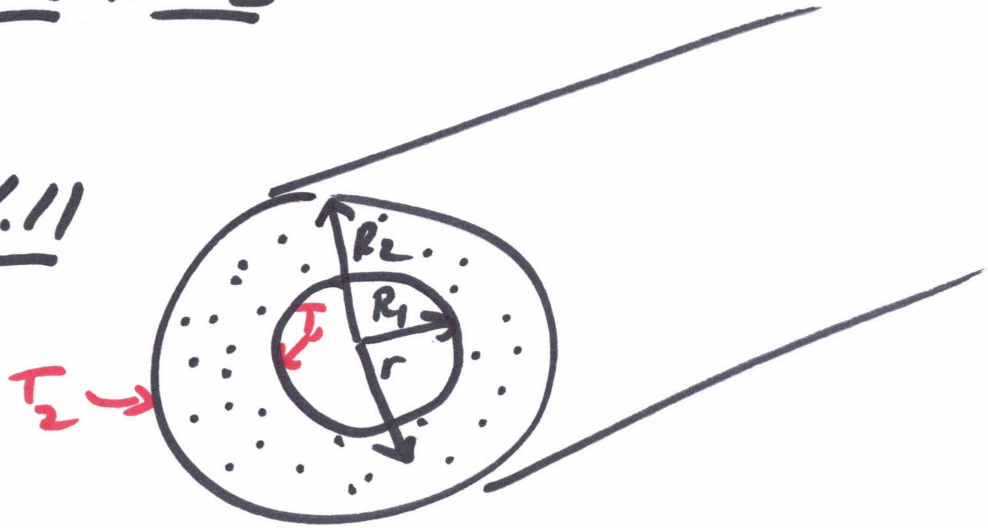
Office Hours 13 JAN 2021

CM3120 (1)
2021

HW 1.11



$R_1 \leq r \leq R_2$

1. Identify as μ E-bal pbm
   (T profile requested)

2. "Slash + Burn"

The **Equation of Energy** for systems with **constant k**

Microscopic energy balance, constant thermal conductivity; Gibbs notation

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + \underline{v} \cdot \nabla T \right) = k \nabla^2 T + S_e$$

Microscopic energy balance, constant thermal conductivity; Cartesian coordinates

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} + v_z \frac{\partial T}{\partial z} \right) = k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + S_e$$

no rxn
no current

Microscopic energy balance, constant thermal conductivity; cylindrical coordinates

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} + v_z \frac{\partial T}{\partial z} \right) = k \left(\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2} \right) + S_e$$

Microscopic energy balance, constant thermal conductivity; spherical coordinates

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} + \frac{v_\phi}{r \sin\theta} \frac{\partial T}{\partial \phi} \right)$$
$$= k \left(\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial \theta} \left(\sin\theta \frac{\partial T}{\partial \theta} \right) + \frac{1}{r^2 \sin^2\theta} \frac{\partial^2 T}{\partial \phi^2} \right) + S_e$$

steady

θ symmetry

LONG

$\underline{v} = 0$

$$\boxed{0 = k \frac{1}{r} \left(\frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) \right)}$$

Reference: F. A. Morrison, "Web Appendix to *An Introduction to Fluid Mechanics*," Cambridge University Press, New York, 2013 (*pages.mtu.edu/~fmorriso/cm310/IFMWebAppendixDMicroEBalanceMorrison.pdf*). This worksheet is on the web at *pages.mtu.edu/~fmorriso/cm310/energy.pdf*.

$$0 = \cancel{k}\,\cancel{\frac{1}{r}}\,\frac{d}{dr}\left(r\frac{dT}{dr}\right)$$

$$0 = \frac{d}{dr}\underbrace{\left(r\frac{dT}{dr}\right)}_{\equiv\,\Phi}$$

$$\frac{d\Phi}{dr} = 0$$

$$r\frac{dT}{dr} = \Phi = c_1$$

$$\frac{dT}{dr} = \frac{c_1}{r}$$

$$T = c_1\int\frac{dr}{r} = \boxed{c_1 \ln r + c_2 = T}$$

④

$$T = c_1 \ln r + c_2$$

Boundary Conditions: $r = R_1 \quad T = T_1$

$r = R_2 \quad T = T_2$

$$\begin{cases} T_1 = c_1 \ln R_1 + c_2 \\ T_2 = c_1 \ln R_2 + c_2 \end{cases}$$

* could substitute in the other eqn too

$$T_1 - T_2 = c_1 \underbrace{(\ln R_1 - \ln R_2)}_{\ln \frac{R_1}{R_2}}$$

$$\boxed{c_1 = \frac{T_1 - T_2}{\ln R_1/R_2}}$$

$$T_1 = \left(\frac{T_1 - T_2}{\ln \frac{R_1}{R_2}}\right) \ln R_1 + C_2$$

$$\boxed{C_2 = T_1 - \left(\frac{T_1 - T_2}{\ln \frac{R_1}{R_2}}\right) \ln R_1}$$

$$T = C_1 \ln r + C_2$$

$$T = c_1 \ln r + T_1 - c_1 \ln R_1$$

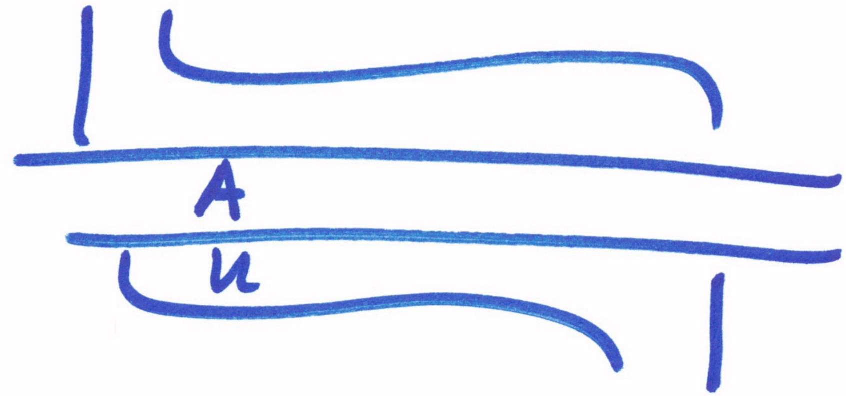
$$T - T_1 = c_1 (\ln r - \ln R_1)$$

$$= c_1 \left(\ln \frac{r}{R_1}\right)$$

$$T - T_1 = \left(\frac{T_1 - T_2}{\ln \frac{R_1}{R_2}} \right) \ln \frac{r}{R_1}$$

$$\boxed{\frac{(T - T_1)}{(T_1 - T_2)} = \frac{\ln \frac{r}{R_1}}{\ln \frac{R_1}{R_2}}}$$

SG(1.3) Objective 3



Functioning of a device $\quad Q = UA(\Delta T_{lm} F_T)$ device

Newton's Law of Cooling (at boundary) $\quad \left|\frac{q_x}{A}\right| = h\left|(T_w - T_b)\right|$ boundary between 2 phases resistance to heat xfer

Fourier's Law (Transport laws) $\quad \frac{q_x}{A} = -k\frac{dT}{dx}$ homogeneous material material prop