CM3120 Transport/Unit Operations 2

STEADY **Heat Transfer—Review**

Professor Faith A. Morrison
Department of Chemical Engineering
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www.chem.mtu.edu/~fmorriso/cm3120/cm3120.html

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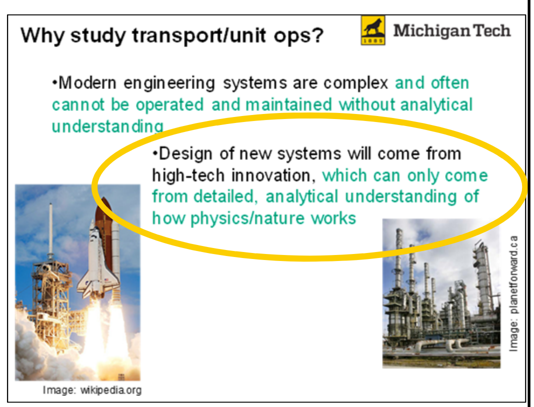
Why study transport/unit ops?

"… detailed, analytical understanding of how physics/nature works …"



How do we apply this understanding?

- **Organize** with dimensional analysis
- **Make operational** by performing experiments leading to data correlations.

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 2

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Topics:

How nature works:

- Mass is conserved
- Momentum is conserved
- Energy is conserved

How engineering operations work:

- Transform one type of mass mixture to others
- Often in a flow environment
- Often with the input or output of energy
- Energy flows are often transient (unsteady)

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How engineer

- Trans
- Ofter
- Ofter
- Energy flows are often transient (unsteady)

Where are we?

How far did we get in CM3110 and other prerequisite courses?

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Why study transport/unit ops?

Where are we?

How far did we get on in CM3110 and other prerequisite courses?

Michigan Tech

CM3110

Transport Processes and Unit Operations I

Part 2:

Professor Faith Morrison

Department of Chemical Engineering
Michigan Technological University

CM3110 - Momentum and Heat Transport
CM3120 – Heat and Mass Transport

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CM3110: Heat Transfer

CM3110 REVIEW

Summary

Within homogeneous phases:

- Microscopic Energy Balances
- 1D **Steady** solutions

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + \underline{v} \cdot \nabla T \right) = k \nabla^2 T + S_e$$

rectangular:

$$\frac{q_x}{A} = C_1$$
$$T = ax + b$$

cylindrical:

$$\frac{q_r}{A} = \frac{C_1}{r}$$
$$T = a \ln x + b$$

- Temperature and *Newton's law of cooling* boundary conditions (if h is supplied; or obtain from literature correlation)

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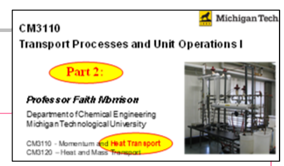
CM3110: Heat Transfer

CM3110 REVIEW



Summary

Across phase boundaries:

- Newton's law of cooling $\frac{q_x}{A} = h|T_{bulk} - T_{wall}|$
- Microscopic Energy, Momentum, and Mass Balances

Micro momentum:

$$\rho \left(\frac{\partial \underline{v}}{\partial t} + \underline{v} \cdot \nabla \underline{v} \right) = -\nabla p + \mu \nabla^2 \underline{v} + \rho \underline{g}$$

Micro energy:

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + \underline{v} \cdot \nabla T \right) = k \nabla^2 T + S_e$$

- Simultaneous effects (complex)
- Solutions are difficult to obtain (and often not really necessary)
  → use ***dimensional analysis*** to obtain ***h***
- h Data correlations for: (use in design)
  - ✓ forced convection (Sieder-Tate)
  - ✓ natural convection
  - ✓ evaporation/condensation
  - ✓ radiation

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CM3110: Heat Transfer

CM3110 REVIEW

Summary

Applied Heat Transfer (including Unit Operations)

- Macroscopic energy balances
- Heat Exchangers
  - ✓ double pipe (ΔT_{lm})
  - ✓ Shell-and-tube ($F_T \Delta T_{lm}$)
  - ✓ Heat exchanger effectiveness (NTU, $Q = \varepsilon(mC_p)_{min}(T_{hi} - T_{ci})$)
- Evaporators/ Condensers
- Ovens (radiation and convection)
- Heat Shields

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Where are we?

Topics:

How nature works:

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How engineering operations work:

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Topics:

How nature works:

- ✓ Mass is conserved
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We are here

How engineering operations work:

- Transform one type of mass mixture to others
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Unsteady State Heat Transfer

CM3120 Transport/Unit Operations 2

Unsteady State Heat Transfer

We begin with a Review:

- Microscopic energy balance
- Fourier's law of heat conduction (k, homogeneous)
- Newton's law of cooling (h, at the boundary)
- Resistances due to k and h
- Solving for the *steady* temperature field $T(x, y, z)$

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Microscopic Energy Balance Review

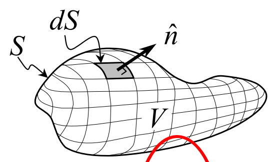
CM3110 REVIEW

The **microscopic** energy balance is an expression of the law of conservation of energy.

It includes consideration of **unsteady** energy flows.

Microscopic Energy Balance:

Equation of Thermal Energy



Microscopic **energy** balance written on an arbitrarily shaped volume, *V*, enclosed by a surface, *S*

Gibbs notation: $$\rho\left(\frac{\partial \hat{E}}{\partial t} + \underline{v}\cdot\nabla\hat{E}\right) = -\nabla\cdot\underline{\tilde{q}} + S_e$$ **general conduction**

Gibbs notation: $$\rho\hat{C}_p\left(\frac{\partial T}{\partial t} + \underline{v}\cdot\nabla T\right) = k\nabla^2 T + S_e$$ **Only Fourier conduction**

(incompressible fluid, constant pressure, neglect $\hat{E}_k, \hat{E}_p$, viscous dissipation)

http://pages.mtu.edu/~fmorriso/cm310/energy_equation.html

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Microscopic Energy Balance Review

CM3110 REVIEW

What ***physics*** determines how rapidly (the *rate*) the heat transfers from the outside stream to the inside stream?

Energy Transport law

Fourier's Law of Heat Conduction

$$\frac{q_x}{A} = -k\frac{dT}{dx}$$

(for a homogeneous phase)

$\frac{q_x}{A}$ –heat flux=energy/area time)
k – thermal conductivity
$\frac{dT}{dx}$ –temperature gradient

(the driving physics of Fourier's law is Brownian motion: energy transports down ∇T due to Brownian motion)

Microscopic Energy Balance Review

CM3110 REVIEW

Heat Transfer Rate law:

Fourier's law of Heat Conduction

makes reference to a coordinate system

$$\frac{q_x}{A} = -k\frac{dT}{dx}$$

Allows you to solve for temperature profiles (also known as temperature *distributions* or *fields*)

Gibbs notation: $\frac{\underline{q}}{A} = -k\nabla T$

Fourier's law in three dimensions

$$\tilde{q} = \frac{\underline{q}}{A} = \begin{pmatrix} -k\frac{\partial T}{\partial x} \\ -k\frac{\partial T}{\partial y} \\ -k\frac{\partial T}{\partial z} \end{pmatrix}_{xyz}$$

- Heat flows **down** a temperature gradient
- Flux is proportional to the magnitude of temperature gradient

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Microscopic Energy Balance Review

CM3110 REVIEW

Equation of Energy
(microscopic energy balance)

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t} + \underline{v}\cdot\nabla T\right) = k\nabla^2 T + S_e$$

- $\frac{\partial T}{\partial t}$: rate of change
- $\underline{v}\cdot\nabla T$: convection
- $k\nabla^2 T$: conduction (all directions)
- S_e: source (energy generated per unit volume per time)

velocity must satisfy equation of motion, equation of continuity

see handout for component notation

http://pages.mtu.edu/~fmorriso/cm310/energy_equation.html

The **Equation of Energy** in Cartesian, cylindrical, and spherical coordinates for Newtonian fluids of constant density, with source term S. Source could be electrical energy due to current flow, chemical energy, etc. Two cases are presented: the general case where thermal conductivity may be a function of temperature (vector flux $\underline{\tilde{q}} = \underline{q}/A$ appears in the equations); and the more usual case, where thermal conductivity is constant.

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Microscopic energy balance, in terms of flux; Gibbs notation

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t} + \underline{v}\cdot\nabla T\right) = -\nabla\cdot\underline{\tilde{q}} + S$$

Microscopic energy balance, in terms of flux; Cartesian coordinates

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t} + v_x\frac{\partial T}{\partial x} + v_y\frac{\partial T}{\partial y} + v_z\frac{\partial T}{\partial z}\right) = -\left(\frac{\partial \tilde{q}_x}{\partial x} + \frac{\partial \tilde{q}_y}{\partial y} + \frac{\partial \tilde{q}_z}{\partial z}\right) + S$$

Microscopic energy balance, in terms of flux; cylindrical coordinates

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t} + v_r\frac{\partial T}{\partial r} + \frac{v_\theta}{r}\frac{\partial T}{\partial \theta} + v_z\frac{\partial T}{\partial z}\right) = -\left(\frac{1}{r}\frac{\partial (r\tilde{q}_r)}{\partial r} + \frac{1}{r}\frac{\partial \tilde{q}_\theta}{\partial \theta} + \frac{\partial \tilde{q}_z}{\partial z}\right) + S$$

Microscopic energy balance, in terms of flux; spherical coordinates

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t} + v_r\frac{\partial T}{\partial r} + \frac{v_\theta}{r}\frac{\partial T}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial T}{\partial \phi}\right) = -\left(\frac{1}{r^2}\frac{\partial (r^2\tilde{q}_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial (\tilde{q}_\theta sin\theta)}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial \tilde{q}_\phi}{\partial \phi}\right) + S$$

Fourier's law of heat conduction, Gibbs notation: $\underline{\tilde{q}} = -k\nabla T$

Fourier's law of heat conduction, Cartesian coordinates: $\begin{pmatrix}\tilde{q}_x\\ \tilde{q}_y\\ \tilde{q}_z\end{pmatrix}_{xyz} = \begin{pmatrix}-k\frac{\partial T}{\partial x}\\ -k\frac{\partial T}{\partial y}\\ -k\frac{\partial T}{\partial z}\end{pmatrix}_{xyz}$

Fourier's law of heat conduction, cylindrical coordinates: $\begin{pmatrix}\tilde{q}_r\\ \tilde{q}_\theta\\ \tilde{q}_z\end{pmatrix}_{xyz} = \begin{pmatrix}-k\frac{\partial T}{\partial r}\\ -\frac{k}{r}\frac{\partial T}{\partial \theta}\\ -k\frac{\partial T}{\partial z}\end{pmatrix}_{r\theta z}$

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The **Equation of Energy** for systems with **constant k**

Microscopic energy balance, constant thermal conductivity; Gibbs notation

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t} + \underline{v}\cdot\nabla T\right) = k\nabla^2 T + S$$

Microscopic energy balance, constant thermal conductivity; Cartesian coordinates

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t} + v_x\frac{\partial T}{\partial x} + v_y\frac{\partial T}{\partial y} + v_z\frac{\partial T}{\partial z}\right) = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) + S$$

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Reference: F. A. Morrison, "Web Appendix to *An Introduction to Fluid Mechanics*," Cambridge University Press, New York, 2013. On the web at www.chem.mtu.edu/~fmorriso/IFM_WebAppendixCD2013.pdf

http://pages.mtu.edu/~fmorriso/cm310/energy_equation.html

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Front side:

- Micro E-balance in terms of flux $\underline{\tilde{q}} \equiv \frac{\underline{q}}{A}$
- Fourier's law, $\underline{\tilde{q}} = -k\nabla T$

Fourier's law of heat conduction, spherical coordinates: $\begin{pmatrix}\tilde{q}_r\\ \tilde{q}_\theta\\ \tilde{q}_\phi\end{pmatrix}_{xyz} = \begin{pmatrix}-k\frac{\partial T}{\partial r}\\ -\frac{k}{r}\frac{\partial T}{\partial \theta}\\ -\frac{k}{r\sin\theta}\frac{\partial T}{\partial \phi}\end{pmatrix}_{r\theta\phi}$

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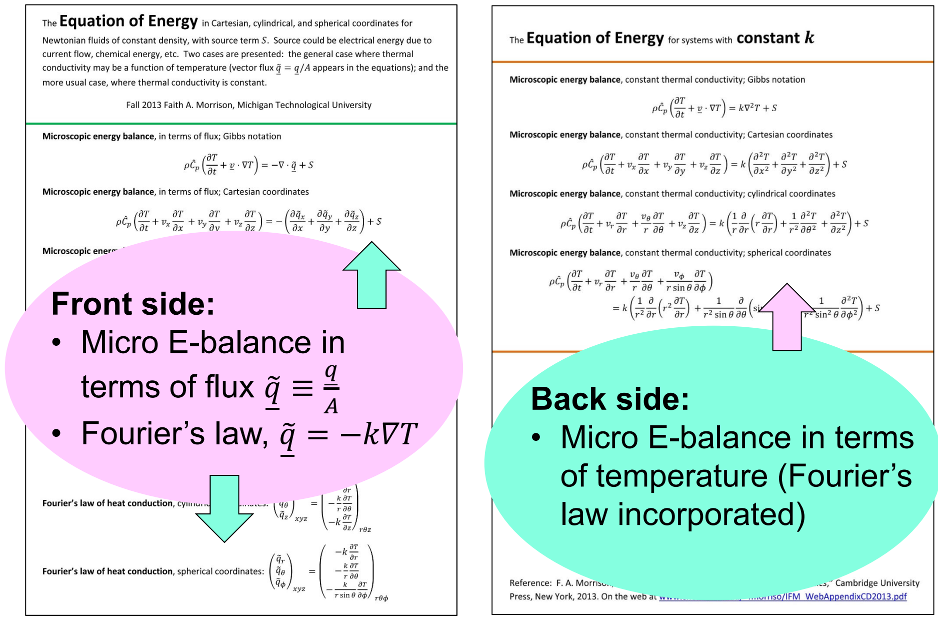

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Microscopic Energy Balance

CM3110 REVIEW

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Fourier's Law of Heat Conduction

CM3110 REVIEW

Fourier's law of heat conduction, Gibbs notation: $\underline{\tilde{q}} = \underline{q}/A = -k\nabla T$

Fourier's law of heat conduction, Cartesian coordinates:
(constant thermal conductivity k)

$$\begin{pmatrix} \tilde{q}_x \\ \tilde{q}_y \\ \tilde{q}_z \end{pmatrix}_{xyz} = \begin{pmatrix} q_x/A \\ q_y/A \\ q_z/A \end{pmatrix}_{xyz} = \begin{pmatrix} -k\frac{\partial T}{\partial x} \\ -k\frac{\partial T}{\partial y} \\ -k\frac{\partial T}{\partial z} \end{pmatrix}_{xyz}$$

$$\tilde{q}_x = \frac{q_x}{A}$$

Fourier's law of heat conduction, cylindrical coordinates:
(constant thermal conductivity k)

$$\begin{pmatrix} \tilde{q}_r \\ \tilde{q}_\theta \\ \tilde{q}_z \end{pmatrix}_{r\theta z} = \begin{pmatrix} q_r/A \\ q_\theta/A \\ q_z/A \end{pmatrix}_{r\theta z} = \begin{pmatrix} -k\frac{\partial T}{\partial r} \\ -\frac{k}{r}\frac{\partial T}{\partial \theta} \\ -k\frac{\partial T}{\partial z} \end{pmatrix}_{r\theta z}$$

Fourier's law of heat conduction, spherical coordinates:
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https://pages.mtu.edu/~fmorriso/cm310/energy.pdf

Microscopic Energy Balance Review

CM3110 REVIEW

Now, *Boundary Conditions* and *Resistances*

Unsteady State Heat Transfer

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Unsteady State Heat Transfer

We begin with a Review:

- Microscopic energy balance
- Fourier's law of heat conduction (k, homogeneous)
- Newton's law of cooling (h, at the boundary)
- Resistances due to k and h
- Solving for the steady temperature field $T(x, y, z)$

Microscopic Energy Balance Review—Boundary Conditions

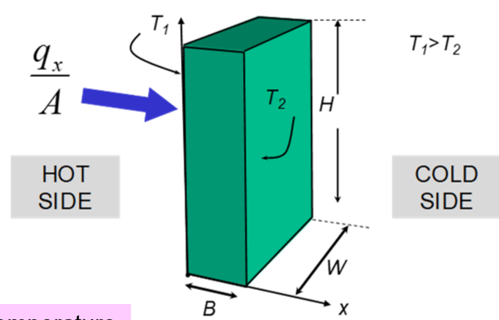
CM3110 REVIEW

We will need **boundary conditions** on temperature to solve the microscopic balances for the temperature distribution.

Example 1: Heat flux in a rectangular solid – Temperature BC

What is the steady state temperature profile in a rectangular slab if one side is held at T_1 and the other side is held at T_2?

Assumptions:
- wide, tall slab
- steady state



We may know the temperature at the boundary.

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Microscopic Energy Balance Review—Boundary Conditions

CM3110 REVIEW

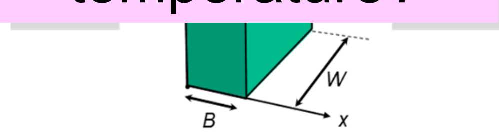
We will need **boundary conditions** on temperature to solve the microscopic balances for the temperature distribution.

Example 1: Heat flux in a rectangular solid – Temperature BC

What is the steady state temperature profile in a rectangular slab if one side is held at T_1 and the other side is held at T_2?

Assumptions:
- wide, tall slab
- steady state

What if we don't know the wall temperature?



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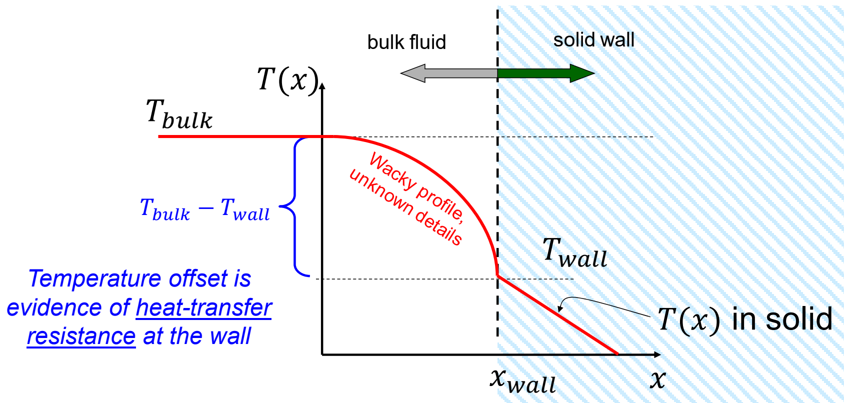
Microscopic Energy Balance Review—Boundary Conditions

CM3110 REVIEW

The interface between the solid and the fluid calls for a new type of **boundary condition**, *Newton's Law of Cooling*.



There may exist a *resistance* to heat transfer due to fluid at the boundary

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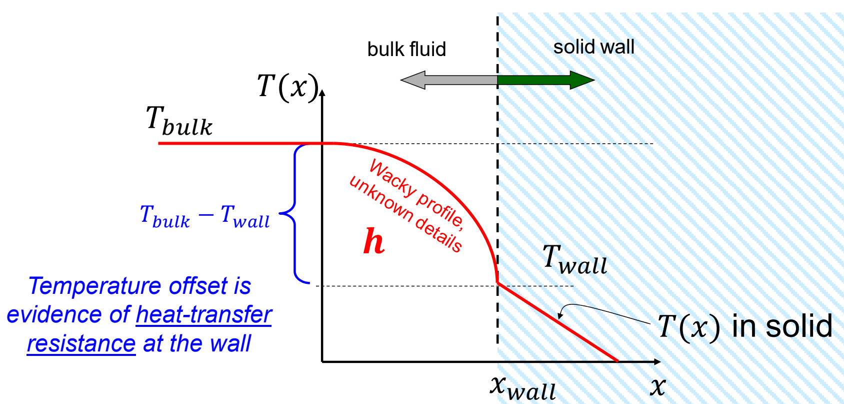
Microscopic Energy Balance Review—Boundary Conditions

CM3110 REVIEW

The interface between the solid and the fluid calls for a new type of **boundary condition**, *Newton's Law of Cooling*.



The temperature difference at the fluid-wall interface is caused by complex phenomena that are lumped together into the heat transfer coefficient, h

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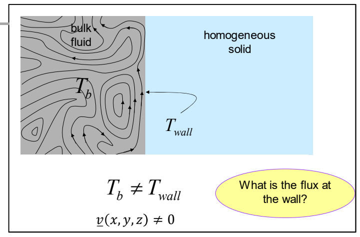
Microscopic Energy Balance Review

The flux at the wall is given by the empirical expression known as **Newton's Law of Cooling**

(a *linear driving-force* model)



This expression serves as the definition of the heat transfer coefficient.

$$\left|\frac{q_x}{A}\right| = h\left|T_{bulk} - T_{wall}\right|$$

***h* depends on:**

- geometry
- fluid velocity field
- fluid properties
- temperature difference

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Microscopic Energy Balance Review

Review so far…

- **Microscopic energy balance**

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t} + \underline{v}\cdot\nabla T\right) = k\nabla^2 T + S_e$$

rate of change ($\frac{\partial T}{\partial t}$); convection ($\underline{v}\cdot\nabla T$); conduction (all directions) ($k\nabla^2 T$); source (S_e)

- **Fourier's law of heat conduction**

Fourier's Law of Heat Conduction

$$\frac{q_x}{A} = -k\frac{dT}{dx}$$

(for a homogeneous phase)

- **Newton's law of cooling** (h, at the boundary)

This expression serves as the definition of the heat transfer coefficient.

$$\left|\frac{q_x}{A}\right| = h\left|T_{bulk} - T_{wall}\right|$$

Pending

- ***Resistances*** due to k and h
- ***Solving*** for the steady temperature field $T(x,y,z)$

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$\mathcal{R}$ = Resistance to Heat Transfer

The language of ***resistance*** to describe the physics of heat transfer will be handy in our study of unsteady state temperature profiles. We encountered this language in CM3110, and we review and summarize now.

$$\frac{q_x}{A} = \frac{\text{driving force}}{\sum \text{resistances}}$$

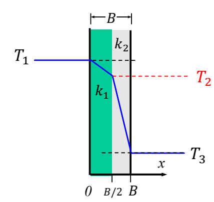
Two limitations create resistance:

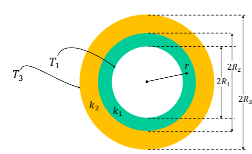
1. *Limited* conductivity within the homogeneous phase (k)
2. *Limited* heat transfer to the homogeneous phase through the boundary (h)

Also, resistances:

1. Are affected by geometry (rectangular versus radial)
2. Can be stacked (that is, added together like electrical resistances)

Note: Geankoplis uses a slightly different definition of resistance; we follow Bird et al. 2002.





Thermal conductivity k and heat transfer coefficient h may be thought of as sources of ***resistance*** $\mathcal{R}$ to heat transfer.

These resistances $\mathcal{R}$ ***stack up*** in a logical way, allowing us to quickly and accurately determine the effect of adding insulating layers, encountering pipe fouling, and other applications.

Using the microscopic energy balance on a test problem, we can solve for the temperature profile and then the heat flux, which is the driving force/resistance.

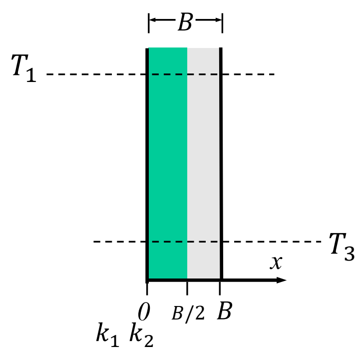
We can then identify the resistances for each test case considered.

$$\frac{q_x}{A} = \frac{\text{driving force}}{\sum \text{resistances}}$$

1

1D Rectangular: Composite Door

For an outside door, a metal is used (k_1) for strength, and a cork (k_2) is used for insulation. Both are the same thickness $B/2$. What is the temperature profile in the door at steady state? What is the energy flux? The inside temperature of the metal is T_1 and the outside temperature of the cork is T_3.



Temperature boundary conditions

See handwritten notes.

https://pages.mtu.edu/~fmorriso/cm310/lectures/handnotes_lec14-15_1d_rect%20temps_composite.pdf

Note: *in the hand notes the temperatures from left to right are T_1, T_3, T_2.*

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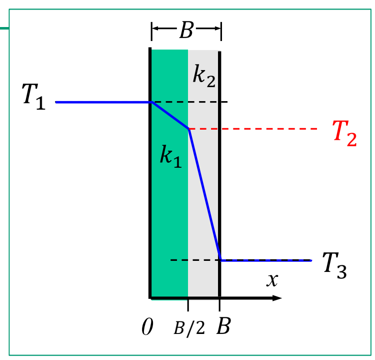
1D Rectangular Slab, Composite Door, two equal width layers



SOLUTION:

$$\frac{q_x}{A} = \frac{(T_1 - T_3)}{\frac{B/2}{k_1} + \frac{B/2}{k_2}}$$

Let: $\mathcal{R}_i \equiv \frac{thickness}{k_i}$

Resistance due to finite thermal conductivity (rectangular)

$$\frac{q_x}{A} = \frac{(T_1 - T_3)}{\mathcal{R}_1 + \mathcal{R}_2} = \frac{\text{driving force}}{\text{resistance}}$$

Each of the layers contributes a resistance, added in *series* (like in electricity).

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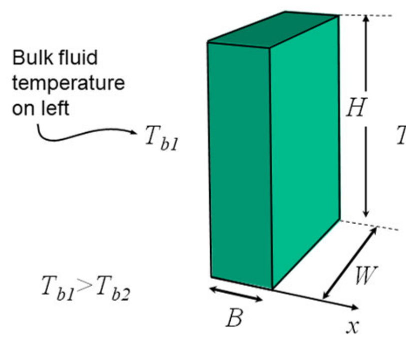
1D Rectangular Heat Transfer – Resistance

2

1D Rectangular: Slab with Exposed to Two Different Fluids

What is the steady state temperature profile in a rectangular slab if the **fluid** on one side is held at T_{b1} and the fluid on the other side is held at T_{b2}?



Newton's law of cooling boundary conditions

Assumptions:
•*h_1 and h_2 are the heat transfer coefficients of the left and right walls*

See handwritten notes.

https://pages.mtu.edu/~fmorriso/cm310/lectures/handnotes_lec14-15_1d_rect_h.pdf

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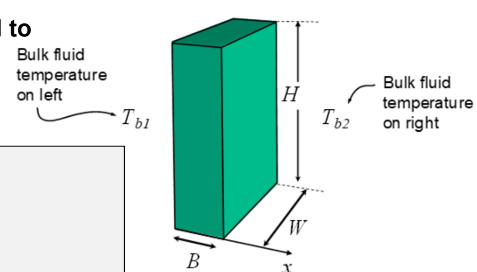
1D Rectangular Heat Transfer – Resistance

1D Rectangular, Slab with Exposed to Two Different Fluids



SOLUTION:

$$\frac{q_x}{A} = \frac{T_{b1} - T_{b2}}{\left(\frac{1}{h_1} + \frac{B}{k} + \frac{1}{h_2}\right)}$$

Let: $\mathcal{R}_i \equiv \frac{1}{h_i}$

Let: $\mathcal{R}_i \equiv \frac{B}{k}$

Resistance due to heat transfer at boundary
Resistance due to finite thermal conductivity

$$\frac{q_x}{A} = \frac{(T_1 - T_3)}{\mathcal{R}_1 + \mathcal{R}_2 + \mathcal{R}_3} = \frac{\text{driving force}}{\text{resistance}}$$

Each of the effects contributes a resistance, added in *series*.

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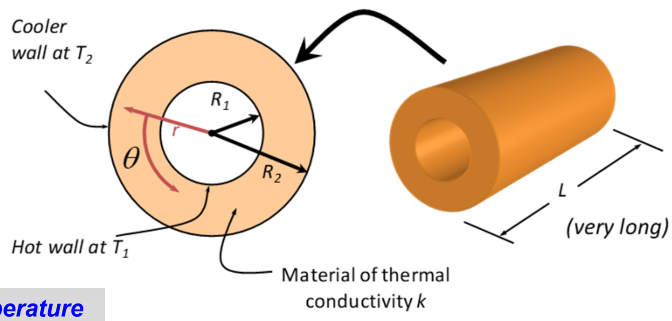
1D Radial Heat Transfer – Resistance

3

1D Radial: Heat flux in a cylindrical shell – Temp BC

What is the steady state temperature profile in a cylindrical shell (pipe) if the inner wall is at T_1 and the outer wall is at T_2, $T_1 > T_2$?



Temperature boundary conditions

See handwritten notes.

https://pages.mtu.edu/~fmorriso/cm310/lectures/handnotes_lec14-15_1d_radial_both%20bc.pdf

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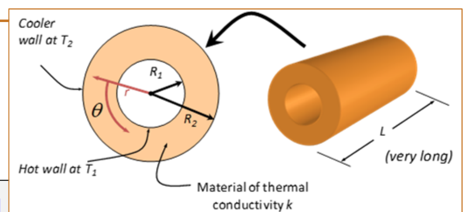
1D Radial Heat Transfer – Resistance

1D Radial: Heat flux in a cylindrical shell – Temp BC



SOLUTION:

$$\frac{q_r}{A} = \left(\frac{(T_1 - T_2)}{\frac{1}{k}\ln\frac{R_2}{R_1}}\right)\frac{1}{r}$$

Radial flux NOT constant

Let: $\mathcal{R}_1 \equiv \frac{1}{k}\ln\frac{R_2}{R_1}$

Resistance due to finite thermal conductivity (radial)

$$\frac{q_r}{A} = \left(\frac{(T_1 - T_2)}{\mathcal{R}_1}\right)\frac{1}{r} = \frac{\text{driving force}}{\text{resistance}}$$

Geometry has an impact on heat-transfer resistance.

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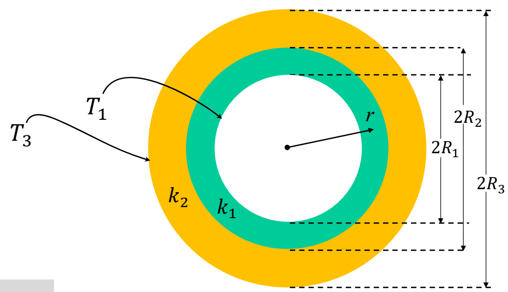
1D Radial Heat Transfer – Resistance

1D Radial: Insulated Pipe (Composite, radial conduction)

4

For a metal pipe carrying a hot liquid (k_1) an insulation layer is added with thermal conductivity k_2. What is the temperature profile in the composite pipe at steady state? What is the flux? The inside temperature of the metal pipe is T_1 and the outside temperature of the insulation is T_3.



Temperature boundary conditions

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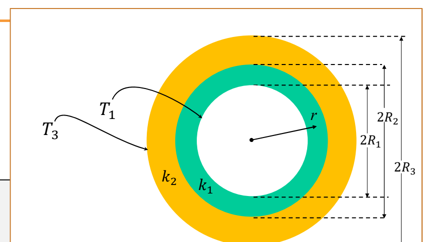
1D Radial Heat Transfer – Resistance

1D Radial: Insulated Pipe

SOLUTION:



$$\frac{q_r}{A} = \left(\frac{(T_1 - T_3)}{\frac{1}{k_1}\ln\frac{R_2}{R_1} + \frac{1}{k_2}\ln\frac{R_3}{R_2}} \right) \frac{1}{r}$$

Radial flux NOT constant

Let: $\mathcal{R}_i \equiv \frac{1}{k_i}\ln\frac{R_{i+1}}{R_i}$

Resistance due to finite thermal conductivity (radial)

$$\frac{q_r}{A} = \left(\frac{(T_1 - T_3)}{\mathcal{R}_1 + \mathcal{R}_2} \right) \frac{1}{r} = \frac{\text{driving force}}{\text{resistance}}$$

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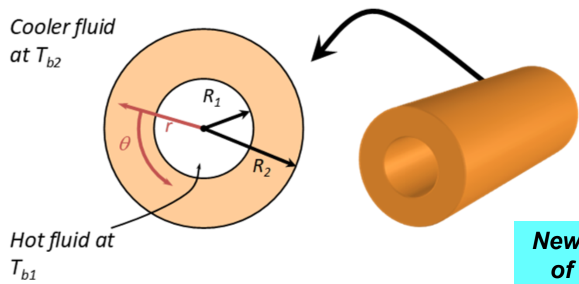
1D Radial Heat Transfer – Resistance

5

1D Radial: Heat flux in a cylindrical shell – Newton's Law BC

What is the steady state temperature profile in a cylindrical shell (pipe) if the fluid on the inside is at T_{b1} and the fluid on the outside is at T_{b2}? ($T_{b1} > T_{b2}$)



Newton's law of cooling boundary conditions

See handwritten notes.
https://pages.mtu.edu/~fmorriso/cm310/lectures/handnotes_lec14-15_1d_radial_both%20bc.pdf

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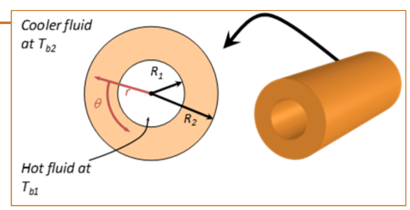
1D Radial Heat Transfer – Resistance

1D Radial: Heat flux in a cylindrical shell – Newton's Law BC



SOLUTION:

$$\frac{q_r}{A} = \frac{(T_{b1} - T_{b2})}{\frac{1}{h_2 R_2} + \frac{1}{k}\ln\left(\frac{R_2}{R_1}\right) + \frac{1}{h_1 R_1}}\left(\frac{1}{r}\right)$$

Radial flux NOT constant

Let: $\mathcal{R}_i \equiv \frac{1}{h_i R_i}$

Let: $\mathcal{R}_3 \equiv \frac{1}{k}\ln\left(\frac{R_2}{R_1}\right)$

Resistance due to heat transfer at boundary
Resistance due to finite thermal conductivity

$$\frac{q_r}{A} = \left(\frac{(T_{b1} - T_{b2})}{\mathcal{R}_1 + \mathcal{R}_2 + \mathcal{R}_3}\right)\frac{1}{r} = \frac{\text{driving force}}{\text{resistance}}$$

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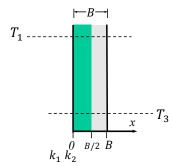
1D Heat Transfer – Resistance

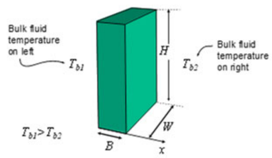
RESISTANCE SUMMARY:

1D Rectangular: Door ($k_1 = k_2$), and Composite Door



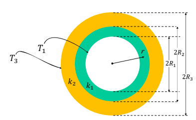
$$\frac{q_x}{A} = \frac{(T_1 - T_3)}{\left(\frac{B/2}{k_1} + \frac{B/2}{k_2}\right)}$$

1D Rectangular: Slab with Newton's law BC



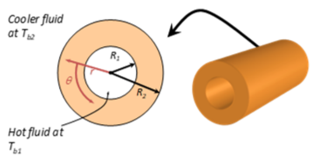
$$\frac{q_x}{A} = \frac{T_{b1} - T_{b2}}{\left(\frac{1}{h_1} + \frac{B}{k} + \frac{1}{h_2}\right)}$$

1D Radial: Pipe ($k_1 = k_2$) and Composite Pipe



$$\frac{q_r}{A} = \frac{(T_1 - T_3)}{\frac{1}{k_1}\ln\frac{R_2}{R_1} + \frac{1}{k_2}\ln\frac{R_3}{R_2}}\left(\frac{1}{r}\right)$$

1D Radial: Pipe with Newton's law BC



$$\frac{q_r}{A} = \frac{(T_{b1} - T_{b2})}{\frac{1}{h_2 R_2} + \frac{1}{k}\ln\left(\frac{R_2}{R_1}\right) + \frac{1}{h_1 R_1}}\left(\frac{1}{r}\right)$$

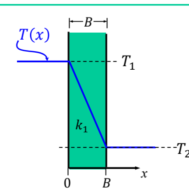
1D Heat Transfer – Resistance

Temperature Boundary Conditions

1D Rectangular

Let: $\mathcal{R} \equiv \frac{B}{k}$

$$\frac{q_x}{A} = \frac{(T_1 - T_2)}{\mathcal{R}} = \frac{\text{driving force}}{\text{resistance}}$$

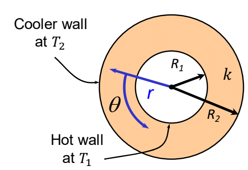


Note: Geankoplis uses a different resistance. For rectangular heat flux:
$R_{Geankoplis} = \mathcal{R}/LW$

1D Radial

Let: $\mathcal{R} \equiv \frac{1}{k}\ln\frac{R_2}{R_1}$

$$\frac{q_r}{A} = \left(\frac{(T_1 - T_2)}{\mathcal{R}}\right)\frac{1}{r} = \frac{\text{driving force}}{\text{resistance}}$$



Note: Geankoplis uses a different resistance. For radial heat flux:
$R_{Geankoplis} = \mathcal{R}/2\pi L$

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1D Heat Transfer – Resistance

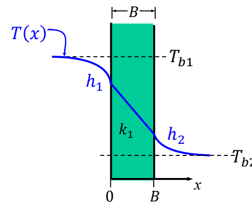
1D Rectangular

Let:

$\mathcal{R}_i \equiv \frac{1}{h_i}$ for $i = 1,2$

$\mathcal{R}_3 \equiv \frac{B}{k}$

$$\frac{q_x}{A} = \frac{(T_{b1} - T_{b2})}{\mathcal{R}_1 + \mathcal{R}_2 + \mathcal{R}_3} = \frac{\text{driving force}}{\text{resistance}}$$



1D Radial

Let:

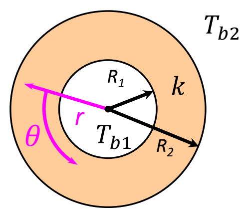
$\mathcal{R}_i \equiv \frac{1}{R_i h_i}$ for $i = 1,2$

$\mathcal{R}_3 \equiv \frac{1}{k} \ln \frac{R_2}{R_1}$

h_1 = inside

h_2 = outside

$$\frac{q_r}{A} = \frac{(T_{b1} - T_{b2})}{\mathcal{R}_1 + \mathcal{R}_2 + \mathcal{R}_3}\left(\frac{1}{r}\right) = \frac{\text{driving force}}{\text{resistance}}$$



Newton's Law of Cooling Boundary Conditions

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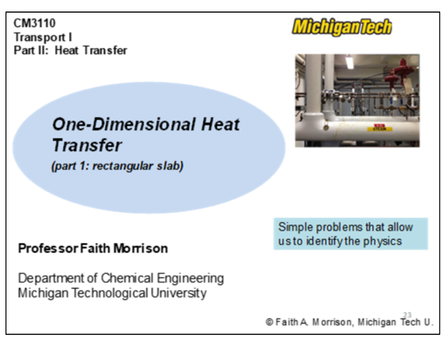
Microscopic Energy Balance—Solve for Temperature Field

CM3110 REVIEW

For review, let's carry out an example of 1D, steady heat transfer



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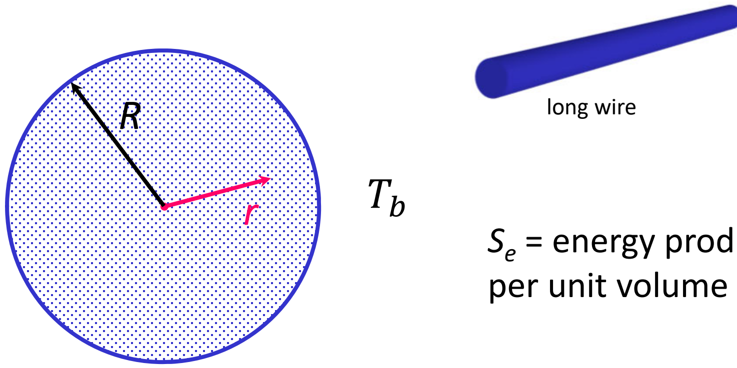
Microscopic Energy Balance—Solve for Temperature Field

Example 3: Heat Conduction with Generation

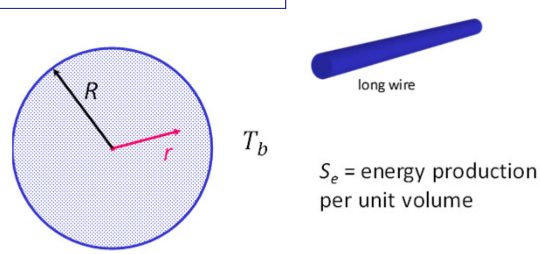
What is the steady state temperature profile in a wire if heat is generated uniformly throughout the wire at a rate of S_e W/m^3 and the bulk fluid surrounding the wire is at T_b? What is the heat flux?



S_e = energy production per unit volume

Microscopic Energy Balance—Solve for Temperature Field

Example: Heat conduction with generation



h = heat transfer coefficient

Let's try.

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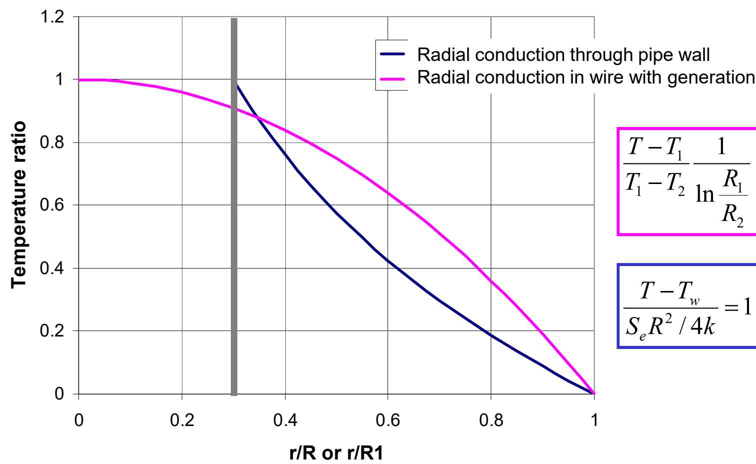
Microscopic Energy Balance—Solve for Temperature Field

Compare radial conduction solutions



$$\frac{T-T_1}{T_1-T_2}\frac{1}{\ln\frac{R_1}{R_2}}=\ln\frac{r}{R_1}$$

$$\frac{T-T_w}{S_eR^2/4k}=1-\left(\frac{r}{R}\right)^2$$

Unsteady State Heat Transfer

CM3110 REVIEW

Heat-Transfer Review Summary:

- Microscopic energy balance
- Fourier's law of heat conduction (k, homogeneous)
- Newton's law of cooling (h, at the boundary)
- **Resistances** due to k and h; vary with boundary conditions (BC) and geometry

	***T* BC**	***h* BC**
1D rectangular	$\frac{B}{k}$	$\frac{1}{h}$
1D radial	$\frac{1}{k}\ln\frac{R_2}{R_1}$	$\frac{1}{Rh}$

Sneak peak: The ratio of T (*internal*) and h (*external*) resistances is the **Biot** number:

$$\boldsymbol{Bi}=\frac{B/k}{1/h}=\frac{\boldsymbol{hB}}{\boldsymbol{k}}$$

This is important in unsteady heat transfer.

- Solving for the *steady* temperature field $T(x,y,z)$, a.k.a. "*Slash and Burn*"

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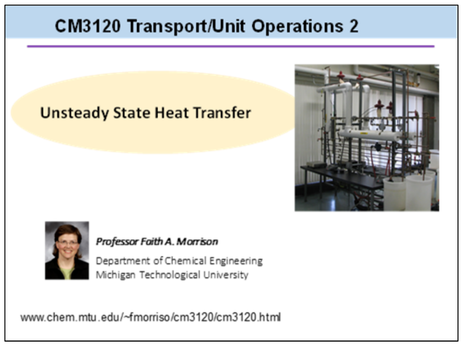
NEXT: Unsteady State Heat Transfer



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