

It's week 7.
Exam 3: Week 9 (after break)

Homework 3A:
Finish Week 6

HW3

Michigan Technological University
Chemical Engineering

CM3120 Transport/Unit Ops 2
Unsteady State Heat Transfer and Diffusion.

HW3a: Unsteady heat transfer (finish week 6)

- Compare the characteristic time we used in laminar/turbulent flow (momentum transport, non-dimensionalization of the microscopic momentum balance) and what we used in unsteady state heat transport (part of the Fourier number). What is the ratio? What does it mean in models we develop for this number to be large or small?
- A large, iron structure (approximate dimensions 3m x 3m x 3m, iron) initially with a uniform temperature of 100°C is exposed to outside air conditions (air temperature = 25°C, heat transfer coefficient = 2.0 x 10⁸ W/m²K). What is the wall temperature after 35 minutes?
- Estimate the temperature at the center of a one-inch diameter solid brass sphere thirty seconds into being subjected to the following treatment: first the sphere is held in a bath and allowed to equilibrate at 10°C; subsequently the sphere is submerged into a vigorously stirred bath at constant temperature 85°C. Assume a heat transfer coefficient of 2300 W/m² K. Answer: 84.6°C or 85°C based on the assumptions made and sig figs.
- Stretch Using Excel, MATLAB, or software of your choosing, create the Heisler chart for the temperature at the center of a sphere as a function of time and Biot number (https://chem.mtu.edu/cm3120/unitops/3120/Heisler_chart_for_week_2019.pdf). Answer: see Appendix F. See TA with questions.
- If we expose a hot dog to the outside winter temperature, it will cool off. How fast it will cool off depends on the wind, how cold it is, and perhaps other factors. Model this process and indicate how you will come to an estimate of how long it will take for a hot dog initially at -200°F to cool to a lukewarm -100°F. Find realistic values for parameters you will need. Answer: Will depend a bit on your assumptions. Our answer was ~ 1s.

Homework 3B:
Finish Week 8

HW3

Michigan Technological University
Chemical Engineering

HW3b: Intro to mass transfer (finish week 8)

- Show that the following relationships for the various versions of the species mass/molar fluxes hold (do not assume Fick's law to show the equalities)
 - $\vec{N}_A + \vec{N}_B = \vec{v}^*$
 - $\vec{N}_A + \vec{N}_B = \vec{v}$
 - $\vec{J}_A + \vec{J}_B = 0$
 - $\vec{J}_A + \vec{J}_B = 0$

In a sentence or two, what are the differences among the various fluxes in the question above? Why have we chosen to use such a variety of nomenclature?

- Species A (gas) is diffusing through stagnant species B (also a gas) at steady state. The situation may be considered to be one-dimensional (1D) diffusion. The steady state flux of species A is 5.0×10^{-8} kmol A/m²s. At one point in the diffusion space, the concentration of A is 0.0008 kmol/m³ and the concentration of B is 0.0016 kmol/m³. What is your estimate of the individual velocities of species A and B along the direction of mass transfer? What is the average molar velocity? Answer: $\vec{v}^* = 0.0012$ m/s.
- Species A (gas) and species B (also a gas) form a binary mixture in which steady equimolar counter diffusion is occurring (see p499 of WRF, posted at https://pages.mtu.edu/~chem3120/unitops/3120/WRF2019_page499-500.pdf). The situation may be considered to be one-dimensional (1D) diffusion. The steady state flux of species A is 5.0×10^{-8} kmol A/m²s. At one point in the diffusion space, the concentration of A is 0.0008 kmol/m³ and the concentration of B is 0.0016 kmol/m³. What is your estimate of the individual velocities of species A and B along the direction of mass transfer? What is the average molar velocity? If it is water and B is nitrogen, what is the mass-average velocity? Answer: $\vec{v}_m = -4.6 \times 10^{-10}$ m/s; $\vec{v}_B = 0$. Comment on the difference.
- Secum album (an important blood protein) is a long-chain polymer that takes on a roughly spherical conformation. If we think of the diffusion of secum albumin as similar to the diffusion of a sphere moving through a solvent (water), we can estimate the size of the molecule. The measured diffusion coefficient of secum albumin is 5.94×10^{-10} cm²/s at 293K. Based on this measured diffusivity, what is the radius of the sphere? Answer: $r = 7.21$ nm.
- A hemispherical drop of liquid is evaporating through a film of air of thickness δ . The temperature and air concentration distribution in the film is shown in the figure below. The evaporation rate of water droplets slowly and steadily evaporates into the air (mostly nitrogen). The evaporation creates a film around the droplets through which the evaporating water diffuses. We can model the diffusion process as shown in the figure below. The temperature in the film is not constant but varies as $T(r)/T(R_2) = (T(R_1)/T(R_2))^{1/2}$; note that this means that both the diffusivity D_{AB} and the concentration $c = P/AT$ are a function of position through their temperature dependences. What is the water mole fraction in the film as a function of radial position? You may assume ideal gas properties for air; you may assume that the diffusivity varies with temperature as follows: $D_{AB}(T)/D_{AB,1} = (T/T(R_2))^{1.75}$

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Diffusion and Mass Transfer

Michigan Technological University
Chemical Engineering

CM3120 Transport/Unit Operations 2

Diffusion and Mass Transfer in MIXTURES

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www.chem.mtu.edu/~fmorrison/cm3120/unitops120.html

It turns out that there are many interesting and applicable problems we can address readily with this form of the species mass balance.

Microscopic species mass balance in a film

$$\frac{d}{dr} \left(-D_{AB} \frac{dc_A}{dr} \right) + \dot{r}_A = 0$$

Let's jump in!

We'll do a "Quick Start" and get into some examples and return to the "why" of it all a bit later.

QUICK START
(to problem solving)

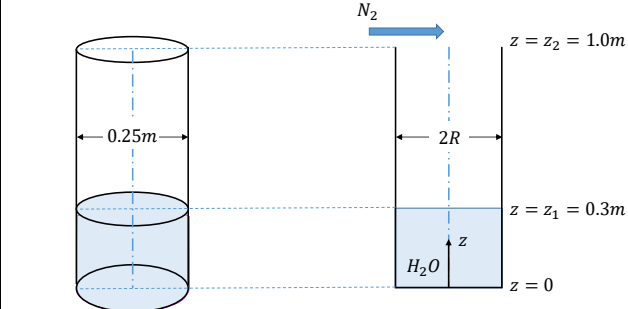
Continuing...

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Last time...

QUICK START

Example 1: Water (40°C , 1.0 atm) slowly and steadily evaporates into nitrogen (40°C , 1.0 atm) from the bottom of a cylindrical tank as shown in the figure below. A stream of dry nitrogen flows slowly past the open tank. The mole fraction of water in the gas at the top opening of the tank is 0.02. **What is the rate of water evaporation?**



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Last time...

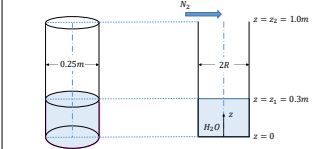
We sought to complete this problem.

The primary goal, however, was (is) **to grow our ability to troubleshoot engineering problems.**

QUICK START

Interrogating the problem:


- Why does the water evaporate?*
- What limits the rate of evaporation?*
- What could be done to accelerate the evaporation?*
- What could be done to slow down the evaporation?*



What is the driving physics?

We take note of the questions that were productive in leading us to the solution.

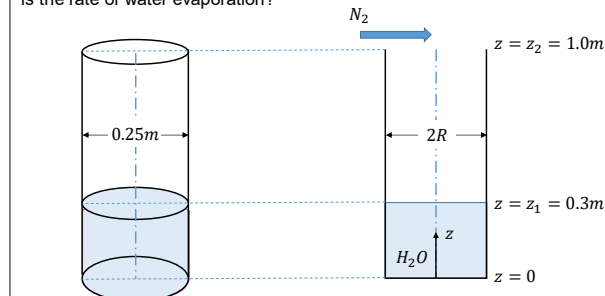
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Last time...

QUICK START

Example 1: Water (40°C, 1.0 atm) slowly and steadily evaporates into nitrogen (40°C, 1.0 atm) from the bottom of a cylindrical tank as shown in the figure below. A stream of dry nitrogen flows slowly past the open tank. The mole fraction of water in the gas at the top opening of the tank is 0.02. The geometry is as shown in the figure. What is water mole fraction as a function of vertical position in the tank? You may assume ideal gas properties. What is the rate of water evaporation?



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Example 1

Seek concentration distribution
 ⇒ microscopic species A mass balance.

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EXAMPLE 1: STEADY DIFFUSION FROM A CYLINDRICAL TANK
 MICRO SPECIES "A" MASS BAL. (cylindrical) see p3

$\frac{dN_{Ae}}{dz} = 0$
 integrate: $N_{Ae} = C_1$

N_{Ae} = combined molar flux in z-direction

Fick's Law: $N_{Ae} = x_A N_{Ae} - c D_{AB} \frac{dx_A}{dz}$
 (cylindrical) see p3
 $N_{Ae} - x_A N_{Ae} = -c D_{AB} \frac{dx_A}{dz}$

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Example 1

$N_{A,z}(1-x_A) = -cD_{AB} \frac{dx_A}{dz}$ ③

from species mass balance $\rightarrow C_1 \rightarrow N_{A,z} dz = -cD_{AB} \frac{dx_A}{(1-x_A)}$

$\int_{z_1}^{z_2} C_1 dz = -cD_{AB} \int_{x_{A1}}^{x_{A2}} \frac{dx_A}{(1-x_A)}$

$C_1 z = +cD_{AB} \ln(1-x_A) + C_2$ $du = -dx_A$

BC: $z = z_2, x_A = 0.02 = x_{A2}$
 $z = z_1, x_A = x_{A1}^* \leftarrow$ Results

To Solve: ④

- substitute
- obtain 2 equations, 2 unknowns
- solve for C_1, C_2

results:
$$\begin{cases} C_1 = \frac{cD_{AB}}{(z_1 - z_2)} \ln\left(\frac{1-x_{A1}}{1-x_{A2}}\right) \\ C_2 = \ln(1-x_{A1}) - \frac{z_1}{(z_1 - z_2)} \ln\left(\frac{1-x_{A1}}{1-x_{A2}}\right) \end{cases}$$

- substitute back. Final answer:
$$\left[\frac{1-x_A}{1-x_{A1}} = \left(\frac{1-x_A}{1-x_{A2}} \right)^{\frac{(z-z_1)/(z_1-z_2)}{1}} \right] //$$

First, we obtained the flux $N_{A,z}$ and the concentration distribution $x_A(z)$.

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Example 1

To Solve: ④

- substitute
- obtain 2 equations, 2 unknowns
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results:
$$\begin{cases} C_1 = \frac{cD_{AB}}{(z_1 - z_2)} \ln\left(\frac{1-x_{A1}}{1-x_{A2}}\right) \\ C_2 = \ln(1-x_{A1}) - \frac{z_1}{(z_1 - z_2)} \ln\left(\frac{1-x_{A1}}{1-x_{A2}}\right) \end{cases}$$

- substitute back. Final answer:
$$\left[\frac{1-x_A}{1-x_{A1}} = \left(\frac{1-x_A}{1-x_{A2}} \right)^{\frac{(z-z_1)/(z_1-z_2)}{1}} \right] //$$

What is the rate of Evaporation? ⑤

Answer: $N_{A,z}$
 $\Rightarrow C_1$

$x_{A1} = \frac{p^*}{P} = 0.0228244$ (see tables for $p^*(H_2O, T)$)

$x_{A2} = 0.02$ (given)

$c = \frac{P}{RT} = 3.891367 \times 10^{-5} \frac{\text{mol}}{\text{cm}^3}$

$P D_{AB} = 2.634 \frac{\text{m}^2 \text{Pa}}{\text{s}} \text{ (HAP)}$

... ANSWER: $N_{A,z} = 0.026 \text{ mol/s} //$

Then, we answered the question:

$$N_{A,z} = C_1 = \frac{cD_{AB}}{(z_1 - z_2)} \ln\left(\frac{1-x_{A1}}{1-x_{A2}}\right)$$

The rate of evaporation $N_{A,z}$ is 0.026 mol/s.

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Example 1

Example: Water (40°C, 1.0 atm) slowly and steadily evaporates into nitrogen (40°C, 1.0 atm) from the bottom of a cylindrical tank as shown in the figure below. A stream of dry nitrogen flows slowly past the open tank. The mole fraction of water in the gas at the top opening of the tank is 0.02. **What is the rate of water evaporation?**

Equation of Species Mass Balance in Terms of Molar Fluxes

① $N_A = c_A v_x$

② $N_A = -c D_{AB} \frac{dx_A}{dz}$

③ $N_A = c_A v_x = -c D_{AB} \frac{dx_A}{dz}$

④ $N_A = c_A v_x = -c D_{AB} \frac{dx_A}{dz}$

⑤ $N_A = c_A v_x = -c D_{AB} \frac{dx_A}{dz}$

Handwritten notes:

① $N_A(1-x_A) = -c D_{AB} \frac{dx_A}{dz}$

② $N_A \frac{dz}{1-x_A} = -c D_{AB} dx_A$

③ $\int_{z=0}^{z=2R} \frac{dz}{1-x_A} = -\frac{c D_{AB}}{N_A} \int_{x_A=0}^{x_A=0.02} dx_A$

④ $\ln(1-x_A) = -\frac{c D_{AB}}{N_A} \frac{z}{R}$

⑤ $\ln(1-0.02) = -\frac{c D_{AB}}{N_A} \frac{2R}{R}$

⑥ $\ln(0.98) = -\frac{2 c D_{AB}}{N_A}$

⑦ $N_A = \frac{2 c D_{AB}}{\ln(0.98)}$

⑧ $N_A = \frac{2 \times 26 \times 10^{-6} \times 3.71 \times 10^5}{\ln(0.98)} = 0.026 \text{ mol/s}$

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The primary goal is **to grow our ability to troubleshoot engineering problems.**

QUICK START

Let's do another problem

Example 2: A water mist forms in an industrial printing operation. Spherical water droplets slowly and steadily evaporate into the air (mostly nitrogen). What is the rate of evaporation and how does the water concentration vary in the gas?

Ver 1

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QUICK START

Example 2: A water mist forms in an industrial printing operation. Spherical water droplets slowly and steadily evaporate into the air (mostly nitrogen). What is the rate of evaporation and how does the water concentration vary in the gas?

Let's Interrogate the problem.

Ver 1

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QUICK START

Example 2: A water mist forms in an industrial printing operation. Spherical water droplets slowly and steadily evaporate into the air (mostly nitrogen). What is the rate of evaporation and how does the water concentration vary in the gas?

Why does the water evaporate?

What limits the rate of evaporation?

What could be done to accelerate the evaporation?

What could be done to slow down the evaporation?

What is the driving physics?

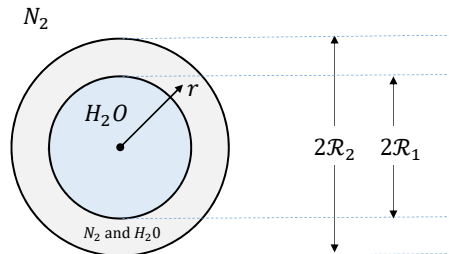
Can we use any ideas from previous experience?

Ver 1

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QUICK START

Example 2: A water mist forms in an industrial printing operation. Spherical water droplets slowly and steadily evaporate into the air (mostly nitrogen). The evaporation creates a film around the droplets through which the evaporating water diffuses. What is the rate of evaporation and how does the water concentration vary as a function distance from the droplet? You may assume ideal gas properties for air.



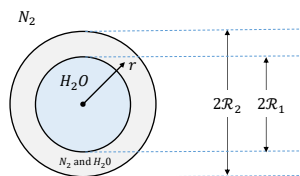
Ver 2

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QUICK START

Example 2: A water mist forms in an industrial printing operation. Spherical water droplets slowly and steadily evaporate into the air (mostly nitrogen). The evaporation creates a film around the droplets through which the evaporating water diffuses. What is the rate of evaporation and how does the water concentration vary as a function distance from the droplet? You may assume ideal gas properties for air.



Ver 2

SOLVE

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QUICK START

SOLUTION:

Example 2: A water mist forms in an industrial printing operation. Spherical water droplets slowly and steadily evaporate into the air (mostly nitrogen). The evaporation creates a film around the droplets through which the evaporating water diffuses. What is the rate of evaporation and how does the water concentration vary as a function distance from the droplet? You may assume ideal gas properties for air.

Ver 2

See hand slides

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QUICK START

Ver 2

Example 2: A water mist forms in an industrial printing operation. Spherical water droplets slowly and steadily evaporate into the air (mostly nitrogen). The evaporation creates a film around the droplets through which the evaporating water diffuses. What is the rate of evaporation and how does the water concentration vary as a function distance from the droplet? You may assume ideal gas properties for air.

Assumptions:

- Film surrounds droplet
- Ideal gas
- Constant temperature

Ver 2

Solution:

$$x_A(r)$$

$$\frac{1 - x_{A1}}{1 - x_{A2}} = \left(\frac{1 - x_{A2}}{1 - x_{A1}} \right) \left(\frac{\frac{1}{R_1} - \frac{1}{r}}{\frac{1}{R_1} - \frac{1}{R_2}} \right)$$

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**More (and varied)
problems**



Continuing...

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