Unsteady Sphere 10 Feb 2020

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The **Equation of Energy** for systems with **constant k**

Microscopic energy balance, constant thermal conductivity; Gibbs notation

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t}+\underline{v}\cdot\nabla T\right)=k\nabla^2 T+S_e$$

Microscopic energy balance, constant thermal conductivity; Cartesian coordinates

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t}+v_x\frac{\partial T}{\partial x}+v_y\frac{\partial T}{\partial y}+v_z\frac{\partial T}{\partial z}\right)=k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right)+S_e$$

Microscopic energy balance, constant thermal conductivity; cylindrical coordinates

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t}+v_r\frac{\partial T}{\partial r}+\frac{v_\theta}{r}\frac{\partial T}{\partial \theta}+v_z\frac{\partial T}{\partial z}\right)=k\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}+\frac{\partial^2 T}{\partial z^2}\right)+S_e$$

$\underline{v}=0$

Microscopic energy balance, constant thermal conductivity; spherical coordinates

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t}+\cancel{v_r\frac{\partial T}{\partial r}}+\cancel{\frac{v_\theta}{r}\frac{\partial T}{\partial \theta}}+\cancel{\frac{v_\phi}{r\sin\theta}\frac{\partial T}{\partial \phi}}\right)$$
$$=k\left(\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)+\cancel{\frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial T}{\partial \theta}\right)}+\cancel{\frac{1}{r^2\sin^2\theta}\frac{\partial^2 T}{\partial \phi^2}}\right)+\cancel{S_e}$$

0 sym

0 sym

no rxn no current

$$\frac{\partial T}{\partial t}=\alpha\,\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)$$

$$\alpha=\frac{k}{\rho c_p}$$

$t=0$ $T=T_0=8C$

$x=R$ Newton's Law

$x=0$ $\frac{\partial T}{\partial r}=0$

Reference: F. A. Morrison, "Web Appendix to *An Introduction to Fluid Mechanics*," Cambridge University Press, New York, 2013 (*pages.mtu.edu/~fmorriso/cm310/IFMWebAppendixDMicroEBalanceMorrison.pdf*). This worksheet is on the web at *pages.mtu.edu/~fmorriso/cm310/energy.pdf*.

$r = R$ $T(r,t)$ 2

$$\boxed{-k\frac{dT}{dr}\bigg|_{r=R} = \frac{q_r}{A} = -h\left(T_b - T\big|_{r=R}\right)}$$

$T_b = 100°C$

Newton's Law of Cooling B.C.

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