HW3

Michigan Technological University
Chemical Engineering

CM3120 Transport/Unit Ops 2

Unsteady State Heat Transfer and Diffusion.

HW3a: Unsteady heat transfer (finish week 6)

1. Compare the characteristic time we used in laminar/turbulent flow (momentum transport, non-dimensionalization of the microscopic momentum balance) and what we used in unsteady state heat transport (part of the Fourier number). What is the ratio? What does it mean in models we develop for this number to be large or small?
2. A large, iron structure (approximate dimensions $3m \times 3m \times 3m$, iron) initially with a uniform temperature of $98^o C$ is exposed to outside air conditions (air temperature $= 25^o C$, heat transfer coefficient $= 2.0 \times 10^1 W/m^2 K$). What is the wall temperature after 15 minutes?
3. Estimate the temperature at the at the center of a one-inch diameter solid brass sphere thirty seconds into being subjected to the following treatment: first the sphere is held in a bath and allowed to equilibrate at $10^o C$; subsequently the sphere is submerged into a vigorously stirred bath at constant temperature $85^o C$. Assume a heat transfer coefficient of $2300\ W/m^2\ K$. Answer: $84.6^o C$ or $85^o C$ based on the assumptions made and sig figs.
4. **Stretch** Using Excel, MATLAB, or software of your choosing, create the Heissler chart for the temperature at the center of a sphere as a function of time and Biot number (pages.mtu.edu/~fmorriso/cm3120/Heissler_Chart_for_web_2019.pdf). Answer: see Appendix F. See TA with questions.
5. If we expose a hot dog to the outside winter temperature, it will cool off. How fast it will cool off depends on the wind, how cold it is, and perhaps other factors. Model this process and indicate how you will come to an estimate of how long it will take for a hot dog initially at $\sim 200^o F$ to cool to a lukewarm $\sim 100^o F$. Find realistic values for parameters you will need. Answer: Will depend a bit on your assumptions. Our answer was $\approx 1s$.

HW3

Michigan Technological University
Chemical Engineering

HW3b: Intro to mass transfer (finish week 8) *updates* 3/5/20

6. Show that the following relationships for the various versions of the species mass/molar fluxes hold (do not assume Fick's law to show the equivalence):
   a. $\underline{N}_A + \underline{N}_B = c\underline{v}^*$
   b. $\underline{n}_A + \underline{n}_B = \rho\underline{v}$
   c. $\underline{j}_A + \underline{j}_B = 0$
   d. $\underline{J}_A^* + \underline{J}_B^* = 0$

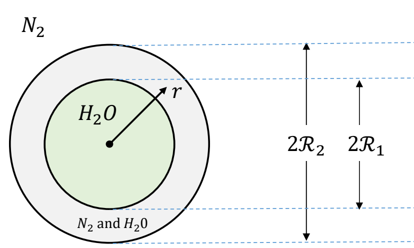
   In a sentence or two, what are the differences among the various fluxes in the question above? Why have we chosen to use such a variety of nomenclature?
7. In a particular region in space, species A (gas) is diffusing through stagnant species B (also a gas) at steady state. The situation may be considered to be one-dimensional (1D) diffusion. The steady state flux of species A (with respect to stationary coordinates) is $5.0 \times 10^{-5}\ kmol\ A\ / m^2 s$. At one point in the diffusion space, the concentration of A is $0.0050\ kmol/m^3$, and the concentration of B is $0.036\ kmol/m^3$. What is your estimate of the individual velocities of species A and B along the direction of mass transfer? What is the average molar velocity? Answer: $\underline{v}^* = 0.0012\ m/s$.
8. In a particular region in space species A (gas) and species B (also a gas) form a binary mixture in which steady equimolar counter diffusion is occurring (see p499 of WRF, posted at https://pages.mtu.edu/~fmorriso/cm3120/WRF2015_pp499-500.pdf). The situation may be considered to be one-dimensional (1D) diffusion. The steady state flux of species A is $5.0 \times 10^{-5}\ kmol\ A\ /m^2 s$ (with respect to stationary coordinates). At one point in the diffusion space, the concentration of A is $0.0050\ kmol/m^3$, and the concentration of B is $0.036\ kmol/m^3$. What is your estimate of the average velocities of molecules of species A and B along the direction of mass transfer? In this region of space, what is the average molar velocity? If A is water and B is nitrogen, what is the mass-average velocity $\underline{v}$? Answer: $v_z = -.4.6 \times 10^{-4} m/s;\ v_z^* \approx 0$. Comment on the difference.
9. ~~Serum albumin (an important blood protein) is a long-chain polymer that takes on a roughly spherical conformation. If we think of the diffusion of serum albumin as similar to the diffusion of a sphere moving through a solvent (water), we can estimate the size of the molecule. The measured diffusion coefficient of serum albumin is $5.94 \times 10^{-7} cm^2/s$ at $293K$. Based on this measured diffusivity, what is your estimate of the diameter of the protein under these conditions? Answer: $D = 7.27 nm$.~~
10. A hemispherical drop of liquid water lies on a flat surface. The water evaporates from the surface through a film of still air near the surface. Within the film, the gas is saturated with water. The temperature and pressure are constant, and the diffusion is slow, so the size of the droplet is almost constant. Model this problem in such a way that we can determine the concentration distribution in the film. How can we calculate the evaporation rate? Answer for concentration distribution in the film: $\left(\frac{1-x_A}{1-x_{A\infty}}\right) = \left(\frac{1-x_{AR}}{1-x_{A\infty}}\right)^{r/R}$; see also notes.

HW3

Michigan Technological University
Chemical Engineering

11. A cold-water pipeline runs through a hot, humid space in a processing plant. Water condenses onto the pipe and drips onto the ground. Model this problem in such a way that we can estimate the rate of water dripping from the pipe.
12. **Stretch:** A water mist forms in an industrial printing operation. Spherical water droplets slowly and steadily evaporate into the air (mostly nitrogen). The evaporation creates a film around the droplets through which the evaporating water diffuses. We can model the diffusion process as shown in the figure below. The temperature in the film is not constant but varies as $T(r)/T(\mathcal{R}_1) = (r/\mathcal{R}_1)^n$; note that this means that both the diffusivity D_{AB} and the concentration $c = P/RT$ are a function of position through their temperature dependences. What is the water mole fraction in the film as a function of radial position? You may assume ideal gas properties for air; you may assume that the diffusivity varies with temperature as follows: BSL2 p550

$$D_{AB}(T)/D_{AB,1} = \left(T/T(\mathcal{R}_1)\right)^{3/2}$$



References:

1. Bird, R. B., W. E. Stewart, and E. N. Lightfoot, Transport Phenomena, 2nd edition, Wiley, NY, 2002. Widely known in the ChemE world as "BSL." Also the 1960 edition is fine too, and available used ("the red book").
2. Christie J. Geankoplis, Transport Processes and Unit Operations, 4th Edition, Prentice Hall, New York (2003). Two hardbound copies are on reserve in the library.
3. Frank P. Incropera, David P. DeWitt, Theodore L. Bergman, Adrienne S. Lavinem, Fundamentals of Heat and Mass Transfer, 6th edition, Wiley, New York (2006).
4. Welty, James R., Gregory L. Rorrer, and David C. Foster, Fundamentals of Momentum, Heat, and Mass Transfer, 6th edition, Wiley, New York, 2015.

HW3

Michigan Technological University
Chemical Engineering

Notes and **Stretch**:

1. This combination of variables already has a name and has appeared in other calculations. Consider the limits of this ratio being very large or very small or equal to one.
2. Always try to solve with lumped parameter method first by checking the Biot number. Does it work? (No, it does not). What is the alternative when the lumped parameter method is not appropriate?
3. We discussed this problem in Lecture 7. I'm asking you to puzzle through how to use a solution from the literature to solve a problem that you have. **Stretch:** Can you compose a realistic engineering/co-op question that could be addressed by the analytical solution to this problem? I suggest you use Excel to evaluate so that you can change the numbers and play with the solution. That's how I did it.
4. The Heissler chart only uses the first term in the series, so it's much simpler than it seems. To find the λ_1 you need to do an iterative calculation (or find them from the literature; they're in Incropera and DeWitt. I have a handout for how to do the iterative calculation with Excel: http://pages.mtu.edu/~fmorriso/RecursiveEqnSolvingWithExcel.pdf.
5. The Heissler chart for a cylinder is in Appendix F. You'll need the definitions on page 286 of WRF.
6. See the text, or BSL (reference at the bottom of my sheet), or my sheets. WRF 24.2
7. Note that B is stagnant. Stretch: If A is water and B is nitrogen, what is $\underline{v}$? Comment on the difference. Answer: $8.2 \times 10^{-4} m/s$ WRF 24.4
8. Equimolar counter diffusion is a classic situation. It's in the next chapter.
9. I ~~noted some sections for you to skim; please take a look at them: http://pages.mtu.edu/~fmorriso/cm3120/Homeworks_Readings.html~~ WRF 24.16
10. Hemisphere diffusion.
    a. The air is stagnant. Include boundary conditions at the surface and far away from the sphere.
    b. The molar flux $\underline{N}_A$ can tell us the evaporation rate at steady state. WRF 25.3
11. Sketch the cold pipe problem and see where diffusion comes into the picture. The molar flux $\underline{N}_A$ can tell us the condensation rate at steady state. Near the water film, the air will be saturated with water (Raoult's law). What temperature would you use? WRF 25.4
12. Note that for an ideal gas $c = \frac{n}{V} = \frac{P}{RT}$ and thus c is a function of temperature.