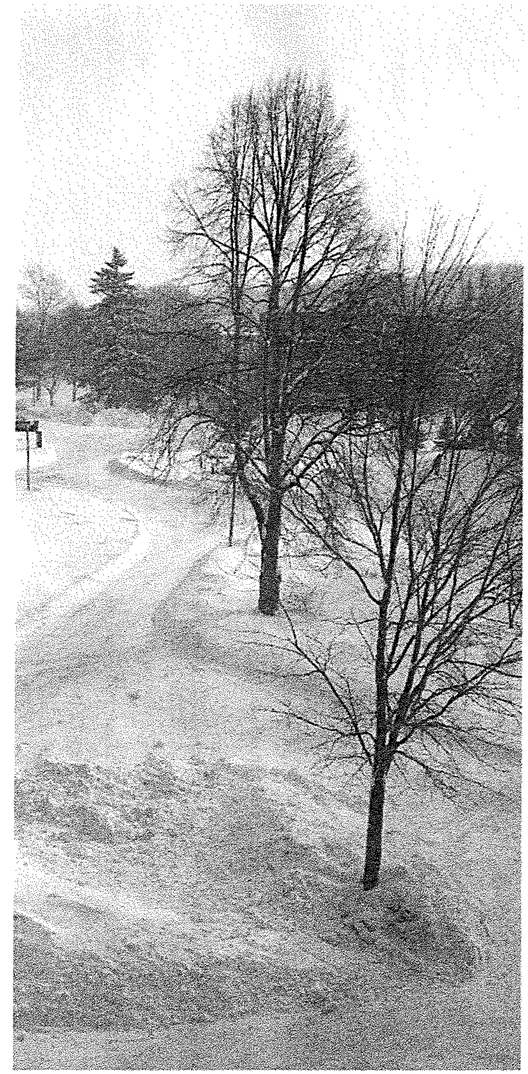
1/29/2020 1
CM3120

Example:
When will my pipes freeze?

The temperature has been 35^{o}F for a while now, sufficient to chill the ground to this temperature for many tens of feet below the surface. Suddenly the temperature drops to -20^{o}F. How long will it take for freezing temperatures (32^{o}F) to reach my pipes, which are 8 *ft* under ground?

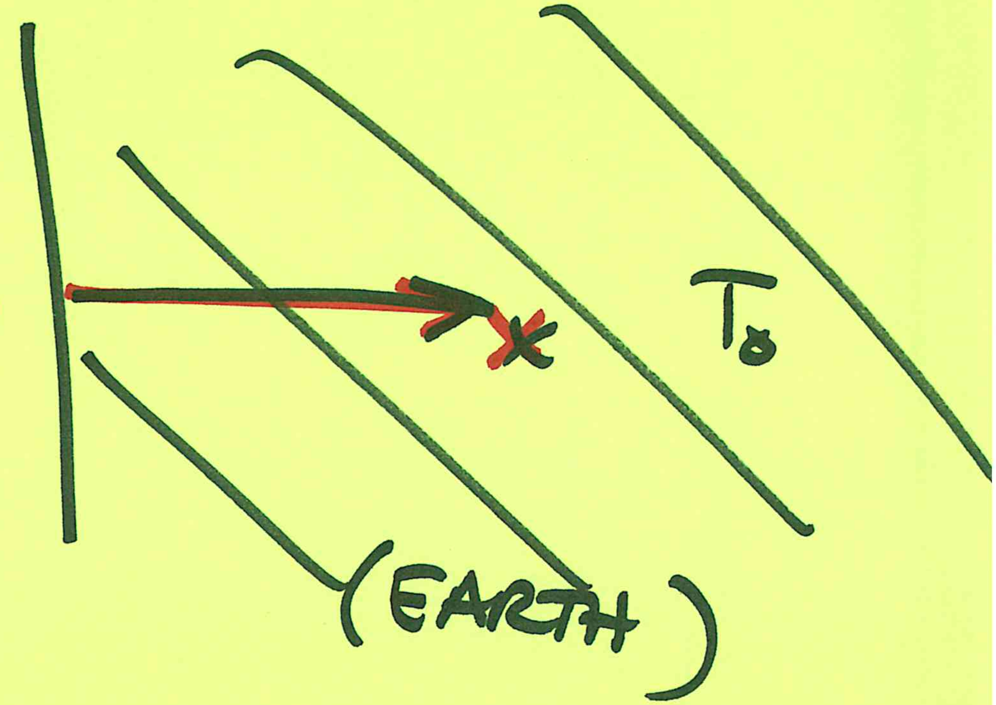
1/30/19



At first... (BUILDING the MODEL) ②

$t < 0$

$\forall x \quad T = T_0 = 55^\circ F$



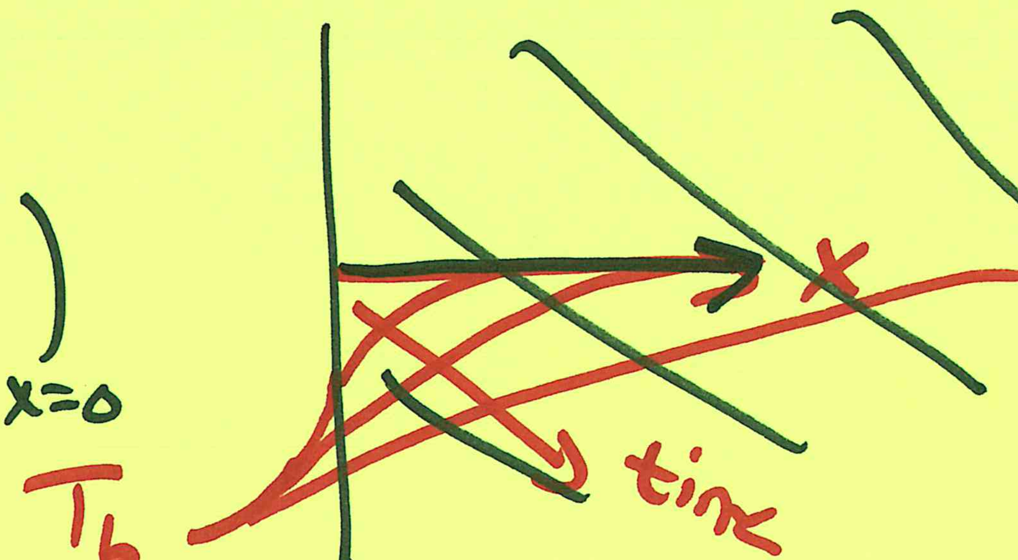
Then suddenly...

$t \geq 0 \quad x = 0$

$$-k\frac{dT}{dx}\Big|_{x=0} = \frac{q_x}{A}\Big|_{x=0} = h\left(T_b - T\right)\Big|_{x=0}$$

(<0)

Newton's Law of Cooling



M

The **Equation of Energy** for systems with **constant k**

Microscopic energy balance, constant thermal conductivity; Gibbs notation

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t}+\underline{v}\cdot\nabla T\right)=k\nabla^2 T+S_e$$

Microscopic energy balance, constant thermal conductivity; Cartesian coordinates

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t}+\cancel{v_x\frac{\partial T}{\partial x}}+\cancel{v_y\frac{\partial T}{\partial y}}+\cancel{v_z\frac{\partial T}{\partial z}}\right)=k\left(\frac{\partial^2 T}{\partial x^2}+\cancel{\frac{\partial^2 T}{\partial y^2}}+\cancel{\frac{\partial^2 T}{\partial z^2}}\right)+\cancel{S_e}$$

no rxn
no elec current

$v=0$

long
wide

Microscopic energy balance, constant thermal conductivity; cylindrical coordinates

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t}+v_r\frac{\partial T}{\partial r}+\frac{v_\theta}{r}\frac{\partial T}{\partial\theta}+v_z\frac{\partial T}{\partial z}\right)=k\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 T}{\partial\theta^2}+\frac{\partial^2 T}{\partial z^2}\right)+S_e$$

Microscopic energy balance, constant thermal conductivity; spherical coordinates

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t}+v_r\frac{\partial T}{\partial r}+\frac{v_\theta}{r}\frac{\partial T}{\partial\theta}+\frac{v_\phi}{r\sin\theta}\frac{\partial T}{\partial\phi}\right)$$
$$=k\left(\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial T}{\partial\theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 T}{\partial\phi^2}\right)+S_e$$

$T(t,x)$

$$\rho\hat{C}_p\frac{\partial T}{\partial t}=k\frac{\partial^2 T}{\partial x^2}$$

$$\frac{\partial T}{\partial t}=\underbrace{\frac{k}{\rho\hat{C}_p}}_{=\alpha\text{ thermal diffusivity}}\frac{\partial^2 T}{\partial x^2}$$

Reference: F. A. Morrison, "Web Appendix to *An Introduction to Fluid Mechanics*," Cambridge University Press, New York, 2013 (*pages.mtu.edu/~fmorriso/cm310/IFMWebAppendixDMicroEBalanceMorrison.pdf*). This worksheet is on the web at *pages.mtu.edu/~fmorriso/cm310/energy.pdf*.

$$\underbrace{\left|\frac{q_x}{A}\right|}_{\frac{q_x}{A}\text{ is negative}} = h\,\underbrace{\left|T_b - T_w\right|}_{\substack{\text{arrange}\\ \text{the temps}\\ \text{to get a}\\ \text{negative \#}}}$$

"initial" (one)

Need: $T(t,x)$

1 initial ¢

2 Boundary conditions

TWO

~~$t=0$~~

$t<0$ $\quad \forall x$ $\quad T = T_0 = 35\,°F$

"boundary" (two)

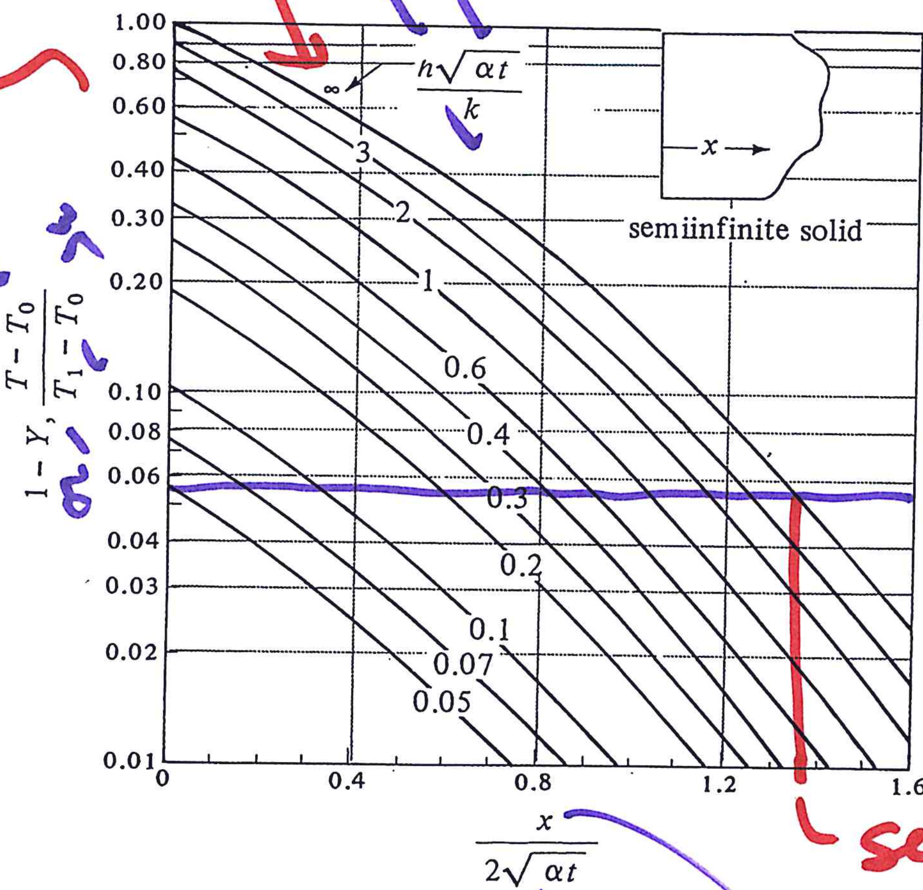
1) $t \geq 0$ $\quad x=0$ $\quad -k \left.\frac{dT}{dx}\right|_{x=0} = h\left(T_b - \left.T\right|_{x=0}\right)$

2) $\forall t$ $\quad x=\infty$ $\quad T=T_0$

6

we know this

guess ∞



$1 - Y = 0.05$

see if this value gives the same t

& t

FIGURE 5.3-3. *Unsteady-state heat conducted in a semi-infinite solid with surface convection. Calculated from Eq. (5.3-7)(S1).*

PURPOSE:
Learn to use solns from the literature - quick.

Geankoplis, 4th edition