartesian coordinates

$$\frac{\partial c_A}{\partial t} = -\left(\frac{\partial N_{A,x}}{\partial x} + \frac{\partial N_{A,y}}{\partial y} + \frac{\partial N_{A,z}}{\partial z}\right) + R_A$$

steady

θ symmetry

Microscopic species mass balance, in terms of combined molar flux; cylindrical coordinates

no z-mass xfr

$$\cancel{\frac{\partial c_A}{\partial t}} = -\left(\frac{1}{r}\frac{\partial (rN_{A,r})}{\partial r} + \cancel{\frac{1}{r}\frac{\partial N_{A,\theta}}{\partial \theta}} + \cancel{\frac{\partial N_{A,z}}{\partial z}}\right) + \cancel{R_A}$$

no homogeneous rxn

Microscopic species mass balance, in terms of combined molar flux; spherical coordinates

$$\frac{\partial c_A}{\partial t} = -\left(\frac{1}{r^2}\frac{\partial (r^2 N_{A,r})}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial (N_{A,\theta}\sin\theta)}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial N_{A,\phi}}{\partial \phi}\right) + R_A$$

Fick's law of diffusion, Gibbs notation: $\underline{N}_A = x_A(\underline{N}_A + \underline{N}_B) - cD_{AB}\nabla x_A \qquad$ WRF 24-22

$$= c_A \underline{v}^* - cD_{AB}\nabla x_A$$

Fick's law of diffusion, Cartesian coordinates:

$$\begin{pmatrix} N_{A,x} \\ N_{A,y} \\ N_{A,z} \end{pmatrix}_{xyz} = \begin{pmatrix} x_A(N_{A,x} + N_{B,x}) - cD_{AB}\frac{\partial x_A}{\partial x} \\ x_A(N_{A,y} + N_{B,y}) - cD_{AB}\frac{\partial x_A}{\partial y} \\ x_A(N_{A,z} + N_{B,z}) - cD_{AB}\frac{\partial x_A}{\partial z} \end{pmatrix}_{xyz}$$

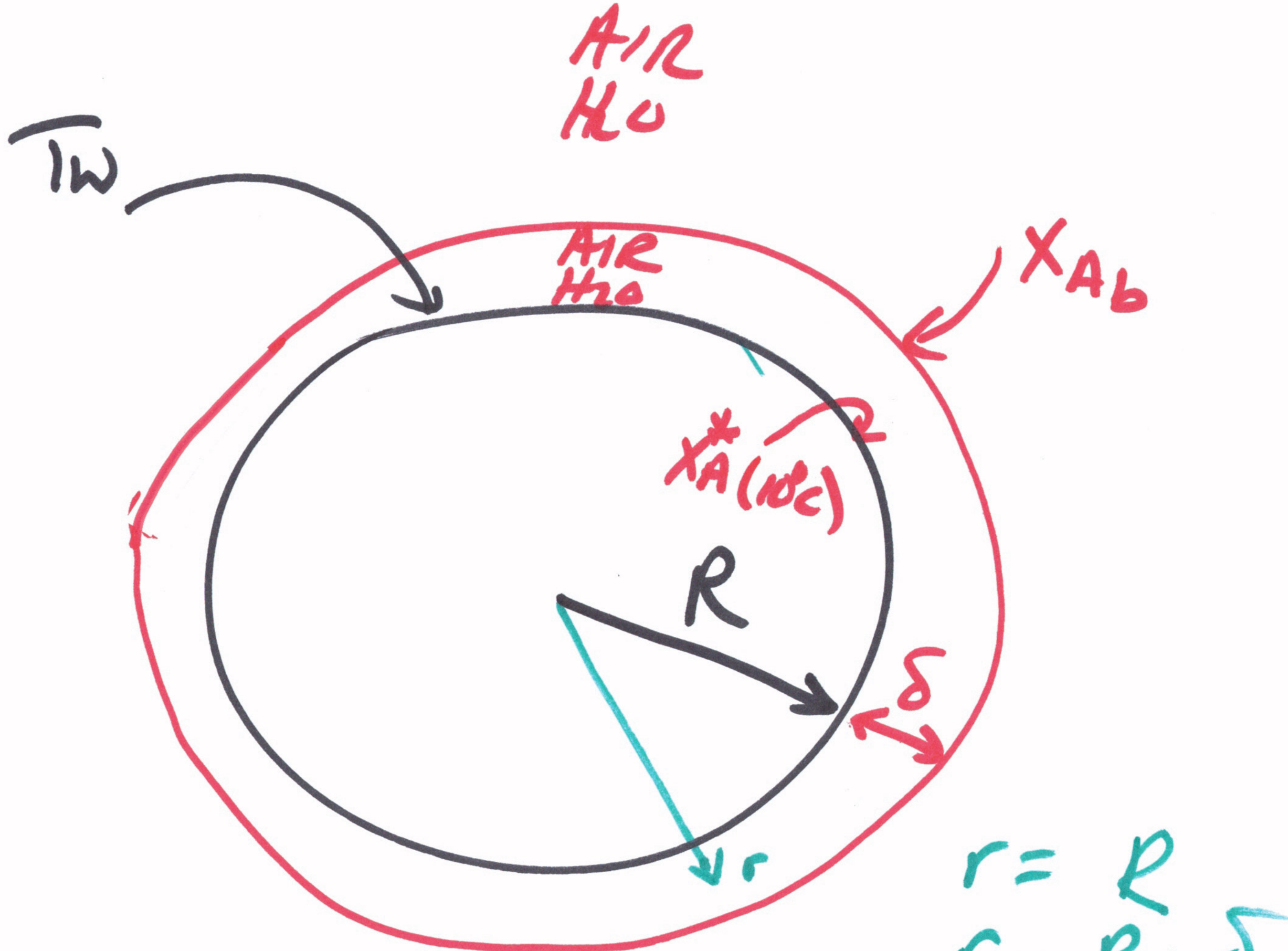
Fick's law of diffusion, cylindrical coordinates:

$$\begin{pmatrix} N_{A,r} \\ \cancel{N_{A,\theta}} \\ \cancel{N_{A,z}} \end{pmatrix}_{r\theta z} = \begin{pmatrix} x_A(N_{A,r} + \cancel{N_{B,r}}) - cD_{AB}\frac{\partial x_A}{\partial r} \\ \cancel{x_A(N_{A,\theta} + N_{B,\theta}) - \frac{cD_{AB}}{r}\frac{\partial x_A}{\partial \theta}} \\ \cancel{x_A(N_{A,z} + N_{B,z}) - cD_{AB}\frac{\partial x_A}{\partial z}} \end{pmatrix}_{r\theta z}$$

stagnant

Fick's law of diffusion, spherical coordinates:

$$\begin{pmatrix} N_{A,r} \\ N_{A,\theta} \\ N_{A,\phi} \end{pmatrix}_{r\theta\phi} = \begin{pmatrix} x_A(N_{A,r} + N_{B,r}) - cD_{AB}\frac{\partial x_A}{\partial r} \\ x_A(N_{A,\theta} + N_{B,\theta}) - \frac{cD_{AB}}{r}\frac{\partial x_A}{\partial \theta} \\ x_A(N_{A,\phi} + N_{B,\phi}) - \frac{cD_{AB}}{r\sin\theta}\frac{\partial x_A}{\partial \phi} \end{pmatrix}_{r\theta\phi}$$


AIR
H2O
Tw
AIR
H2O
X_{Ab}
$X_A^*(10°C)$
R
δ
r
$r = R$
$r = R + \delta$

$$\underbrace{N_{Ar}}_{\frac{c_1}{r}}(1-x_A) = -cD_{AB}\frac{dx_A}{dr}$$

$$\left(\frac{c_1}{r}\frac{1}{-cD_{AB}}\right)dr = \frac{dx_A}{1-x_A}$$

$$\int \underbrace{\left(\frac{c_1}{-cD_{AB}}\right)}_{\equiv\alpha}\frac{dr}{r} = \int\frac{dx_A}{1-x_A}$$

Q: is α constant WRT Temp? Across the domain?

$$\alpha\int\frac{dr}{r} = (-1)\int\frac{-dx_A}{1-x_A}$$

$u = 1-x_A$

$du = -dx_A$

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$$0 = \cancel{\frac{1}{r}} \frac{\partial}{\partial r}(\underbrace{r N_{a,r}}_{\equiv \Phi})$$

$$\frac{d\Phi}{dr} = 0$$

$$r N_{A,r} = \Phi = c_1$$

$$\boxed{N_{A,r} = \frac{c_1}{r}}$$

Fick's Law: $N_{Ar} = x_A(N_{Ar} + \underbrace{\cancel{N_{Br}}}_{\text{Stagnant}}) - c\mathcal{D}_{AB}\dfrac{dx_A}{dr}$

$$N_{Ar} - x_A N_{Ar} = -c\mathcal{D}_{AB}\frac{dx_A}{dr}$$

$$\alpha \ln r = -\ln(1-x_A) + C_2$$

$$\left(\frac{c_1}{-cD_{AB}}\right)\ln r = -\ln(1-x_A) + C_2$$

BC: $r = R \quad x_A = x_A^*$

$r = R+\delta \quad x_A = x_{A_b}$

solve for c_1, C_2

What was the question?

$$N_{A,r} = \frac{\text{moles of } A}{\text{area time}} \text{ in } r \text{ direction}$$

$$N_{Ar}\Big|_{r=R} = \left(\frac{C_1}{r}\right)_R$$

$$\text{moles of A entering the sink} = \frac{C_1 2\pi R L}{\cancel{R}} = \frac{\text{moles}}{\text{time}}$$

$$= \boxed{C_1 2\pi L \ \frac{\text{moles}}{\text{time}}}$$

Answer