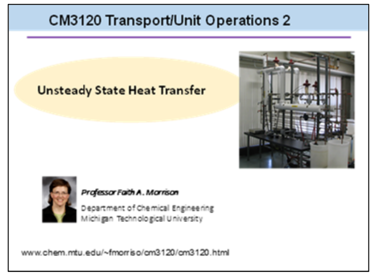
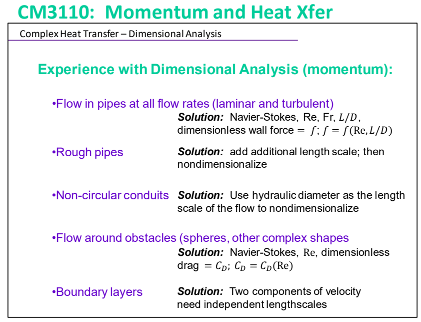
To understand and more complex heat transfer units, we turn now to…



Dimensional Analysis


CM3110: Momentum and Heat Xfer

Complex Heat Transfer – Dimensional Analysis

Experience with Dimensional Analysis (momentum):

- Flow in pipes at all flow rates (laminar and turbulent) — ***Solution:*** Navier-Stokes, Re, Fr, L/D, dimensionless wall force $= f$; $f = f(\mathrm{Re}, L/D)$
- Rough pipes — ***Solution:*** add additional length scale; then nondimensionalize
- Non-circular conduits — ***Solution:*** Use hydraulic diameter as the length scale of the flow to nondimensionalize
- Flow around obstacles (spheres, other complex shapes — ***Solution:*** Navier-Stokes, Re, dimensionless drag $= C_D$; $C_D = C_D(\mathrm{Re})$
- Boundary layers — ***Solution:*** Two components of velocity need independent lengthscales

1

CM3120 Transport/Unit Operations 2

Dimensional Analysis

Towards Understanding Unsteady State Heat Transfer (and more)

Professor Faith A. Morrison
Department of Chemical Engineering
Michigan Technological University

Includes review

www.chem.mtu.edu/~fmorriso/cm3120/cm3120.html

2

Heat Transfer: Steady vs. Unsteady

What is our usual strategy for complex phenomena?

Answer: Dimensional Analysis

CM3110: Momentum and Heat Xfer

Complex Heat Transfer – Dimensional Analysis

Experience with Dimensional Analysis (momentum):

- Flow in pipes at all flow rates (laminar and turbulent) — ***Solution:*** Navier-Stokes, Re, Fr, L/D, dimensionless wall force $= f$; $f = f(\text{Re}, L/D)$
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- Boundary layers — ***Solution:*** Two components of velocity need independent lengthscales

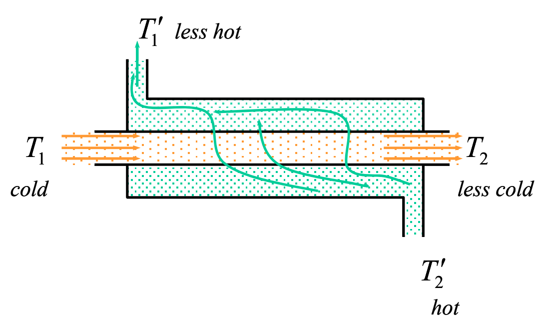
Let's review

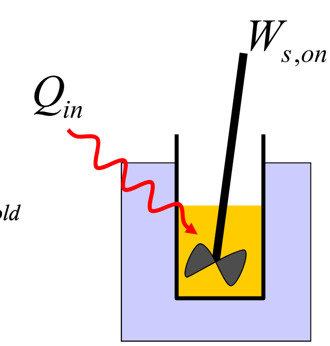
3

Complex Heat Transfer (CM3110)

CM3110 REVIEW

How do we handle complex geometries, complex flows, complex machinery?





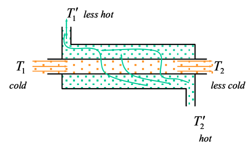
Process scale

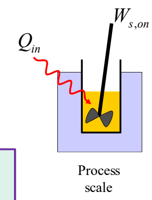
4

Complex Heat Transfer – Dimensional Analysis

CM3110 REVIEW

(***Answer***: *Use the same techniques we have been using in fluid mechanics)*





Engineering Modeling (complex systems)

- Choose an idealized problem and solve it
- From insight obtained from **ideal** problem, identify governing equations of **real** problem
- Nondimensionalize the governing equations; deduce dimensionless scale factors (e.g. Re, Fr for fluids)
- Design experiments to test modeling thus far
- Revise modeling (structure of dimensional analysis, identity of scale factors, e.g. add roughness lengthscale)
- Design additional experiments
- Iterate until useful correlations result

5

Complex Heat Transfer – Dimensional Analysis

CM3110 REVIEW

Experience with Dimensional Analysis (momentum):

- Flow in pipes at all flow rates (laminar and turbulent)
  Solution: Navier-Stokes, Re, Fr, L/D, dimensionless drag $= f$; $f = f(\mathrm{Re}, L/D)$
- Rough pipes — ***Solution:*** add additional length scale; then nondimensionalize
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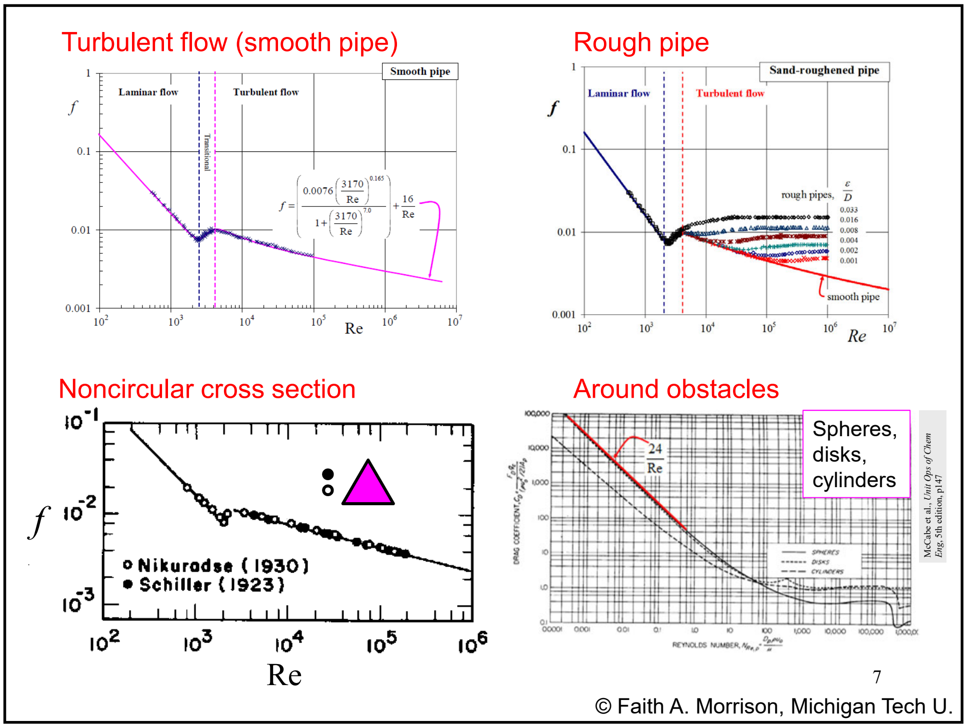
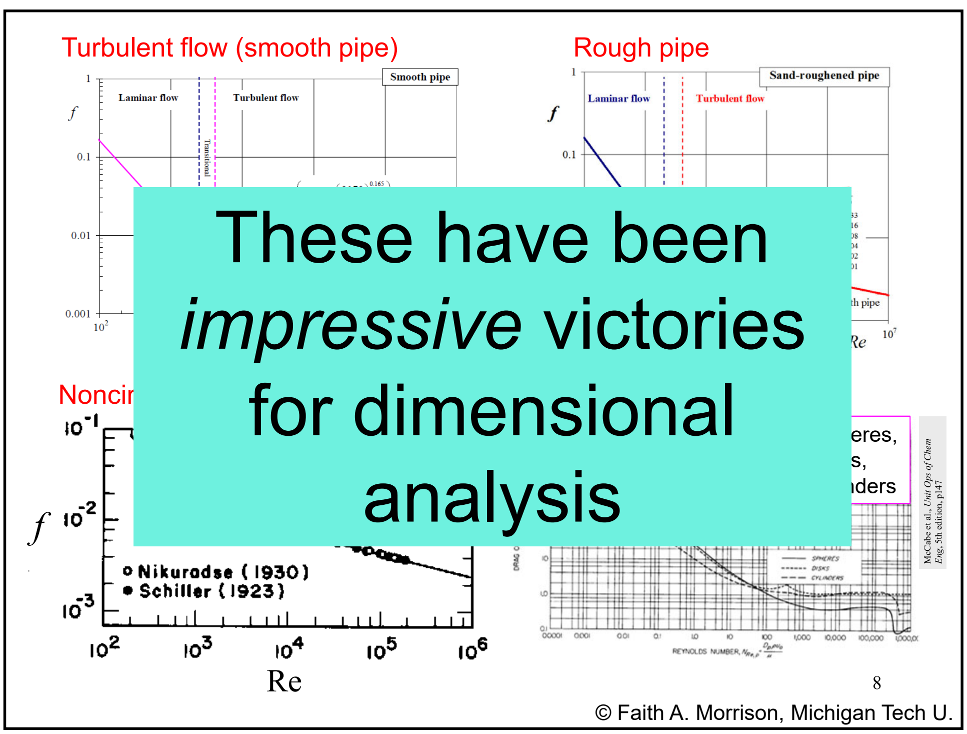
  Solution: Navier-Stokes, Re, dimensionless drag $= C_D$; $C_D = C_D(\mathrm{Re})$
- Boundary layers — ***Solution:*** Two components of velocity need independent lengthscales

6


Turbulent flow (smooth pipe)
Smooth pipe
Laminar flow
Turbulent flow
Transitional
Re
Rough pipe
Sand-roughened pipe
Laminar flow
Turbulent flow
rough pipes
smooth pipe
Re
Noncircular cross section
Nikuradse (1930)
Schiller (1923)
Re
Around obstacles
Spheres, disks, cylinders
24/Re
McCabe et al., Unit Ops of Chem Eng, 5th edition, p147
7
© Faith A. Morrison, Michigan Tech U.


Turbulent flow (smooth pipe)
Rough pipe
These have been *impressive* victories for dimensional analysis
Nikuradse (1930)
Schiller (1923)
Re
8
© Faith A. Morrison, Michigan Tech U.

Heat Transfer: Steady vs. Unsteady

CM3110 REVIEW

How did Dimensional Analysis work for ***steady heat transfer***?

Answer: Here's the method:

- Choose an idealized problem and solve it
- From insight obtained from **ideal** problem, identify governing equations of **real** problem
- Nondimensionalize the governing equations; deduce dimensionless scale factors (e.g. Re, Fr for fluids)
- Design experiments to test modeling thus far
- Revise modeling (structure of dimensional analysis, identity of scale factors, e.g. add roughness lengthscale)
- Design additional experiments
- Iterate until useful correlations result

9

Complex Heat Transfer – Dimensional Analysis

CM3110 REVIEW

Complex Heat transfer Problems to Solve:

- Forced convection heat transfer from fluid to wall

  Solution: ?

- Natural convection heat transfer from fluid to wall

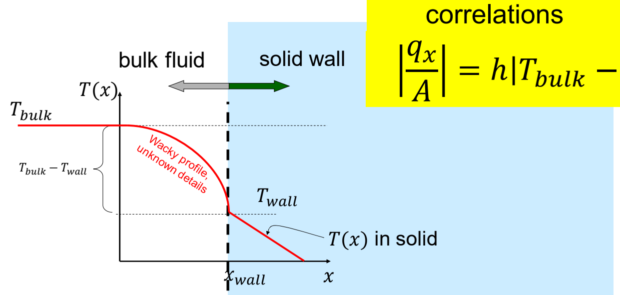
  Solution: ?

- Radiation heat transfer from solid to fluid

  Solution: ?

Nondimensionalize h and produce data correlations

$$\left|\frac{q_x}{A}\right| = h\left|T_{bulk} - T_{wall}\right|$$



10

Complex Heat Transfer – Dimensional Analysis

CM3110 REVIEW

Complex Heat transfer Problems to Solve:

•Forced convection heat transfer from fluid to wall
Solution: ?

•Natural convection heat transfer from fluid to wall
Solution: ?

•Radiation heat transfer from solid to fluid
Solution: ?

- The functional form of h will be different for these three situations (different physics)
- Investigate simple problems in each category, model them, take data, correlate

11

Complex Heat Transfer – Dimensional Analysis

CM3110 REVIEW

Complex Heat transfer Problems to Solve:

•Forced convection heat transfer from fluid to wall
Solution: ?

•Natural convection heat transfer from fluid to wall
Solution: ?

•Radiation heat transfer from solid to fluid
Solution: ?

- The functional form of h will be different for these three situations (different physics)
- Investigate simple problems in each category, model them, take data, correlate

Let's look at forced convection in a pipe. There are three pieces to the physics:

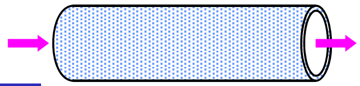
Pipe flow

Energy

Boundary conditions

12

Forced Convection Heat Transfer



CM3110 REVIEW

Pipe flow

z-component of the Navier-Stokes Equation:

$$\rho\left(\frac{\partial v_z}{\partial t}+v_r\frac{\partial v_z}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta}+v_z\frac{\partial v_z}{\partial z}\right)$$

$$=-\frac{\partial P}{\partial z}+\mu\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2}+\frac{\partial^2 v_z}{\partial z^2}\right)+\rho g_z$$

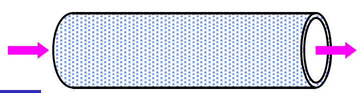
Choose:

D = characteristic length
V = characteristic velocity
D/V = characteristic time
ρV^2 = characteristic pressure

- Choose "typical" values (*scale factors*)
- Use them to scale the equations
- Deduce which terms dominate

13

Forced Convection Heat Transfer



CM3110 REVIEW

Pipe flow

non-dimensional variables:

time:

$$t^* \equiv \frac{tV}{D}$$

position:

$$r^* \equiv \frac{r}{D}$$

$$z^* \equiv \frac{z}{D}$$

velocity:

$$v_z^* \equiv \frac{v_z}{V}$$

$$v_r^* \equiv \frac{v_r}{V}$$

$$v_\theta^* \equiv \frac{v_\theta}{V}$$

driving force:

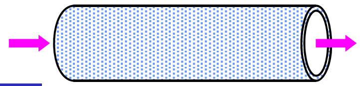
$$P^* \equiv \frac{P}{\rho V^2}$$

$$g_z^* \equiv \frac{g_z}{g}$$

- Choose "typical" values (*scale factors*)
- Use them to scale the equations
- Deduce which terms dominate

14

Forced Convection Heat Transfer



CM3110 REVIEW

Energy

Microscopic energy balance:

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} + v_z \frac{\partial T}{\partial z} \right) = k \left(\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2} \right) + S$$

non-dimensional variables:

position:

$$r^* \equiv \frac{r}{D}$$

$$z^* \equiv \frac{z}{D}$$

temperature:

$$T^* \equiv \frac{T - T_o}{(T_1 - T_o)}$$

source:

$$S^* \equiv \frac{S}{S_0}$$

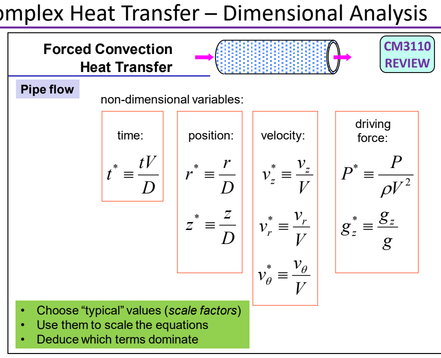
Choose:
T – use a characteristic **interval** (since distance from absolute zero is not part of this physics)
S – use a reference source, S_0

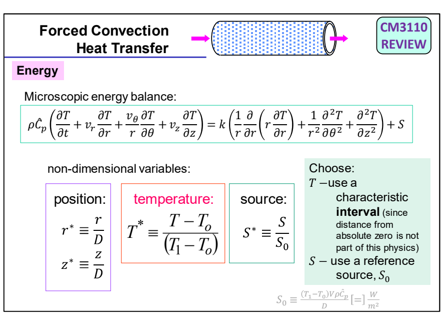
$$S_0 \equiv \frac{(T_1 - T_0) V \rho \hat{C}_p}{D} [=] \frac{W}{m^2}$$

15

Complex Heat Transfer – Dimensional Analysis

CM3110 REVIEW





Substitute all these definitions,

$$(t^*, r^*, z^*, p^*, g^*, v_r^*, v_\theta^*, v_z^*, T^*, S^*)$$

into the ***governing equations*** and simplify…

16

Complex Heat Transfer – Dimensional Analysis

CM3110 REVIEW

Non-dimensional Energy Equation

$$\left(\frac{\partial T^*}{\partial t^*} + v_r^*\frac{\partial T^*}{\partial r^*} + \frac{v_\theta^*}{r^*}\frac{\partial T^*}{\partial \theta} + v_z^*\frac{\partial T^*}{\partial z^*}\right) = \frac{1}{\mathrm{Pe}}\left(\frac{1}{r^*}\frac{\partial}{\partial r^*}\left(r^*\frac{\partial T^*}{\partial r^*}\right) + \frac{1}{r^{*2}}\frac{\partial^2 T^*}{\partial \theta^2} + \frac{\partial^2 T^*}{\partial z^2}\right) + S^*$$

$$\mathrm{Pe} = \mathrm{Pr}\,\mathrm{Re} = \frac{\hat{C}_p\mu}{k}\frac{\rho V D}{\mu}$$

Non-dimensional Navier-Stokes Equation

$$\frac{Dv_z^*}{Dt^*} = -\frac{\partial P^*}{\partial z^*} + \frac{1}{\mathrm{Re}}\left(\nabla^2 v_z\right)^* + \frac{1}{Fr}g^*$$

$$\mathrm{Pr} \equiv \frac{\hat{C}_p\mu}{k}$$

Non-dimensional Continuity Equation

$$\frac{\partial v_x^*}{\partial x^*} + \frac{\partial v_y^*}{\partial y^*} + \frac{\partial v_z^*}{\partial z^*} + = 0$$

$$\frac{Dv_z}{Dt} \equiv \frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}$$

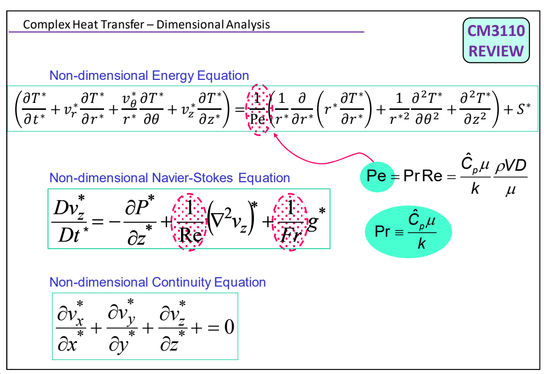
17

Complex Heat Transfer – Dimensional Analysis

CM3110 REVIEW

How do we get to the "*engineering quantities of interest*" ?

What are the "*engineering quantities of interest*" ?

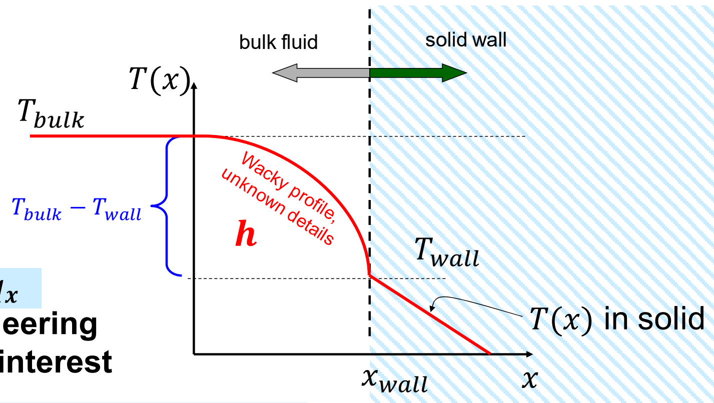

Complex Heat Transfer – Dimensional Analysis
CM3110 REVIEW
Non-dimensional Energy Equation
Non-dimensional Navier-Stokes Equation
Non-dimensional Continuity Equation

18

Microscopic Energy Balance Review—Boundary Conditions

CM3110 REVIEW

The interface between the solid and the fluid calls for a new type of boundary condition, ***Newton's Law of Cooling***.



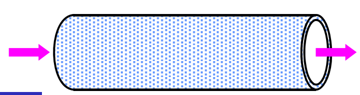
$Q = q_x$

is the engineering quantity of interest

$$\left|\frac{q_x}{A}\right| = h\left|T_{bulk} - T_{wall}\right|$$

19

Forced Convection Heat Transfer



CM3110 REVIEW

Linear driving force model

$$\left|\frac{q_x}{A}\right| = h\left|T_1 - T_0\right|$$

Apply at the surface:

$$(2\pi RL)(h)(T_1 - T_0) = Q = \iint_S \left[\hat{e}_r \cdot \underline{\tilde{q}}\right]_{surface} dS$$

$$(2\pi RL)(h)(T_1 - T_0) = Q = \int_0^{2\pi}\int_0^{L} -k\left.\frac{\partial T}{\partial r}\right|_{r=R} R\,dz\,d\theta$$

Now, non-dimensionalize this expression as well.

20

Complex Heat Transfer – Dimensional Analysis

CM3110 REVIEW

$$h(\pi DL)(T_1 - T_o) = \int_0^{2\pi}\int_0^{L/D} -k \left.\frac{\partial T^*}{\partial r^*}\right|_{r^*=1/2} \frac{(T_1 - T_o)}{D}\frac{D^2}{2} dz^* d\theta$$

$$2\pi\left(\frac{hD}{k}\right)\left(\frac{L}{D}\right) = \int_0^{2\pi}\int_0^{L/D} -\left.\frac{\partial T^*}{\partial r^*}\right|_{r^*=1/2} dz^* d\theta$$

Nusselt number, Nu (dimensionless heat-transfer coefficient)

$$Nu = Nu\left(T^*, \frac{L}{D}\right)$$

one additional dimensionless group

21

Complex Heat Transfer – Dimensional Analysis

CM3110 REVIEW

$$h(\pi DL)(T_1 - T_o) = \int_0^{2\pi}\int_0^{L/D} -k \left.\frac{\partial T^*}{\partial r^*}\right|_{r^*=1/2} \frac{(T_1 - T_o)}{D}\frac{D^2}{2} dz^* d\theta$$

This is a function of Re and Pr through fluid $\underline{v}$ distribution and energy balance

$$2\pi\left(\frac{hD}{k}\right)\left(\frac{L}{D}\right) = \int_0^{2\pi}\int_0^{L/D} -\left.\frac{\partial T^*}{\partial r^*}\right|_{r^*=1/2} dz^* d\theta$$

Nusselt number, Nu (dimensionless heat-transfer coefficient)

$$Nu = Nu\left(T^*, \frac{L}{D}\right)$$

one additional dimensionless group

22

Complex Heat Transfer – Dimensional Analysis

CM3110 REVIEW

According to our **dimensional analysis** calculations, the dimensionless heat transfer coefficient should be found to be a function of ~~four~~ dimensionless groups:
three

no free surfaces

$$Nu = Nu\left(\mathrm{Re}, \mathrm{Pr}, \cancel{\mathrm{Fr}}, \frac{L}{D}\right)$$

Peclet number

$$\mathrm{Pe} \equiv \frac{\rho \hat{C}_p V D}{k} = \frac{\hat{C}_p \mu}{k} \frac{\rho V D}{\mu}$$

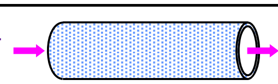
Prandtl number

$$\mathrm{Pr} \equiv \frac{\hat{C}_p \mu}{k}$$

Now, do the experiments.

23

Complex Heat Transfer – Dimensional Analysis



CM3110 REVIEW

Now, do the experiments.

Forced Convection Heat Transfer

- Build apparatus (*several* actually, with different D, L)
- Run fluid through the inside (at different $\underline{v}$; for different fluids $\rho, \mu, \hat{C}_p, k$)
- Measure T_{bulk} on inside; T_{wall} on inside
- Measure rate of heat transfer, Q
- Calculate h: $|Q| = hA|T_{bulk} - T_{wall}|$
- Report h values in terms of dimensionless correlation:

$$\mathrm{Nu} = \frac{hD}{k} = f\left(\mathrm{Re}, \mathrm{Pr}, \frac{L}{D}\right)$$

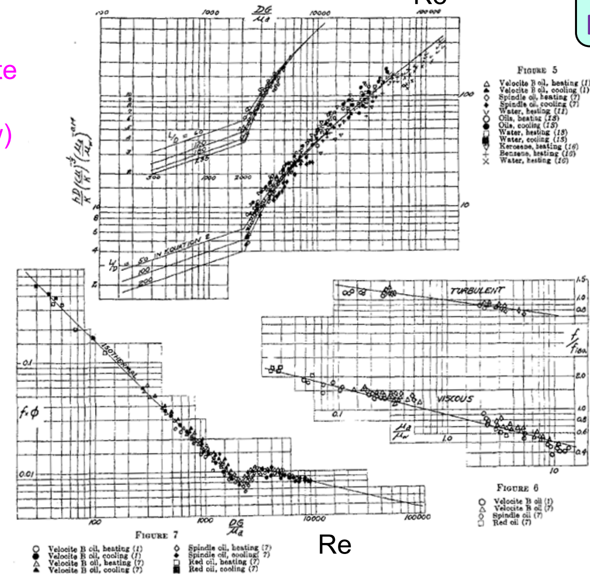
It should only be a function of these dimensionless numbers (***if*** our Dimensional Analysis is correct…..)

24

Complex Heat Transfer – Dimensional Analysis

CM3110 REVIEW

Sieder and Tate equation (turbulent flow)



Sieder and Tate equation (laminar flow)

Effect of viscosity ratio

Sieder & Tate, *Ind. Eng. Chem.* **28** 227 (1936)

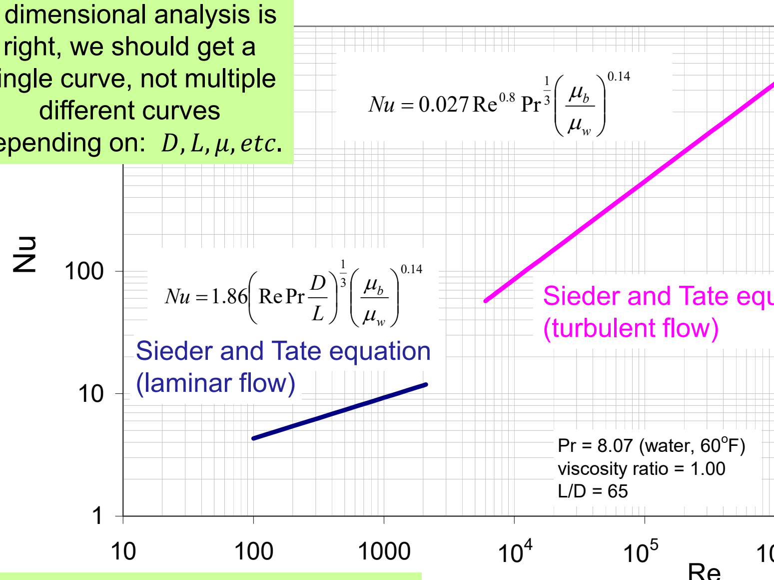
25

Complex Heat Transfer – Dimensional Analysis

Correlations for Forced Convection Heat Transfer Coefficients

CM3110 REVIEW

If dimensional analysis is right, we should get a single curve, not multiple different curves depending on: $D, L, \mu, etc.$



Sieder & Tate, *Ind. Eng. Chem.* **28** 227 (1936)
Geankoplis, 4th ed. eqn 4.5-4, page 260

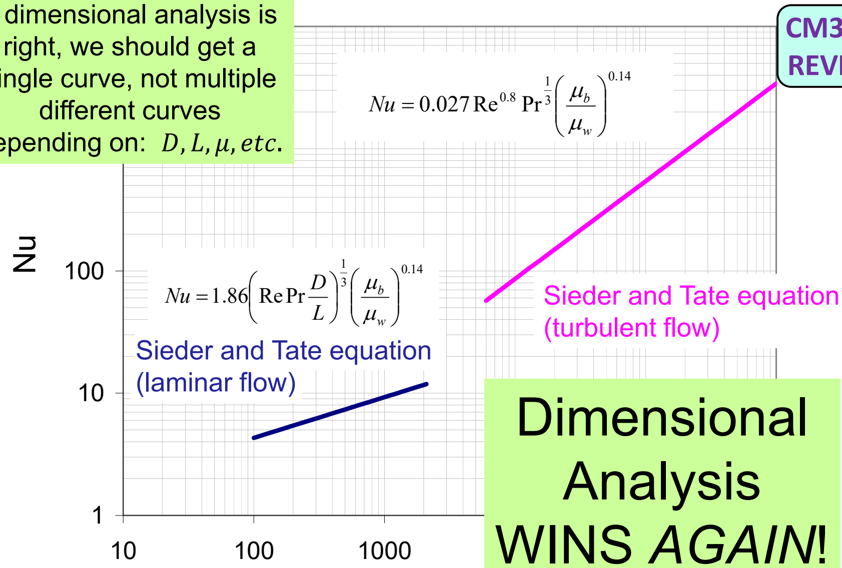
26

Complex Heat Transfer – Dimensional Analysis

Correlations for Forced Convection Heat Transfer Coefficients

CM3110 REVIEW

If dimensional analysis is right, we should get a single curve, not multiple different curves depending on: $D, L, \mu, etc.$



Dimensional Analysis WINS *AGAIN*!

Sieder & Tate, *Ind. Eng. Chem.* **28** 227 (1936)
Geankoplis, 4th ed. eqn 4.5-4, page 260

27

Complex Heat Transfer – Dimensional Analysis

CM3110 REVIEW

Forced convection
Heat Transfer in Laminar flow in pipes

$$\mathrm{Nu}_a = \frac{h_a D}{k} = 1.86\left(\mathrm{RePr}\frac{D}{L}\right)^{\frac{1}{3}}\left(\frac{\mu_b}{\mu_w}\right)^{0.14}$$

Sieder-Tate equation (laminar flow)

Forced convection
Heat Transfer in Turbulent flow in pipes

$$\mathrm{Nu}_{lm} = \frac{h_{lm} D}{k} = 0.027\mathrm{Re}^{0.8}\mathrm{Pr}^{\frac{1}{3}}\left(\frac{\mu_b}{\mu_w}\right)^{0.14}$$

Sieder-Tate equation (turbulent flow)

Physical Properties evaluated at:

$$\frac{T_{b,in} + T_{b,out}}{2}$$

May have to be estimated

$$\frac{T_{b,in} + T_{b,out}}{2}$$

bulk mean temperature

Fine print matters!

- all physical properties (except μ_w) evaluated at the **bulk mean temperature**
- Laminar or turbulent flow

28

Complex Heat Transfer – Dimensional Analysis

CM3110 REVIEW

Heat transfer data correlations from the literature (real, complex systems)

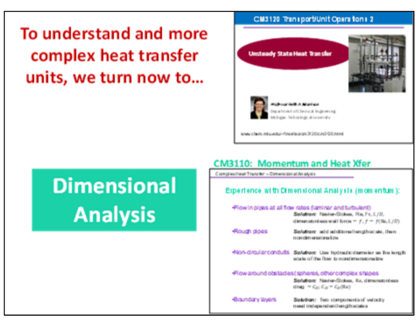
Example of *partial* summary of correlations from the literature

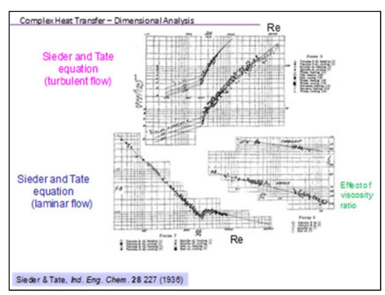
CM3110 REVIEW

Laminar flow in pipes	$Nu_a = \frac{h_a D}{k} = 1.86\left(\text{Re}\,\text{Pr}\frac{D}{L}\right)^{\frac{1}{3}}\left(\frac{\mu_b}{\mu_w}\right)^{0.14}$	Re<2100, (RePrD/L)>100, horizontal pipes, eqn 4.5-4, page 238; all properties evaluated at the temperature of the bulk fluid except μ_w which is evaluated at the wall temperature.
Turbulent flow in smooth pipes	$Nu_{lm} = \frac{h_{lm} D}{k} = 0.027\,\text{Re}^{0.8}\,\text{Pr}^{\frac{1}{3}}\left(\frac{\mu_b}{\mu_w}\right)^{0.14}$	Re>6000, 0.7 <Pr <16,000, L/D>60 , eqn 4.5-8, page 239; all properties evaluated at the mean temperature of the bulk fluid except μ_w which is evaluated at the wall temperature. The mean is the average of the inlet and outlet bulk temperatures; not valid for liquid metals.
Air at 1atm in turbulent flow in pipes	$h_{lm} = \frac{3.52V(m/s)^{0.8}}{D(m)^{0.2}}$ $h_{lm} = \frac{0.5V(ft/s)^{0.8}}{D(ft)^{0.2}}$	equation 4.5-9, page 239
Water in turbulent flow in pipes	$h_{lm} = 1429\left(1+0.0146T\left({}^{o}C\right)\right)\frac{V(m/s)^{0.8}}{D(m)^{0.2}}$ $h_{lm} = 150\left(1+0.011T\left({}^{o}F\right)\right)\frac{V(ft/s)^{0.8}}{D(ft)^{0.2}}$	4 < T(°C)<105, equation 4.5-10, page 239

(studied in CM3110)

Complex Heat Transfer – Dimensional Analysis





$$\text{Nu} = \text{Nu}\left(\text{Re}, \text{Pr}, \frac{L}{D}\right)$$

Summary

- Dimensional analysis works as well in heat transfer as in momentum transfer
- We should use it (and probably also in mass transfer, but…)
- These dimensionless numbers are stacking up (and…)
- What do they really mean?

Dimensional Analysis

These numbers tell us about the relative importance of the terms they precede.

Dimensionless numbers from the
Equations of Change *(microscopic balances)*

momentum

Non-dimensional Navier-Stokes Equation

$$\left(\frac{\partial v_z^*}{\partial t^*} + \underline{v}^* \cdot \nabla^* v_z^*\right) = -\frac{\partial P^*}{\partial z^*} + \frac{1}{\mathrm{Re}}(\nabla^{*2} v_z^*) + \frac{1}{\mathrm{Fr}} g^*$$

Re – Reynolds
Fr – Froude

energy

Non-dimensional Energy Equation

$$\left(\frac{\partial T^*}{\partial t^*} + \underline{v}^* \cdot \nabla^* T^*\right) = \frac{1}{\mathrm{RePr}}(\nabla^{*2} T^*) + S^*$$

Pe – Péclet$_h$ = RePr
Pr – Prandtl

mass

Non-dimensional Continuity Equation (species A)

$$\left(\frac{\partial x_A^*}{\partial t^*} + \underline{v}^* \cdot \nabla^* x_A^*\right) = \frac{1}{\mathrm{ReSc}}(\nabla^{*2} x_A^*)$$

Pe – Péclet$_m$ = ReSc
Sc – Schmidt

ref: BSL1, p581, 644

31

Dimensionless Numbers

Dimensionless numbers from the
Equations of Change

Re – Reynolds $= \frac{\rho V D}{\mu} = \frac{VD}{\nu}$

Fr – Froude $= \frac{V^2}{gD}$

Pe – Péclet$_h$ $= \mathrm{RePr} = \frac{\hat{C}_p \rho V D}{k} = \frac{VD}{\alpha}$

Pe – Péclet$_m$ $= \mathrm{ReSc} = \frac{VD}{D_{AB}}$

These numbers tell us about the **relative importance of the terms** they precede in the microscopic balances (*scenario properties*).

Pr – Prandtl $= \frac{\hat{C}_p \mu}{k} = \frac{\nu}{\alpha}$

Sc – Schmidt $= \mathrm{LePr} = \frac{\mu}{\rho D_{AB}} = \frac{\nu}{D_{AB}}$

Le – Lewis $= \frac{\alpha}{D_{AB}}$

These numbers compare the **magnitudes of the diffusive transport coefficients** ν, α, D_{AB} (*material properties*).

32

Dimensionless Numbers

Dimensionless numbers from the
Equations of Change

$\text{Re} - \text{Reynolds} = \frac{\rho V D}{\mu} = \frac{VD}{\nu}$

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These numbers compare the **magnitudes of the diffusive transport coefficients** ν, α, D_{AB} (*material properties*).

Transport coefficients

33

Dimensional Analysis

These numbers are defined to help us build **transport data correlations** based on the fewest number of grouped (dimensionless) variables (*scenario property*).

Dimensionless numbers from the
Engineering Quantities of Interest

momentum

Dimensionless Force on the Wall (Drag)

$$f = \frac{1}{\pi}\frac{D}{L}\frac{1}{\text{Re}} \int_0^{\frac{L}{D}} \int_0^{2\pi} \left(\frac{\partial v_z^*}{\partial r^*} \right)\Bigg|_{r^*=\frac{1}{2}} d\theta dz^*$$

(Fanning)

f — Friction Factor

$\frac{L}{D}$ — Aspect Ratio

$$f = \frac{\mathcal{F}_{drag}}{\left(\frac{1}{2}\rho V^2\right) A_c}$$

energy

Newton's Law of Cooling

$$Nu = \frac{1}{2\pi L/D} \int_0^{2\pi} \int_0^{L/D} -\frac{\partial T^*}{\partial r^*}\Bigg|_{r^*=1/2} dz^* d\theta$$

Nu — Nusselt

$\frac{L}{D}$ — Aspect Ratio

$$\text{Nu} = \frac{hD}{k}$$

mass xfer

Dimensionless Mass Transfer Coefficient

$$Sh = \frac{1}{2\pi}\frac{D}{L} \int_0^{\frac{L}{D}} \int_0^{2\pi} \left(-\frac{\partial x_A^*}{\partial r^*} \right)\Bigg|_{r^*=\frac{1}{2}} d\theta dz^*$$

Sh — Sherwood

$\frac{L}{D}$ — Aspect Ratio

$$\text{Sh} = \frac{k_m D}{D_{AB}}$$

34

momentum
energy
mass

Dimensionless Numbers

Re – Reynolds $= \frac{\rho V D}{\mu} = \frac{VD}{\nu}$

Fr – Froude $= \frac{V^2}{gD}$

Pe – Péclet$_h$ = RePr $= \frac{\hat{C}_p \rho V D}{k} = \frac{VD}{\alpha}$

Pe – Péclet$_m$ = ReSc $= \frac{VD}{D_{AB}}$

These numbers from the governing equations tell us about the relative importance of the terms they precede in the microscopic balances (***scenario*** *properties*).

Pr – Prandtl $= \frac{\hat{C}_p \mu}{k} = \frac{\nu}{\alpha}$

Sc – Schmidt = LePr $= \frac{\mu}{\rho D_{AB}} = \frac{\nu}{D_{AB}}$

Le – Lewis $= \frac{\alpha}{D_{AB}}$

These numbers compare the magnitudes of the diffusive transport coefficients ν, α, D_{AB} (***material*** *properties*).

f – Friction Factor $= \frac{\mathcal{F}_{drag}}{\left(\frac{1}{2}\rho V^2\right) A_c}$

Nu – Nusselt $= \frac{hD}{k}$

Sh – Sherwood $= \frac{k_m D}{D_{AB}}$

These numbers are defined to help us build transport data correlations based on the fewest number of grouped (dimensionless) variables (***scenario*** *properties*).

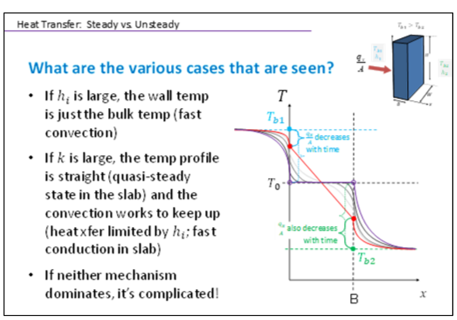
35

Unsteady State Heat Transfer: Dimensional Analysis

NEW STUFF!

***Question*: What now?**

Answer: Let's apply Dimensional Analysis to something new, unsteady state heat transfer, to sort out the various effects.



Engineering Modeling (complex systems)

- Choose an idealized problem and solve it
- From insight obtained from **ideal** problem, identify governing equations of **real** problem
- Nondimensionalize the governing equations; deduce dimensionless scale factors (e.g. Re, Fr for fluids)
- Design experiments to test modeling thus far
- Revise modeling (structure of dimensional analysis, identity of scale factors, e.g. add roughness lengthscale)
- Design additional experiments
- Iterate until useful correlations result

36