

Last time,

Unsteady State Heat Transfer

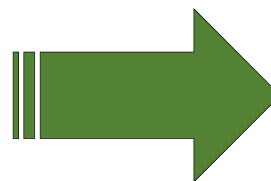
CM3120 Transport/Unit Operations 2

More complex Systems:
Unsteady State Heat Transfer
(Analytical Solutions)



Professor Faith A. Morrison
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Michigan Technological University

Exploring the "lay of the land"



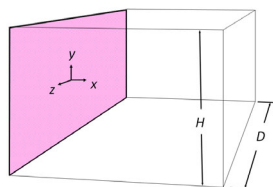
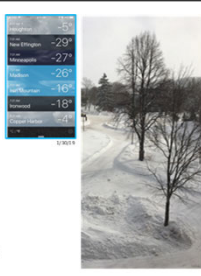
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Solution Summary:

Example:

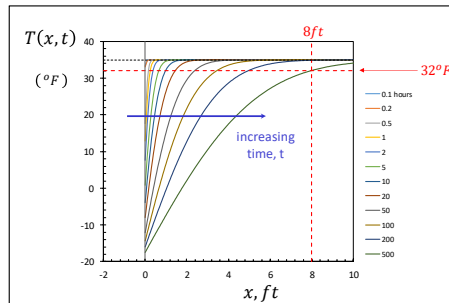
When will my pipes freeze?

The temperature has been 35°F for a while now, sufficient to chill the ground to this temperature for many tens of feet below the surface. Suddenly the temperature drops to -20°F. How long will it take for freezing temperatures (32°F) to reach my pipes, which are 8 ft under ground?



Answer:

$t = 509 \text{ hours} \approx 21 \text{ days}$



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Dimensional Analysis, Unsteady State Convection

Non-dimensionalize (eqns, BCs)

$$\frac{\partial T}{\partial t} = \alpha \left(\frac{\partial^2 T}{\partial x^2} \right)$$

$$q_x = -k \frac{\partial T}{\partial x} = hA(T_1 - T)$$

non-dimensional variables:

position:

$$x^* \equiv \frac{x}{D}$$

temperature:

$$Y \equiv \frac{(T_1 - T)}{(T_1 - T_0)}$$

time:

$$t^* \equiv \frac{\alpha t}{D^2}$$

Note:

$D = D_{char}$ and this length scale is not always "diameter"

This dimensionless time is called Fourier number Fo.

Fo – Fourier Number = $\frac{\alpha t}{D^2}$

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In dimensionless form, we see that this problem reduces to

$$Y = Y \left(\frac{x}{D}, Fo, Bi \right)$$

Dimensionless quantities:

$$Y = \frac{(T_1 - T)}{(T_1 - T_0)}$$

$$t^* = \text{Fo} = \frac{\alpha t}{D^2}$$

$$x^* = \frac{x}{D}$$

$$\text{Bi} = \frac{hD}{k}$$

Y (dimensionless temperature interval)

Fourier number (dimensionless time based on thermal diffusion)

Biot number (pronounced BEE-OH)
Ratio of heat transfer resistance at the boundary to resistance in the solid. This is a *transport issue*.

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Bi – Biot Number = $\frac{hD}{k}$

$Bi = \frac{D_{char}/k}{1/h}$

At high Bi, the surface temperature equals the bulk temperature; heat transfer is limited by conduction in the body.

At moderate Bi, heat transfer is affected by both conduction in the body and the rate of heat transfer to the surface.

At low Bi, the temperature is uniform in a finite body; heat transfer is limited by rate of heat transfer to the surface (h).

Quantifies the tradeoffs between the rate of internal heat flux (by conduction, k) and the rate of heat delivery to the boundary (by convection, h)

High Bi:
low k ,
high h

Moderate Bi:
neither process
dominates

Low Bi:
high k ,
low h

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low h

When the temperature is uniform in the body, we can do a macroscopic energy balance to solve many problems of interest. This is called a “lumped parameter analysis.”

$T = T(t)$

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Bi – Biot Number = $\frac{hD}{k}$

$Bi = \frac{D_{char}/k}{1/h}$

Quantifies the tradeoffs between the rate of internal heat flux (by conduction, k) and the rate of heat delivery to the boundary (by convection, h)

$T = T(x, y, z, t)$, easy BC

At high Bi, the surface temperature equals the bulk temperature; heat transfer is limited by conduction in the body.

High Bi:
low k ,
high h

At moderate Bi, heat transfer is affected by both conduction in the body and the rate of heat transfer to the surface.

Moderate Bi:

When the wall temperature and the bulk temperature are equal, the microscopic energy balance is easier to carry out (temperature boundary conditions).

At low Bi, the temperature is uniform in a finite body; heat transfer is limited by rate of heat transfer to the surface (h).

Low Bi:
high k ,
low h

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Bi – Biot Number = $\frac{hD}{k}$

$Bi = \frac{D_{char}/k}{1/h}$

Quantifies the tradeoffs between the rate of internal heat flux (by conduction, k) and the rate of heat delivery to the boundary (by convection, h)

When both processes affect the outcomes, the full solution may be necessary. For uniform starting temperatures, the solutions are published.

$T = T(x, y, z, t)$, hard BC

At moderate Bi, heat transfer is affected by both conduction in the body and the rate of heat transfer to the surface.

Moderate Bi:
neither process dominates

At low Bi, the temperature is uniform in a finite body; heat transfer is limited by rate of heat transfer to the surface (h).

Low Bi:
high k ,
low h

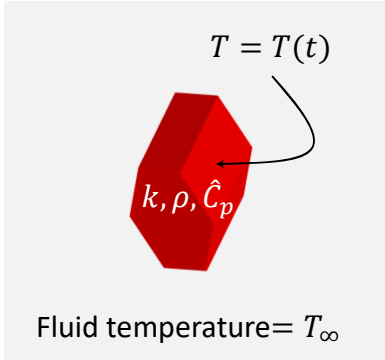
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Unsteady State Heat Transfer: Low Biot Number

Example: Quench cooling of a manufactured part.

If a piece of steel with $T = T_0$ is dropped into a large, well stirred reservoir of fluid at bulk temperature T_∞ , what is the temperature of the steel as a function of time?

- $k = \text{large}$, which means that there is no internal resistance to heat transfer in the part
- Therefore, we are NOT calculating a temperature profile (internal T is uniform)
- \Rightarrow **Use Unsteady, Macroscopic Energy Balance**



Fluid temperature = T_∞

$$Bi_{LP} = \frac{hV_{sys}}{kA_{sys}}$$

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Unsteady State Heat Transfer: Low Biot Number

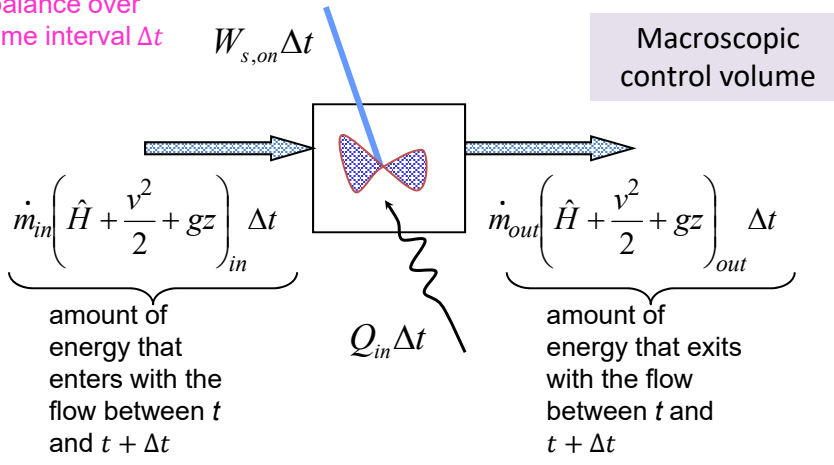
Unsteady Macroscopic Energy Balance

see Felder and Rousseau or Himmelblau

balance over
time interval Δt

$W_{s,on} \Delta t$

Macroscopic
control volume



$\dot{m}_{in} \left(\hat{H} + \frac{v^2}{2} + gz \right)_{in} \Delta t$

amount of energy that enters with the flow between t and $t + \Delta t$

$\dot{m}_{out} \left(\hat{H} + \frac{v^2}{2} + gz \right)_{out} \Delta t$

amount of energy that exits with the flow between t and $t + \Delta t$

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Unsteady State Heat Transfer: Low Biot Number

Unsteady Macroscopic Energy Balance

accumulation = input – output

$$\frac{d}{dt} (U_{sys} + E_{k,sys} + E_{p,sys}) = -\Delta H - \Delta E_k - \Delta E_p + Q_{in} + W_{s,on}$$

Background:
pages.mtu.edu/~fmorriso/cm310/IFMWeb
AppendixDMicroEBalanceMorrison.pdf

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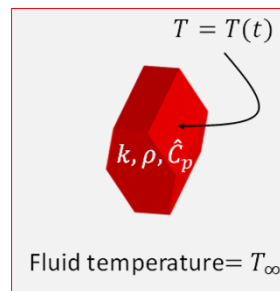
Unsteady State Heat Transfer: Low Biot Number

Unsteady Macroscopic Energy Balance

accumulation = input – output

$$\frac{d}{dt} (U_{sys} + E_{k,sys} + E_{p,sys}) = -\Delta H - \Delta E_k - \Delta E_p + Q_{in} + W_{s,on}$$

How do we apply
this balance to our
current problem?



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Unsteady State Heat Transfer: Low Biot Number

Unsteady Macroscopic Energy Balance

accumulation = input – output

$$\frac{d}{dt}(U_{sys} + E_{k,sys} + E_{p,sys}) = -\Delta H - \Delta E_k - \Delta E_p + Q_{in} + W_{s,on}$$

You try.

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Unsteady State Heat Transfer: Low Biot Number

Unsteady Macroscopic Energy Balance

accumulation = input – output

$$\frac{d}{dt}(U_{sys} + E_{k,sys} + E_{p,sys}) = -\Delta H - \Delta E_k - \Delta E_p + Q_{in} + W_{s,on}$$

negligible
no flow
no shafts

For negligible changes in E_p and E_k , no flow, no phase change, no chemical rxn, and no shafts:

$$\frac{dU_{sys}}{dt} = Q_{in}$$

$$\rho V_{sys} \hat{C}_v \frac{dT_{sys}}{dt} = Q_{in}$$

$\hat{C}_v \approx \hat{C}_p$ for liquids, solids

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Unsteady State Heat Transfer: Low Biot Number

Unsteady Macroscopic Energy Balance

How do we quantify the heat in Q_{in} ?

Fluid temperature = T_∞

$$\frac{d}{dt}(U_{sys} + E_{k,sys} + E_{p,sys}) = -\Delta H - \Delta E_k - \Delta E_p + Q_{in} + W_{s,on}$$

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Unsteady Macroscopic Energy Balance

accumulation = input - output

Q_{in} = Heat **into** the chosen macroscopic control volume

$$\frac{d}{dt}(U_{sys} + E_{k,sys} + E_{p,sys}) = -\Delta H - \Delta E_k - \Delta E_p + Q_{in} + W_{s,on}$$

$Q_{in} = \sum_i q_{in,i}$ comes from a variety of sources:

- Thermal conduction: $q_{in} = -kA \frac{dT}{dx}$
- Convection heat xfer: $|q_{in}| = |hA(T_b - T)|$
- Radiation: $q_{in} = \epsilon \sigma A (T_{surroundings}^4 - T_{surface}^4)$
- Electric current: $q_{in} = I^2 R_{elec} L$
- Chemical Reaction: $q_{in} = S_{rxn} V_{sys}$

$S [=] \frac{\text{energy}}{\text{time volume}}$

pages.mtu.edu/~fmorriso/cm310/IFMWebAppendixDMicroEBalanceMorrison.pdf
Incropera and DeWitt, 6th edition p18

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Unsteady Macroscopic Energy Balance

*accumulation =
input – output*

$Q_{in} = \text{Heat } \mathbf{into} \text{ the chosen macroscopic control volume}$

$$\frac{d}{dt}(U_{sys} + E_{k,sys} + E_{p,sys}) = -\Delta H - \Delta E_k - \Delta E_p + Q_{in} + W_{s,on}$$

Signs must match transfer from outside (bulk fluid) to inside (metal)

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Unsteady Macroscopic Energy Balance

$\frac{d}{dt}(U_{sys} + E_{k,sys} + E_{p,sys}) = -\Delta H - \Delta E_k - \Delta E_p + Q_{in} + W_{s,on}$

$Q_{in} = \sum_i q_{in,i}$ comes from a variety of sources:

- **Thermal conduction:** $q_{in} = -kA \frac{dT}{dx}$
e.g. device held by bracket; a solid phase that extends through boundaries of control volume
- **Convection heat xfer:** $|q_{in}| = |hA(T_b - T)|$
e.g. device dropped in stirred liquid; forced air stream flows past, natural convection occurs outside system; phase change at boundary
- **Radiation:** $q_{in} = \epsilon\sigma A(T_{surroundings}^4 - T_{surface}^4)$
e.g. device at high temp. exposed to a gas/vacuum; hot enough to produce nat. conv.=possibly hot enough for radiation
- **Electric current:** $q_{in} = I^2 R_{elec} L$
e.g. if electric current is flowing within the device/control volume/system
- **Chemical Reaction:** $q_{in} = S_{rxn} V_{sys}$
e.g. if a homogeneous reaction is taking place throughout the device/ control volume/system

S-B constant:
 $\sigma = 5.676 \times 10^{-8} \frac{W}{m^2 K^4}$

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Unsteady Macroscopic Energy Balance

accumulation =
input – output

$Q_{in} = \text{Heat } \mathbf{into} \text{ the chosen macroscopic control volume}$

$$\frac{d}{dt}(U_{sys} + E_{k,sys} + E_{p,sys}) = -\Delta H - \Delta E_k - \Delta E_p + Q_{in} + W_{s,on}$$

$Q_{in} = \sum_i q_{in,i}$ comes from a variety of sources:

- ✘ • Thermal conduction: $q_{in} = -kA \frac{dT}{dx}$
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Unsteady State Heat Transfer: Low Biot Number

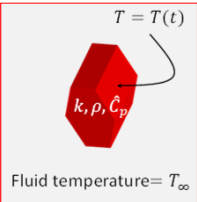
Unsteady Macroscopic Energy Balance Applied to cooling steel part:

$$\rho V_{sys} \hat{C}_p \frac{dT_{sys}}{dt} = Q_{in}$$

The temperature changes in the part are due to the heat loss

The heat loss depends on the heat-transfer coefficient from the part to the environment

$$Q_{in} = Ah(T_{\infty} - T)$$



$\hat{C}_v \approx \hat{C}_p$ for liquids, solids

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Unsteady State Heat Transfer: Low Biot Number

Unsteady Macroscopic Energy Balance Applied to cooling steel part:

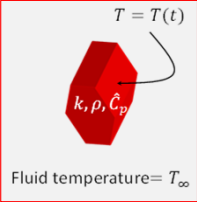
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The heat loss depends on the heat-transfer coefficient from the part to the environment

$$\rho V_{sys} \hat{C}_p \frac{dT_{sys}}{dt} = Q_{in}$$

$$Q_{in} = Ah(T_{\infty} - T)$$

You solve.



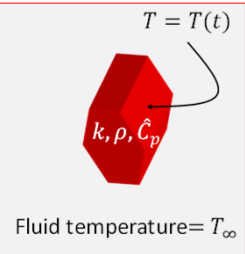
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Unsteady State Heat Transfer: Low Biot Number

Unsteady Macroscopic Energy Balance Applied to cooling steel part:

$$\frac{(T_{\infty} - T)}{(T_{\infty} - T_0)} = e^{-\left(\frac{hA}{\rho \hat{C}_p V}\right) t}$$

$$\ln\left(\frac{(T_{\infty} - T)}{(T_{\infty} - T_0)}\right) = -\left(\frac{hA}{\rho \hat{C}_p V}\right) t$$



$V_{sys} = V$

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Unsteady State Heat Transfer: Low Biot Number

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$$\ln\left(\frac{(T_\infty - T)}{(T_\infty - T_0)}\right) = -\left(\frac{hA}{\rho\hat{C}_pV}\right)t$$

In dimensionless form? ➔

Fluid temperature = T_∞

$V_{sys} = V$

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Unsteady State Heat Transfer: Low Biot Number

Unsteady Macroscopic Energy Balance Applied to cooling steel part:

$$Bi_{LP} = \frac{hV_{sys}}{kA_{sys}}$$

Fluid temperature = T_∞

$$\frac{(T_\infty - T)}{(T_\infty - T_0)} = e^{-\left(\frac{hA}{\rho\hat{C}_pV}\right)t}$$

$$\frac{hAt}{\rho\hat{C}_pV} = \left(\frac{h}{k}\right)\left(\frac{k}{\rho\hat{C}_p}\right)\left(\frac{A}{V}\right)t = \left(\frac{Bi}{D_{char}}\right)\alpha\left(\frac{t}{D_{char}}\right) = Bi\,Fo$$

LP good for:
 $Bi_{LP} < 0.1$

Bi – Biot Number = $\frac{hD_{char}}{k}$

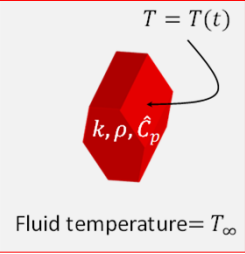
Fo – Fourier Number = $\frac{\alpha t}{D_{char}^2}$

$D_{char} \equiv \frac{\text{volume}}{\text{area}} = \frac{V}{A}$ thermal diffusivity
 $\alpha \equiv \frac{k}{\rho\hat{C}_p}$

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Unsteady State Heat Transfer: Low Biot Number

Unsteady Macroscopic Energy Balance Applied to cooling steel part:

$$Bi_{LP} = \frac{hV_{sys}}{kA_{sys}}$$


Fluid temperature = T_∞

$$Y = \frac{(T_\infty - T)}{(T_\infty - T_0)} = e^{-Bi Fo}$$

WRF p279

Lumped parameter analysis

https://en.wikipedia.org/wiki/Lumped_element_model

Lumped parameter analysis:

LP good for:
 $Bi_{LP} < 0.1$

Bi – Biot Number = $\frac{hD_{char}}{k} = \frac{hV}{kA}$

Fo – Fourier Number = $\frac{\alpha t}{D_{char}^2}$

$D_{char} \equiv \frac{\text{volume}}{\text{surf. area}} = \frac{V}{A}$ thermal diffusivity

$\alpha \equiv \frac{k}{\rho \hat{C}_p}$

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Unsteady State Heat Transfer

Summary

$$Bi_{LP} = \frac{hV_{sys}}{kA_{sys}}$$

LP good for:
 $Bi_{LP} < 0.1$

Quantifies the tradeoffs between the rate of internal heat flux (by conduction, k) and the rate of heat delivery to the boundary (by convection, h)

At high Bi, the surface temperature equals the bulk temperature: heat transfer is limited by rate of heat transfer to the surface.

High Bi: dominates

When the temperature is uniform in the body, we can do a macroscopic energy balance to solve many problems of interest. This is called a "lumped parameter analysis."

At low Bi, the temperature is uniform in a finite body; heat transfer is limited by rate of heat transfer to the surface (h).

Low Bi: high k , low h

Low Bi: no internal temperature variation \Rightarrow Lumped parameter analysis (macroscopic energy balance, unsteady); $Bi_{LP} = hV/kA < 0.1$

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2. Negligible Surface Resistance, $\frac{1}{h}$

High Bi:
low k ,
high h

Bi – Biot Number = $\frac{hD}{k}$ Quantifies the tradeoffs between the rate of internal heat flux (by conduction, k) and the rate of heat delivery to the boundary (by convection, h)

At high Bi, the surface temperature equals the bulk temperature; heat transfer is limited by conduction in the body.

At moderate Bi, heat transfer is affected by both the body and the surface.

At low Bi, the temperature is uniform in the body; heat transfer is limited by rate of heat transfer to the surface (h).

High Bi:
low k ,
high h

Moderate Bi:

Low Bi:
high k ,
low h

When the wall temperature and the bulk temperature are equal, the microscopic energy balance is easier to carry out (temperature boundary conditions).

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low k ,
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Low Bi:
high k ,
low h

When the wall temperature and the bulk temperature are equal, the microscopic energy balance is easier to carry out (temperature boundary conditions).

At high Bi, the surface temperature equals the bulk temperature; heat transfer is limited by conduction in the body.

We have done many examples with constant temperature boundary conditions.

$$\lim_{h \rightarrow \infty} |T_{bulk} - T_{wall}| = 0$$

$\Rightarrow T_{wall} = T_{bulk}$

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Unsteady State Heat Transfer

Summary

High Bi: dominated by internal temperature variation \Rightarrow solve with temperature boundary conditions; $Bi = hD_{char}/k$ (D_{char} varies with the problem)

Low Bi: no internal temperature variation \Rightarrow Lumped parameter analysis (macroscopic energy balance, unsteady); $Bi_{LP} = hV/kA < 0.1$

Bi – Biot Number = $\frac{hD}{k}$ Quantifies the tradeoffs between the rate of internal heat flux (by conduction, k) and the rate of heat delivery to the boundary (by convection, h)

At high Bi, the surface temperature equals the bulk temperature; heat transfer is limited by conduction in the body. **High Bi: low k , high h**

At moderate Bi, heat transfer is affected by both conduction in the body and the rate of heat transfer to the surface. **Moderate Bi: neither process dominates**

At low Bi, the temperature is uniform in a finite body; heat transfer is limited by rate of heat transfer to the surface (h). **Low Bi: high k , low h**

When the wall temperature and the bulk temperature are equal, the microscopic energy balance is easier to carry out (temperature boundary conditions).

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3. No Mechanism Dominates

**Moderate Bi:
nether process dominates**

At moderate Bi, heat transfer is affected by both conduction in the body and the rate of heat transfer to the surface.

Bi – Biot Number = $\frac{hD}{k}$ Quantifies the tradeoffs between the rate of internal heat flux (by conduction, k) and the rate of heat delivery to the boundary (by convection, h)

When both processes affect the outcomes, the full solution may be necessary; for uniform starting temperatures there are charts.

At moderate Bi, heat transfer is affected by both conduction in the body and the rate of heat transfer to the surface. **Moderate Bi: nether process dominates**

At low Bi, the temperature is uniform in a finite body; heat transfer is limited by rate of heat transfer to the surface (h). **Low Bi: high k , low h**

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3. No Mechanism Dominates

Bi – Biot Number = $\frac{hD}{k}$ Quantifies the tradeoffs between the rate of internal heat flux (by conduction, k) and the rate of heat delivery to the boundary (by convection, h)

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Moderate Bi:
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At low Bi, the temperature is uniform in a finite body; heat transfer is limited by rate of heat transfer to the surface (h).

Low Bi:
high k ,
low h

At moderate Bi, heat transfer is affected by both conduction in the body and the rate of heat transfer to the surface.

This is the most complicated set of cases.

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