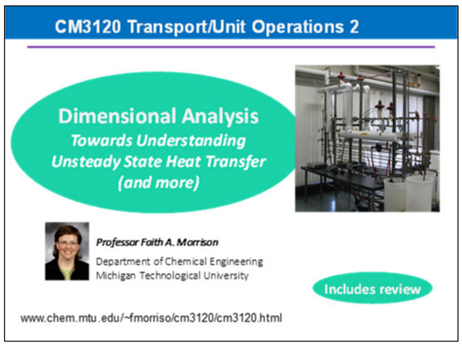
Last time,
Dimensional Analysis for Heat Transfer



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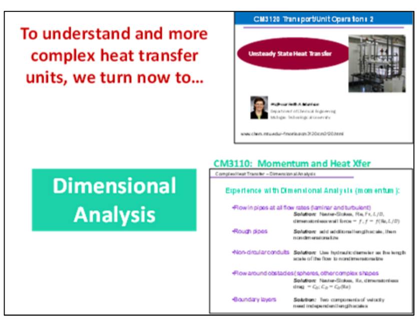

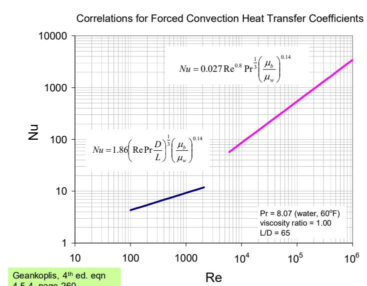

$$\text{Nu} = \text{Nu}\left(\text{Re}, \text{Pr}, \frac{L}{D}\right)$$

Summary

- Dimensional analysis works as well in heat transfer as in momentum transfer
- We should use it (and probably also in mass transfer, but…)
- These dimensionless numbers are stacking up (and…)
- What do they really mean?

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Dimensional Analysis

These numbers tell us about the relative importance of the terms they precede.

Dimensionless numbers from the **Equations of Change** *(microscopic balances)*

momentum — Non-dimensional Navier-Stokes Equation

$$\left(\frac{\partial v_z^*}{\partial t^*} + \underline{v}^* \cdot \nabla^* v_z^*\right) = -\frac{\partial P^*}{\partial z^*} + \frac{1}{\mathrm{Re}}\left(\nabla^{*2} v_z^*\right) + \frac{1}{\mathrm{Fr}} g^*$$

Re – Reynolds
Fr – Froude

energy — Non-dimensional Energy Equation

$$\left(\frac{\partial T^*}{\partial t^*} + \underline{v}^* \cdot \nabla^* T^*\right) = \frac{1}{\mathrm{RePr}}\left(\nabla^{*2} T^*\right) + S^*$$

Pe – Péclet$_h$ = RePr
Pr – Prandtl

mass — Non-dimensional Continuity Equation (species A)

$$\left(\frac{\partial x_A^*}{\partial t^*} + \underline{v}^* \cdot \nabla^* x_A^*\right) = \frac{1}{\mathrm{ReSc}}\left(\nabla^{*2} x_A^*\right)$$

Pe – Péclet$_m$ = ReSc
Sc – Schmidt

ref: BSL1, p581, 644

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Dimensionless Numbers

Dimensionless numbers from the **Equations of Change**

Re – Reynolds $= \frac{\rho V D}{\mu} = \frac{VD}{\nu}$

Fr – Froude $= \frac{V^2}{gD}$

Pe – Péclet$_h$ = RePr $= \frac{\hat{C}_p \rho V D}{k} = \frac{VD}{\alpha}$

Pe – Péclet$_m$ = ReSc $= \frac{VD}{D_{AB}}$

These numbers tell us about the **relative importance of the terms** they precede in the microscopic balances (*scenario properties*).

Pr – Prandtl $= \frac{\hat{C}_p \mu}{k} = \frac{\nu}{\alpha}$

Sc – Schmidt = LePr $= \frac{\mu}{\rho D_{AB}} = \frac{\nu}{D_{AB}}$

Le – Lewis $= \frac{\alpha}{D_{AB}}$

These numbers compare the **magnitudes of the diffusive transport coefficients** ν, α, D_{AB} (*material properties*).

thermal diffusivity $\alpha \equiv \frac{k}{\rho \hat{C}_p}$

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Dimensional Analysis

These numbers are defined to help us build **transport data correlations** based on the fewest number of grouped (dimensionless) variables (*scenario property*).

Dimensionless numbers from the **Engineering Quantities of Interest**

momentum

Dimensionless Force on the Wall (Drag)

$$f = \frac{1}{\pi}\frac{D}{L}\frac{1}{\text{Re}}\int_0^{\frac{L}{D}}\int_0^{2\pi}\left(\frac{\partial v_z^*}{\partial r^*}\right)\Bigg|_{r^*=\frac{1}{2}} d\theta dz^*$$

f – Friction Factor (*Fanning*)

$\frac{L}{D}$ – Aspect Ratio

$$f = \frac{\mathcal{F}_{drag}}{\left(\frac{1}{2}\rho V^2\right) A_c}$$

energy

Newton's Law of Cooling

$$Nu = \frac{1}{2\pi L/D}\int_0^{2\pi}\int_0^{L/D} -\frac{\partial T^*}{\partial r^*}\Bigg|_{r^*=1/2} dz^* d\theta$$

Nu – Nusselt

$\frac{L}{D}$ – Aspect Ratio

$$\text{Nu} = \frac{hD}{k}$$

mass xfer

Dimensionless Mass Transfer Coefficient

$$Sh = \frac{1}{2\pi}\frac{D}{L}\int_0^{\frac{L}{D}}\int_0^{2\pi}\left(-\frac{\partial x_A^*}{\partial r^*}\right)\Bigg|_{r^*=\frac{1}{2}} d\theta dz^*$$

Sh – Sherwood

$\frac{L}{D}$ – Aspect Ratio

$$\text{Sh} = \frac{k_m D}{D_{AB}}$$

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Dimensionless Numbers

momentum
energy
mass

Re – Reynolds $= \frac{\rho V D}{\mu} = \frac{VD}{\nu}$

Fr – Froude $= \frac{V^2}{gD}$

Pe – Péclet$_h$ = RePr $= \frac{\hat{C}_p \rho V D}{k} = \frac{VD}{\alpha}$

Pe – Péclet$_m$ = ReSc $= \frac{VD}{D_{AB}}$

These numbers from the governing equations tell us about the relative importance of the terms they precede in the microscopic balances (***scenario*** *properties*).

Pr – Prandtl $= \frac{\hat{C}_p \mu}{k} = \frac{\nu}{\alpha}$

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f – Friction Factor $= \frac{\mathcal{F}_{drag}}{\left(\frac{1}{2}\rho V^2\right) A_c}$

Nu – Nusselt $= \frac{hD}{k}$

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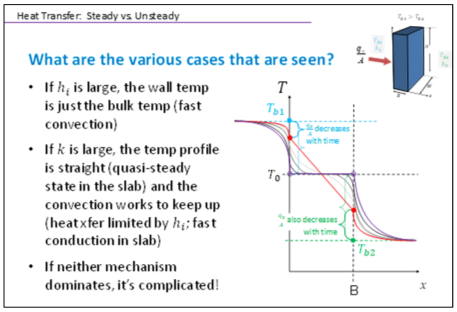
Unsteady State Heat Transfer: Dimensional Analysis

NEW STUFF!

***Question*: What now?**

Answer: Let's apply Dimensional Analysis to something new, unsteady state heat transfer, to sort out the various effects.



Engineering Modeling (complex systems)

- Choose an idealized problem and solve it
- From insight obtained from **ideal** problem, identify governing equations of **real** problem
- Nondimensionalize the governing equations; deduce dimensionless scale factors (e.g. Re, Fr for fluids)
- Design experiments to test modeling thus far
- Revise modeling (structure of dimensional analysis, identity of scale factors, e.g. add roughness lengthscale)
- Design additional experiments
- Iterate until useful correlations result

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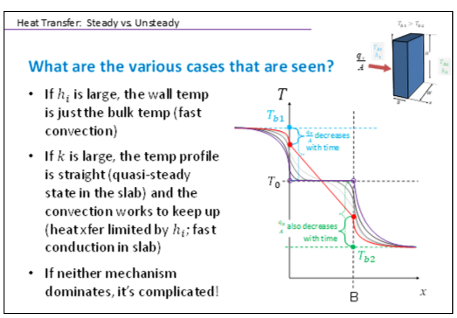

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SPOILER ALERT:
There'll be some new dimensionless numbers!

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CM3120 Transport/Unit Operations 2

More complex Systems:
Unsteady State Heat Transfer (Analytical Solutions)

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Department of Chemical Engineering
Michigan Technological University

www.chem.mtu.edu/~fmorriso/cm3120/cm3120.html

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We model the dynamics of unsteady state heat transfer because there are very practical problems that we can solve with such models.

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Example:
When will my pipes freeze?

The temperature has been 35°F for a while now, sufficient to chill the ground to this temperature for many tens of feet below the surface. Suddenly the temperature drops to -20°F. How long will it take for freezing temperatures (32°F) to reach my pipes, which are 8 *ft* under ground?

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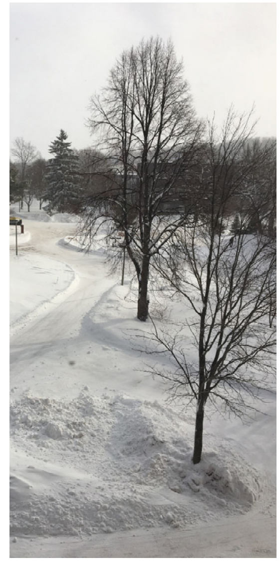

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Unsteady State Heat Transfer: Dimensional Analysis

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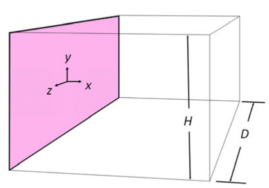
STEP ONE:

Idealized problem:
1D heat transfer in a semi-infinite solid



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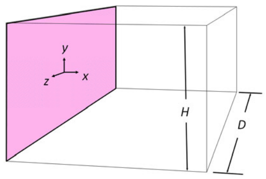
Unsteady State Heat Transfer: Dimensional Analysis

Develop a model:



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Example 1: **Unsteady Heat Conduction in a Semi-infinite solid**

A very long, very wide, very tall slab is initially at a temperature T_0. At time $t = 0$, the left face of the slab is exposed to a vigorously mixed gas at temperature T_1. What is the time-dependent temperature profile in the slab?

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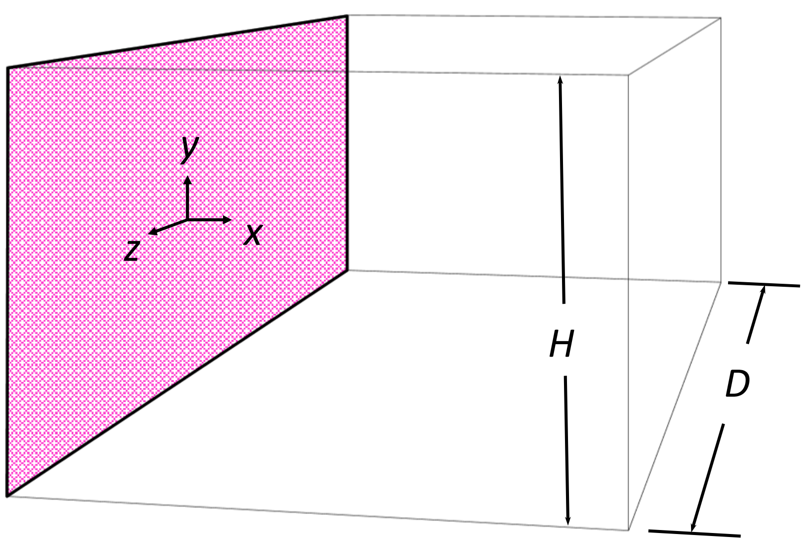
Unsteady State Heat Transfer

Example: Unsteady Heat Conduction in a Semi-infinite solid



H, D, very large

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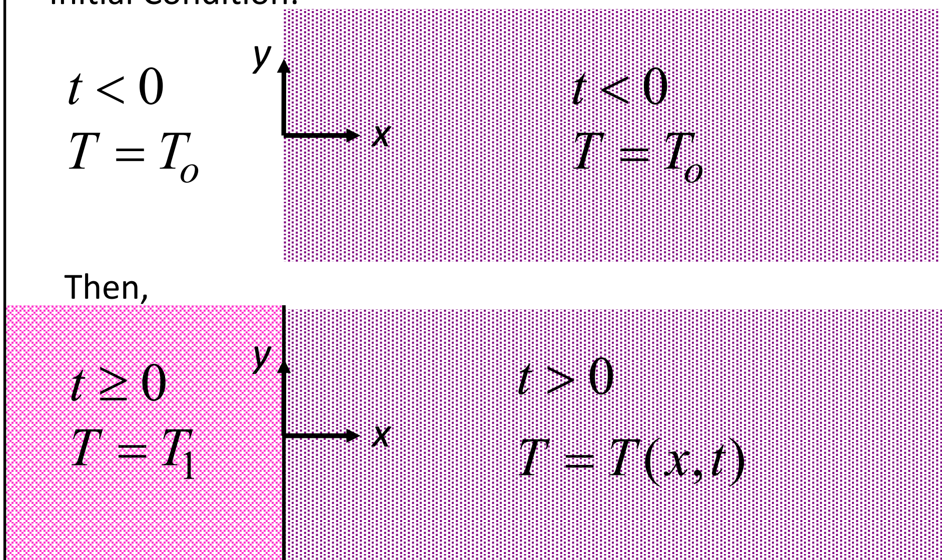
1D Heat Transfer: Unsteady State

Initial Condition:



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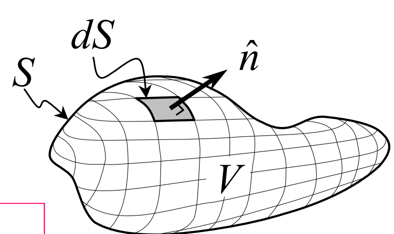
1D Heat Transfer: Unsteady State

General Energy Transport Equation

(microscopic energy balance)

As for the derivation of the microscopic momentum balance, the microscopic energy balance is derived on an arbitrary volume, *V*, enclosed by a surface, *S*.



Gibbs notation:

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + \underline{v} \cdot \nabla T \right) = k \nabla^2 T + S$$

see handout for component notation

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1D Heat Transfer: Unsteady State

General Energy Transport Equation

(microscopic energy balance)

convection

source

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + \underline{v} \cdot \nabla T \right) = k \nabla^2 T + S$$

(energy generated per unit volume per time)

rate of change

conduction (all directions)

velocity must satisfy equation of motion, equation of continuity

see handout for component notation

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Equation of energy for Newtonian fluids of constant density, ρ, and thermal conductivity, k, with source term (source could be viscous dissipation, electrical energy, chemical energy, etc., with units of energy/(volume time)).

Source: R. B. Bird, W. E. Stewart, and E. N. Lightfoot, *Transport Processes*, Wiley, NY, 1960, page 319.

www.chem.mtu.edu/~fmorriso/cm310/energy2013.pdf

Gibbs notation (vector notation)

thermal diffusivity $\alpha \equiv \frac{k}{\rho \hat{C}_p}$

$$\left(\frac{\partial T}{\partial t} + \underline{v} \cdot \nabla T \right) = \frac{k}{\rho \hat{C}_p} \nabla^2 T + \frac{S}{\rho \hat{C}_p}$$

Cartesian (xyz) coordinates:

$$\frac{\partial T}{\partial t} + v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} + v_z \frac{\partial T}{\partial z} = \frac{k}{\rho \hat{C}_p} \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + \frac{S}{\rho \hat{C}_p}$$

Cylindrical (rθz) coordinates:

$$\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} + v_z \frac{\partial T}{\partial z} = \frac{k}{\rho \hat{C}_p} \left(\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2} \right) + \frac{S}{\rho \hat{C}_p}$$

Spherical (rθϕ) coordinates:

$$\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} + \frac{v_\phi}{r \sin\theta} \frac{\partial T}{\partial \phi} = \frac{k}{\rho \hat{C}_p} \left(\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial \theta} \left(\sin\theta \frac{\partial T}{\partial \theta} \right) + \frac{1}{r^2 \sin^2}$$

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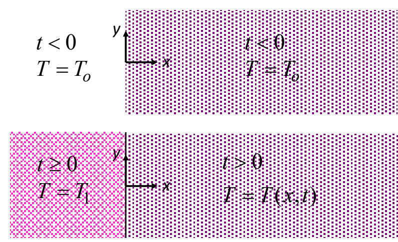
1D Heat Transfer: Unsteady State

Example 1: **Unsteady Heat Conduction in a Semi-infinite solid**

A very long, very wide, very tall slab is initially at a temperature T_0. At time $t = 0$, the left face of the slab is exposed to a vigorously mixed gas at temperature T_1. What is the time-dependent temperature profile in the slab?

Newton's law of cooling BC's:

$$|q_x| = hA\left|T_{bulk} - T_{surface}\right|$$



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1D Heat Transfer: Unsteady State

Microscopic Energy Equation in Cartesian Coordinates

$$\frac{\partial T}{\partial t} + v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} + v_z \frac{\partial T}{\partial z} = \frac{k}{\rho \hat{C}_p}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) + \frac{S}{\rho \hat{C}_p}$$

$$\alpha \equiv \frac{k}{\rho \hat{C}_p} =$$ thermal diffusivity

what are the boundary conditions? initial conditions?

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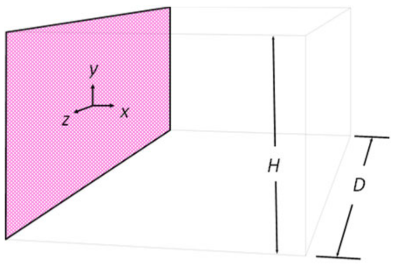
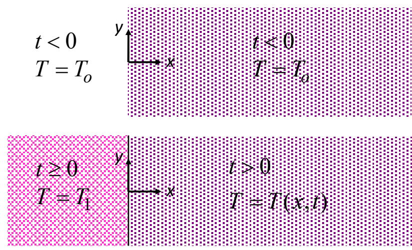
1D Heat Transfer: Unsteady State

Example: Unsteady Heat Conduction in a Semi-infinite solid

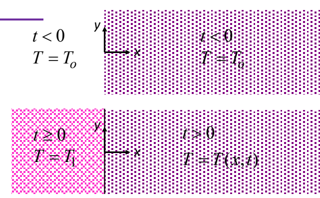




You try.

1D Heat Transfer: Unsteady State

Unsteady State Heat Conduction in a Semi-Infinite Slab



$$\frac{\partial T}{\partial t} = \frac{k}{\rho \hat{C}_p}\left(\frac{\partial^2 T}{\partial x^2}\right) = \alpha\left(\frac{\partial^2 T}{\partial x^2}\right)$$

thermal diffusivity

$$\alpha \equiv \frac{k}{\rho \hat{C}_p}$$

Initial condition: $t = 0 \quad T = T_0 \quad \forall\ x$

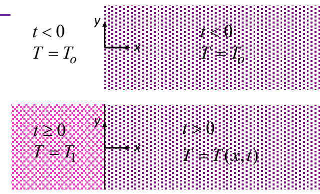
Boundary conditions:

$$x = 0 \quad \frac{q_x}{A} = -k\frac{dT}{dx} = h(T_1 - T) \quad t > 0$$

$$x = \infty \quad T = T_0 \quad \forall\ t$$

1D Heat Transfer: Unsteady State

Unsteady State Heat Conduction in a Semi-Infinite Slab



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"for all t"

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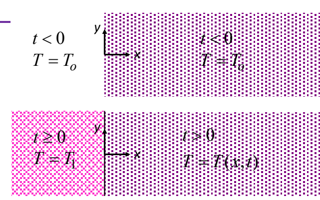
1D Heat Transfer: Unsteady State

Unsteady State Heat Conduction in a Semi-Infinite Slab



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$$x = \infty \qquad T = T_0 \qquad \forall\ t$$

The solution of the PDE is obtained by combination of variables.

$$\beta \equiv \frac{h\sqrt{\alpha\, t}}{k} \qquad \zeta \equiv \frac{x}{2\sqrt{\alpha\, t}}$$

See text WRF p284

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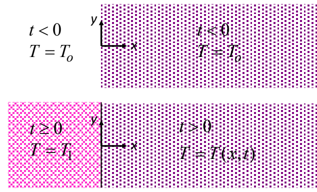
Unsteady State Heat Conduction in a Semi-Infinite Slab



Solution:

$$\frac{T-T_0}{T_1-T_0} = \operatorname{erfc}\zeta - e^{\beta(2\zeta+\beta)}\operatorname{erfc}(\zeta+\beta)$$

$$Y \equiv \frac{(T_1-T)}{(T_1-T_0)}$$

$$1-Y = \frac{(T-T_0)}{(T_1-T_0)}$$

$$\beta \equiv \frac{h\sqrt{\alpha\, t}}{k} \qquad \zeta \equiv \frac{x}{2\sqrt{\alpha\, t}}$$

complementary error function of y (*a standard function in Excel*)

$$\operatorname{erfc}(y) \equiv 1 - \operatorname{erf}(y)$$

error function of y

$$\operatorname{erf}(y) \equiv \frac{2}{\sqrt{\pi}}\int_0^y e^{-(y')^2}\, dy'$$

- **Geankoplis 4th ed., eqn 5.3-7, page 363**
- **WRF, eqn 18-21, page 286**

thermal diffusivity $\alpha \equiv \frac{k}{\rho \hat{C}_p}$

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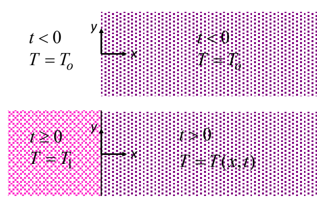
Unsteady State Heat Conduction in a Semi-Infinite Slab



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To make this solution easier to use, we can plot it.

complementary error function of y

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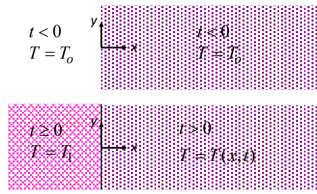
Unsteady State Heat Conduction in a Semi-Infinite Slab



This:

$$\frac{T-T_0}{T_1-T_0}=\operatorname{erfc}\zeta-e^{\beta(2\zeta+\beta)}\operatorname{erfc}(\zeta+\beta)$$

Versus this: $\zeta\equiv\dfrac{x}{2\sqrt{\alpha\, t}}$

To make this solution easier to use, we can plot it.

At various values of this:

$$\beta\equiv\frac{h\sqrt{\alpha\, t}}{k}$$

$$Y\equiv\frac{(T_1-T)}{(T_1-T_0)}$$

$$1-Y=\frac{(T-T_0)}{(T_1-T_0)}$$

thermal diffusivity $\alpha\equiv\frac{k}{\rho\hat{C}_p}$

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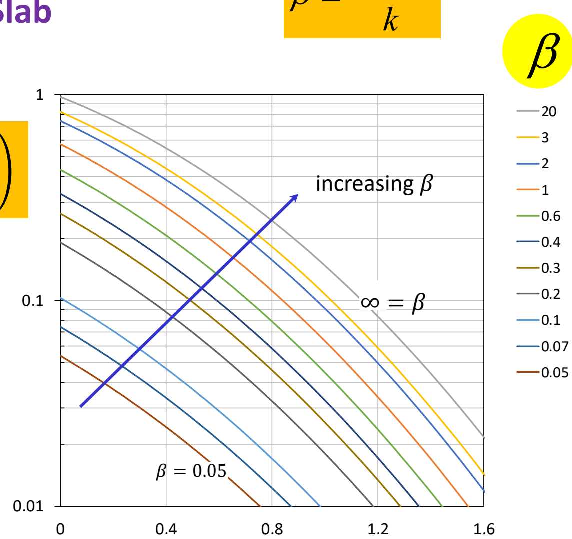
Unsteady State Heat Conduction in a Semi-Infinite Slab

$$\beta\equiv\frac{h\sqrt{\alpha\, t}}{k}$$

$$1-Y=\left(\frac{T-T_0}{T_1-T_0}\right)$$



$$\zeta=\frac{x}{2\sqrt{\alpha\, t}}$$

$$Y\equiv\frac{(T_1-T)}{(T_1-T_0)}$$

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Plot design after Geankoplis 4th ed., Figure 5.3-3, page 364

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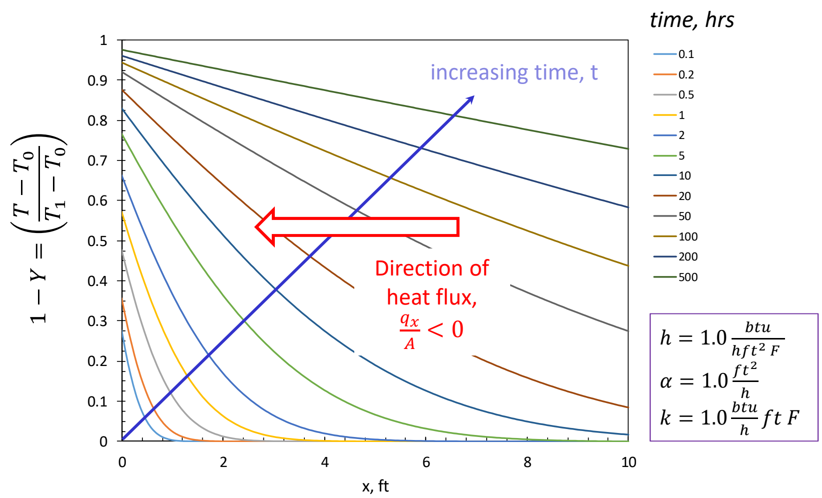
1D Heat Transfer: Unsteady State Heat Conduction in a Semi-Infinite Slab

With modern tools, we can plot the solution directly (evaluated in Excel)

Unsteady State Heat Conduction in a Semi-Infinite Slab



thermal diffusivity $\alpha \equiv \frac{k}{\rho \hat{C}_p}$

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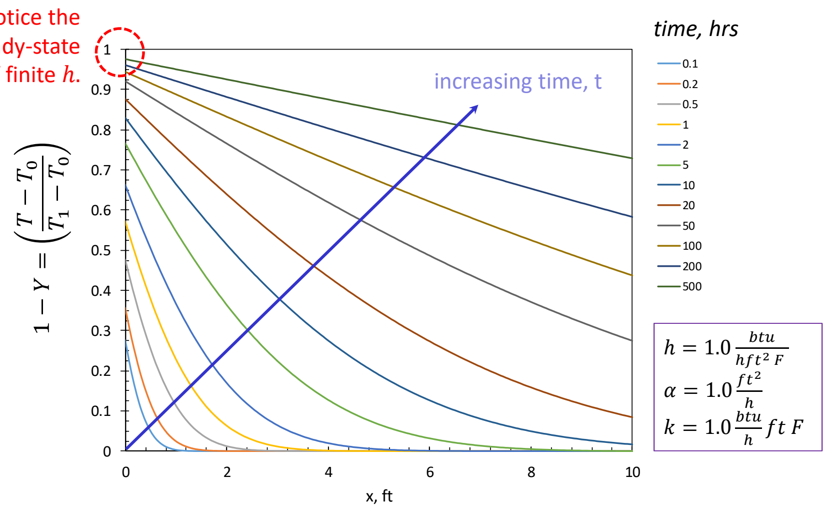
1D Heat Transfer: Unsteady State Heat Conduction in a Semi-Infinite Slab

With modern tools, we can plot the solution directly (evaluated in Excel)

Unsteady State Heat Conduction in a Semi-Infinite Slab

Notice the steady-state effect of finite h.



thermal diffusivity $\alpha \equiv \frac{k}{\rho \hat{C}_p}$

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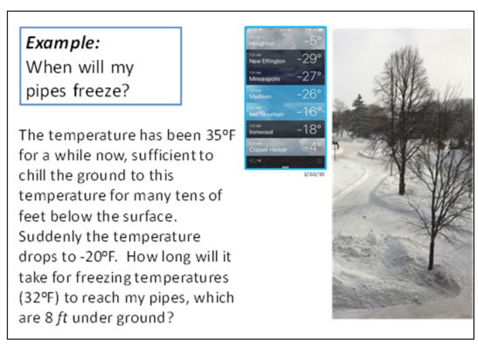
1D Heat Transfer: Unsteady State Heat Conduction in a Semi-Infinite Slab

We need the appropriate physical property data for the soil.

$$h = 2.0 \frac{BTU}{h\ ft^2\ {}^oF}$$

$$\alpha_{soil} = 0.018 \frac{ft^2}{h}$$

$$k_{soil} = 0.5 \frac{BTU}{h\ ft\ {}^oF}$$


Example:
When will my pipes freeze?
The temperature has been 35ºF for a while now, sufficient to chill the ground to this temperature for many tens of feet below the surface. Suddenly the temperature drops to -20ºF. How long will it take for freezing temperatures (32ºF) to reach my pipes, which are 8 *ft* under ground?

thermal diffusivity $\alpha \equiv \frac{k}{\rho \hat{C}_p}$

Geankoplis 4th ed.

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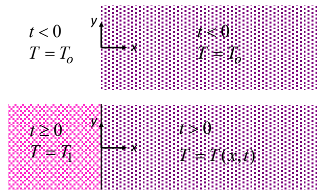
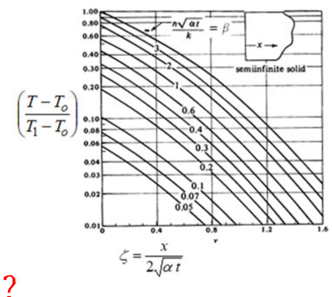
Example: When will my pipes freeze?

1D Heat Transfer: Unsteady State Heat Conduction in a Semi-Infinite Slab



$$\frac{T-T_0}{T_1-T_0}=\operatorname{erfc}\zeta-e^{\beta(2\zeta+\beta)}\operatorname{erfc}(\zeta+\beta)$$



$$\zeta\equiv\frac{x}{2\sqrt{\alpha\, t}}$$

$$\beta\equiv\frac{h\sqrt{\alpha\, t}}{k}$$

Both ζ and β depend on time

$T_0=?$

$T_1=?$

$T=?$

$$\frac{T-T_0}{T_1-T_0}=?$$

$$Y\equiv\frac{(T_1-T)}{(T_1-T_0)}$$

$$1-Y=\frac{(T-T_0)}{(T_1-T_0)}$$

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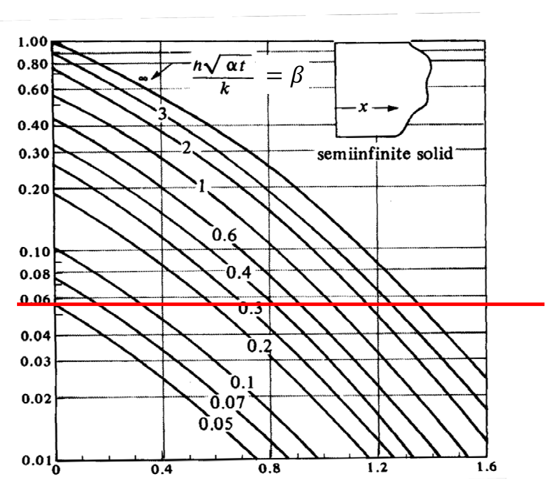
Example: When will my pipes freeze?

1D Heat Transfer: Unsteady State Heat Conduction in a Semi-Infinite Slab

$$1-Y=\left(\frac{T-T_0}{T_1-T_0}\right)$$

You try.



$$\zeta=\frac{x}{2\sqrt{\alpha\, t}}$$

Geankoplis 4th ed., Figure 5.3-3, page 364

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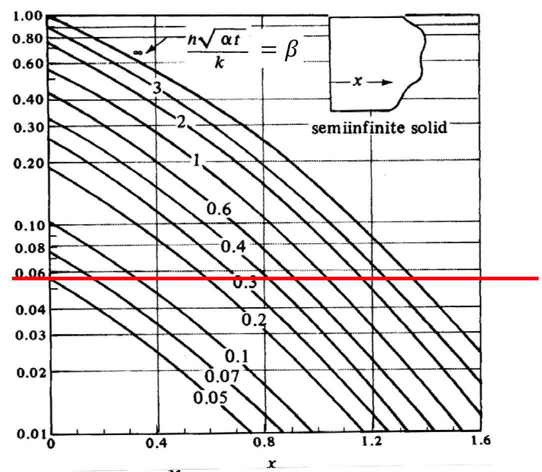
Example: When will my pipes freeze?

1D Heat Transfer: Unsteady State Heat Conduction in a Semi-Infinite Slab

$$1 - Y = \left(\frac{T - T_0}{T_1 - T_0}\right)$$

Solution:

Guess large β)
(Interative solution)



$$\zeta = \frac{x}{2\sqrt{\alpha t}}$$

Geankoplis 4th ed., Figure 5.3-3, page 364

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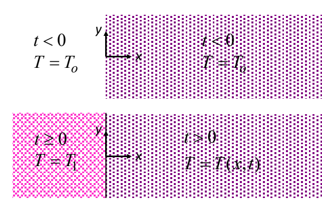
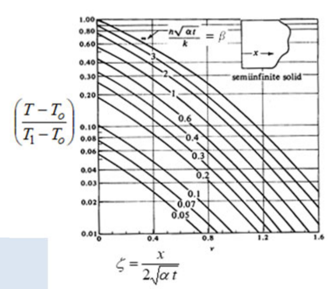
Example: When will my pipes freeze?

1D Heat Transfer: Unsteady State Heat Conduction in a Semi-Infinite Slab



$$\frac{T - T_0}{T_1 - T_0} = \text{erfc}\,\zeta - e^{\beta(2\zeta+\beta)}\,\text{erfc}(\zeta + \beta)$$



Answer:
$t \approx 480\ hours \approx 20\ days$

$$Y \equiv \frac{(T_1 - T)}{(T_1 - T_0)}$$

$$1 - Y = \frac{(T - T_0)}{(T_1 - T_0)}$$

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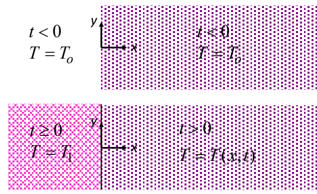
Example: When will my pipes freeze?

1D Heat Transfer: Unsteady State Heat Conduction in a Semi-Infinite Slab



Or, use Excel. (How exactly?)

$$\frac{T-T_0}{T_1-T_0}=\operatorname{erfc}\zeta-e^{\beta(2\zeta+\beta)}\operatorname{erfc}(\zeta+\beta)$$

$$\zeta\equiv\frac{x}{2\sqrt{\alpha\,t}}$$

$$\beta\equiv\frac{h\sqrt{\alpha\,t}}{k}$$

You try.

T0=		
T1=		
T=		
h=		
alpha=		
k=		
x=		

Answer:

$t=21.2\ days$

$\beta=12.1$

pages.mtu.edu/~fmorriso/cm310/2019PracticeProblemsInHeatTransfer%28Geankoplis%29.pdf

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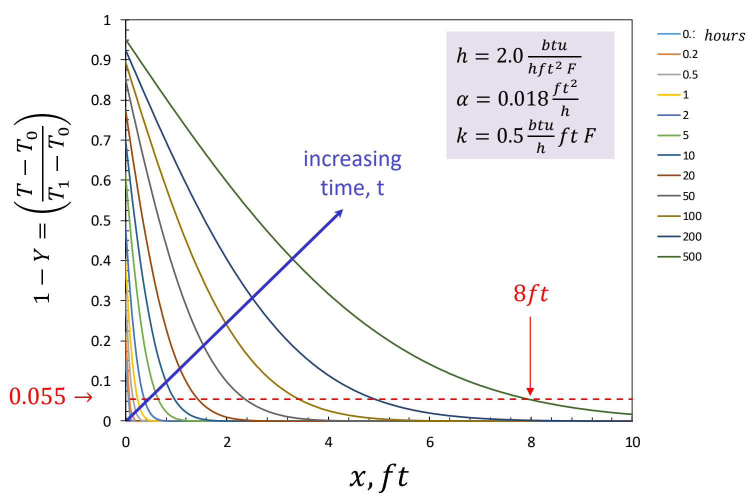
Example: When will my pipes freeze?

1D Heat Transfer: Unsteady State Heat Conduction in a Semi-Infinite Slab

With modern tools, we can plot the evolution of the model directly (evaluated in Excel)



aa_solutions/Unsteady semi infinite solid plots.xlsx

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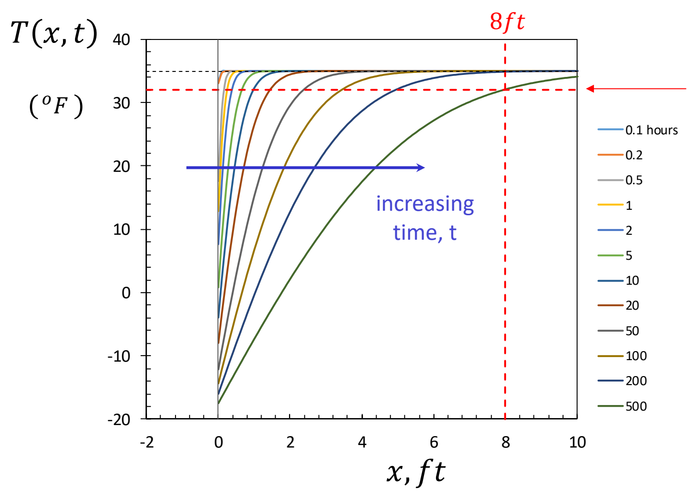
Example: When will my pipes freeze?

1D Heat Transfer: Unsteady State Heat Conduction in a Semi-Infinite Slab

With modern tools, we can plot the evolution of the model directly (evaluated in Excel)



$h = 2.0 \frac{btu}{hft^2\,F}$

$\alpha = 0.018 \frac{ft^2}{h}$

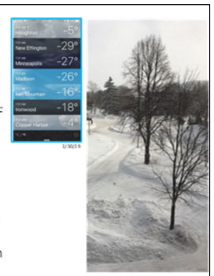
$k = 0.5 \frac{btu}{h} ft\, F$

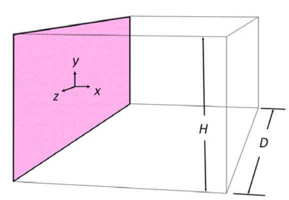
aa_solutions/Unsteady semi infinite solid plots.xlsx

Solution Summary:

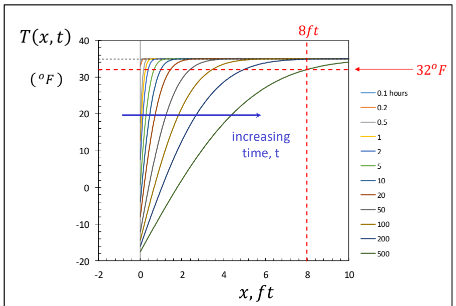
Example:
When will my pipes freeze?

The temperature has been 35°F for a while now, sufficient to chill the ground to this temperature for many tens of feet below the surface. Suddenly the temperature drops to -20°F. How long will it take for freezing temperatures (32°F) to reach my pipes, which are 8 *ft* under ground?





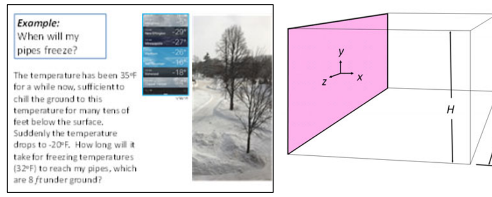
Answer:
$t = 509\ hours \approx 21\ days$

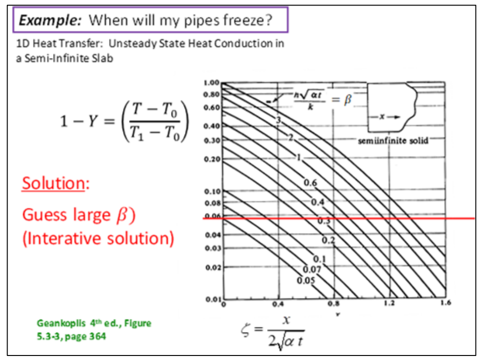


TAKEAWAY:

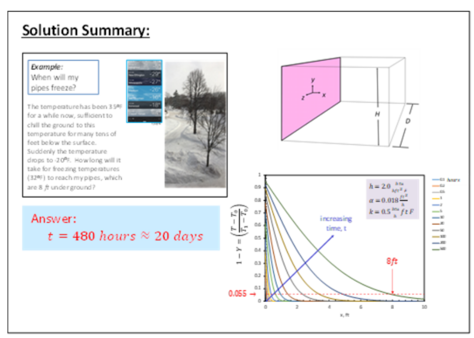
We're not making the case that this ONE solution is so important to learn. . .

Rather, knowing that solutions are available, *and* being able to set up and walk yourself through the published graph (chart, plot, equation) to answer a question of interest, *is* **an important engineering thinking skill**.





We used unsteady state heat transfer modeling to solve one practical problem.



What can we do to extend these methods to a wider class of problems?