CM3120 Transport/Unit Operations 2

Unsteady State Heat Transfer

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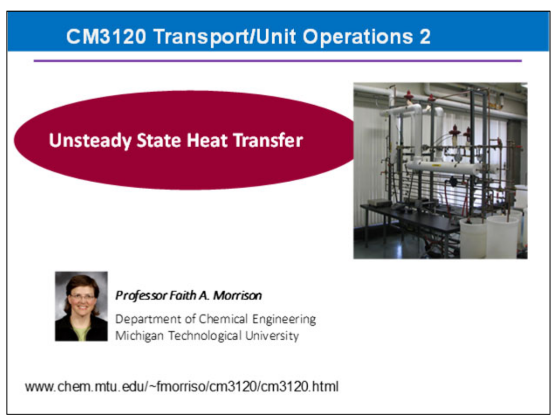
www.chem.mtu.edu/~fmorriso/cm3120/cm3120.html

1

Where to start?



We seek to study **unsteady state heat transfer**.
Let's start by looking over several subjects that form the foundation for what we hope to study.

These basic concepts are familiar. Cycling back can deepen our understanding (and help us put new concepts in context)

2

CM3120 Transport/Unit Operations 2

Energy Balance Review

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www.chem.mtu.edu/~fmorriso/cm3120/cm3120.html

3

CM2110/CM2120 - Review

Macroscopic Energy Balances

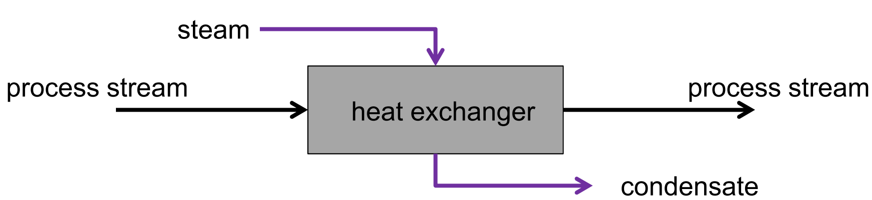
The physics:
- Mass conservation
- Energy conservation

Open system energy balance

$$\Delta E_p + \Delta E_k + \Delta H = Q_{in} + W_{s,on} \qquad \text{(out-in)}$$

Closed system energy balance

$$\Delta E_p + \Delta E_k + \Delta U = Q_{in} + W_{on} \qquad \text{(final-initial)}$$



www.chem.mtu.edu/~fmorriso/cm310/Energy_Balance_Notes_2008.pdf

4

CM2110/CM2120 - Review

Review:

How do we decide what equations to use for what?

- Closed system E-bal (first law of thermo)
- Open system E-bal ($H = U + PV$, flowing systems)
- Mechanical energy balance (SISO, steady, isothermal, no rxn, no phase change, little heat transferred)

Macroscopic Energy Balances

Open system energy balance

$$\Delta E_p + \Delta E_k + \Delta H = Q_{in} + W_{s,on} \qquad \text{(out-in)}$$

Closed system energy balance

$$\Delta E_p + \Delta E_k + \Delta U = Q_{in} + W_{on} \qquad \text{(final-initial)}$$

$$\text{MEB:} \quad \frac{\Delta p}{\rho} + \frac{\Delta(v^2)}{2\alpha} + g\Delta z + F = \frac{W_{s,on}}{\dot{m}}$$

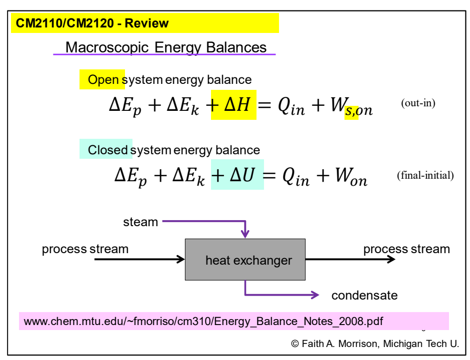
Notes:
1. Δ has different meanings in the three e-balances
2. Open-system:
$$\Delta H = \sum_{outs} m_i \hat{H}_i - \sum_{ins} m_i \hat{H}_i$$
3. Use MEB if you can (easy); but not if it does not appy!

Knowing what assumptions we are making means we understand our models. See handout for summary:

www.chem.mtu.edu/~fmorriso/cm310/Energy_Balance_Notes_2008.pdf

5

What ***physics*** determines how rapidly the heat transfers from the outside stream to the inside stream?



Fourier's Law of Heat Conduction

$$\frac{q_x}{A} = -k\frac{dT}{dx}$$

(for a homogeneous phase)

$\frac{q_x}{A}$ –heat flux=energy/area time)

k – thermal conductivity

$\frac{dT}{dx}$ –temperature gradient

(the driving physics is Brownian motion: energy transports down ∇T due to Brownian motion)

CM3110 - Review

CM3110 REVIEW

Heat Transfer Rate law: Fourier's law of Heat Conduction:

makes reference to a coordinate system

Allows you to solve for temperature profiles

$$\frac{q_x}{A} = -k\frac{dT}{dx}$$

Gibbs notation: $\frac{\underline{q}}{A} = -k\nabla T$

Fourier's law 3D

$$\tilde{\underline{q}} = \frac{\underline{q}}{A} = \begin{pmatrix} -k\frac{\partial T}{\partial x} \\ -k\frac{\partial T}{\partial y} \\ -k\frac{\partial T}{\partial z} \end{pmatrix}_{xyz}$$

- Heat flows **down** a temperature gradient
- Flux is proportional to magnitude of temperature gradient

7

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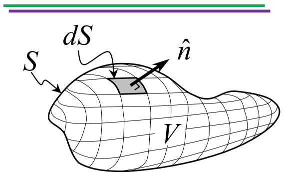
CM3110 REVIEW

To use the heat transport law (Fourier's law) we need to use the **microscopic** energy balance.

Microscopic Energy Balance:

Equation of Thermal Energy



Microscopic **energy** balance written on an arbitrarily shaped volume, *V*, enclosed by a surface, *S*

Gibbs notation: $\rho\left(\frac{\partial \hat{E}}{\partial t} + \underline{v}\cdot\nabla\hat{E}\right) = -\nabla\cdot\tilde{\underline{q}} + S_e$ **general conduction**

Gibbs notation: $\rho\hat{C}_p\left(\frac{\partial T}{\partial t} + \underline{v}\cdot\nabla T\right) = k\nabla^2 T + S_e$ **Only Fourier conduction**

(incompressible fluid, constant pressure, neglect $\hat{E}_k, \hat{E}_p$, viscous dissipation)

http://pages.mtu.edu/~fmorriso/cm310/energy_equation.html

8

CM3110 - Review

CM3110 REVIEW

Equation of Energy
(microscopic energy balance)

convection

source

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + \underline{v} \cdot \nabla T \right) = k \nabla^2 T + S_e$$

(energy generated per unit volume per time)

rate of change

conduction (all directions)

velocity must satisfy equation of motion, equation of continuity

see handout for component notation

http://pages.mtu.edu/~fmorriso/cm310/energy_equation.html

The **Equation of Energy** in Cartesian, cylindrical, and spherical coordinates for Newtonian fluids of constant density, with source term S. Source could be electrical energy due to current flow, chemical energy, etc. Two cases are presented: the general case where thermal conductivity may be a function of temperature (vector flux $\tilde{\underline{q}} = \underline{q}/A$ appears in the equations); and the more usual case, where thermal conductivity is constant.

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Microscopic energy balance, in terms of flux; Gibbs notation

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + \underline{v} \cdot \nabla T \right) = -\nabla \cdot \tilde{\underline{q}} + S$$

Microscopic energy balance, in terms of flux; Cartesian coordinates

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} + v_z \frac{\partial T}{\partial z} \right) = -\left(\frac{\partial \tilde{q}_x}{\partial x} + \frac{\partial \tilde{q}_y}{\partial y} + \frac{\partial \tilde{q}_z}{\partial z} \right) + S$$

Microscopic energy balance, in terms of flux; cylindrical coordinates

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} + v_z \frac{\partial T}{\partial z} \right) = -\left(\frac{1}{r} \frac{\partial (r \tilde{q}_r)}{\partial r} + \frac{1}{r} \frac{\partial \tilde{q}_\theta}{\partial \theta} + \frac{\partial \tilde{q}_z}{\partial z} \right) + S$$

Microscopic energy balance, in terms of flux; spherical coordinates

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} + \frac{v_\phi}{r \sin\theta} \frac{\partial T}{\partial \phi} \right) = -\left(\frac{1}{r^2} \frac{\partial (r^2 \tilde{q}_r)}{\partial r} + \frac{1}{r \sin\theta} \frac{\partial (\tilde{q}_\theta sin\theta)}{\partial \theta} + \frac{1}{r \sin\theta} \frac{\partial \tilde{q}_\phi}{\partial \phi} \right) + S$$

Fourier's law of heat conduction, Gibbs notation: $\tilde{\underline{q}} = -k \nabla T$

Fourier's law of heat conduction, Cartesian coordinates: $\begin{pmatrix} \tilde{q}_x \\ \tilde{q}_y \\ \tilde{q}_z \end{pmatrix}_{xyz} = \begin{pmatrix} -k \frac{\partial T}{\partial x} \\ -k \frac{\partial T}{\partial y} \\ -k \frac{\partial T}{\partial z} \end{pmatrix}_{xyz}$

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The **Equation of Energy** for systems with **constant k**

Microscopic energy balance, constant thermal conductivity; Gibbs notation

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + \underline{v} \cdot \nabla T \right) = k \nabla^2 T + S$$

Microscopic energy balance, constant thermal conductivity; Cartesian coordinates

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} + v_z \frac{\partial T}{\partial z} \right) = k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + S$$

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Reference: F. A. Morrison, "Web Appendix to *An Introduction to Fluid Mechanics*," Cambridge University Press, New York, 2013. On the web at www.chem.mtu.edu/~fmorriso/IFM_WebAppendixCD2013.pdf

http://pages.mtu.edu/~fmorriso/cm310/energy_equation.html

10

The **Equation of Energy** in Cartesian, cylindrical, and spherical coordinates for Newtonian fluids of constant density, with source term S. Source could be electrical energy due to current flow, chemical energy, etc. Two cases are presented: the general case where thermal conductivity may be a function of temperature (vector flux $\underline{\tilde{q}} = \underline{q}/A$ appears in the equations); and the more usual case, where thermal conductivity is constant.

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Microscopic energy balance, in terms of flux; Gibbs notation

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Microscopic ener...

Front side:

- Micro E-bal in terms of flux $\underline{\tilde{q}} \equiv \frac{\underline{q}}{A}$
- Fourier's law, $\underline{\tilde{q}} = -k \nabla T$

Fourier's law of heat conduction, cylindrical coordinates: $\begin{pmatrix} \tilde{q}_r \\ \tilde{q}_\theta \\ \tilde{q}_z \end{pmatrix}_{xyz} = \begin{pmatrix} -k \frac{\partial T}{\partial r} \\ -\frac{k}{r} \frac{\partial T}{\partial \theta} \\ -k \frac{\partial T}{\partial z} \end{pmatrix}_{r\theta z}$

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11

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Microscopic energy balance, constant thermal conductivity; spherical coordinates

Back side:

- Micro E-bal in terms of temperature (Fourier's law incorporated)

http://pages.mtu.edu/~fmorriso/cm310/energy_equation.html

12

Microscopic Energy Balance

CM3110 REVIEW

The **Equation of Energy** for systems with **constant k**

Microscopic energy balance, constant thermal conductivity; Gibbs notation

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Microscopic energy balance, constant thermal conductivity; Cartesian coordinates

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Fourier's Law of Heat Conduction

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Fourier's law of heat conduction, Gibbs notation: $\underline{\tilde{q}} = -k\nabla T$

Fourier's law of heat conduction, Cartesian coordinates:
$$\begin{pmatrix}\tilde{q}_x\\ \tilde{q}_y\\ \tilde{q}_z\end{pmatrix}_{xyz} = \begin{pmatrix}-k\frac{\partial T}{\partial x}\\ -k\frac{\partial T}{\partial y}\\ -k\frac{\partial T}{\partial z}\end{pmatrix}_{xyz}$$

$$\tilde{q}_x = \frac{q_x}{A}$$

Fourier's law of heat conduction, cylindrical coordinates:
$$\begin{pmatrix}\tilde{q}_r\\ \tilde{q}_\theta\\ \tilde{q}_z\end{pmatrix}_{xyz} = \begin{pmatrix}-k\frac{\partial T}{\partial r}\\ -\frac{k}{r}\frac{\partial T}{\partial \theta}\\ -k\frac{\partial T}{\partial z}\end{pmatrix}_{r\theta z}$$

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http://pages.mtu.edu/~fmorriso/cm310/energy_equation.html

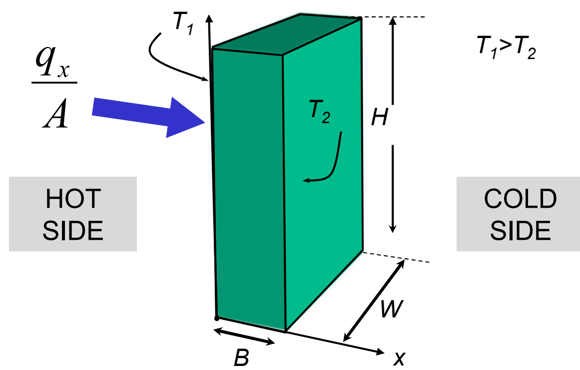
We will need **boundary conditions** on temperature

CM3110 REVIEW

Example 1: Heat flux in a rectangular solid – Temperature BC

What is the steady state temperature profile in a rectangular slab if one side is held at T_1 and the other side is held at T_2?

Assumptions:
•wide, tall slab
•steady state



15

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CM3110 REVIEW

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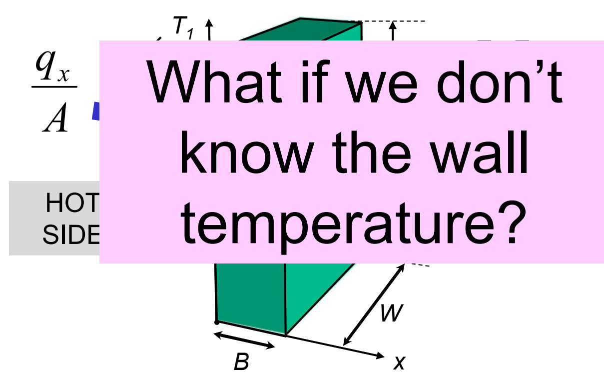
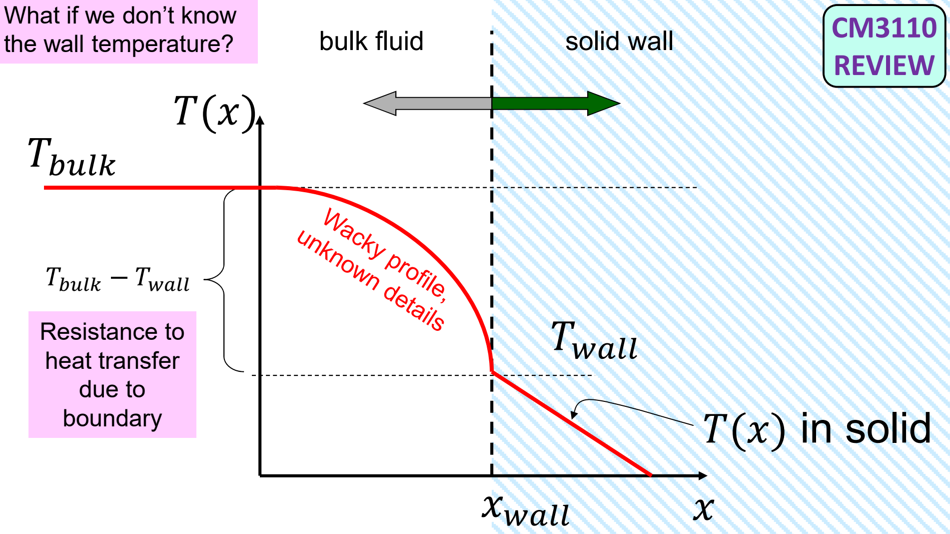


What if we don't know the wall temperature?

16

What if we don't know the wall temperature?

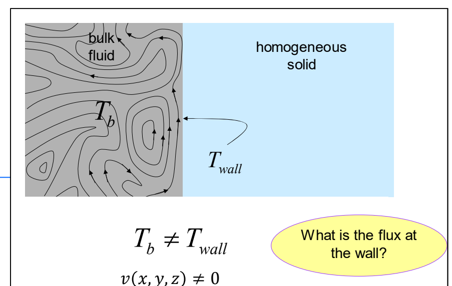
CM3110 REVIEW



The temperature difference at the fluid-wall interface is caused by complex phenomena that are lumped together into the heat transfer coefficient, *h*

17

The flux at the wall is given by the empirical expression known as **Newton's Law of Cooling**



This expression serves as the definition of the heat transfer coefficient.

$$\left|\frac{q_x}{A}\right| = h\left|T_{bulk} - T_{wall}\right|$$

Resistance to heat transfer due to boundary

***h* depends on:**

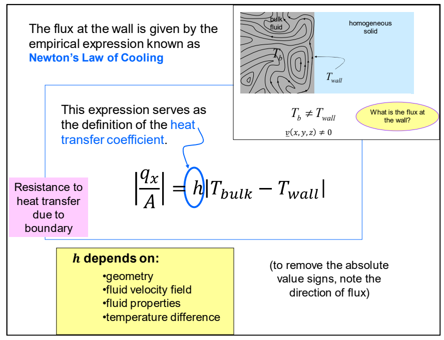
- geometry
- fluid velocity field
- fluid properties
- temperature difference

(to remove the absolute value signs, note the direction of flux)

18

Note:

Large values of h are associated with small $|T_{bulk} - T_{wall}|$



$$\left|\frac{q_x}{A}\right| = h|T_{bulk} - T_{wall}|$$

19

Newton's law of cooling BC

CM3110 REVIEW

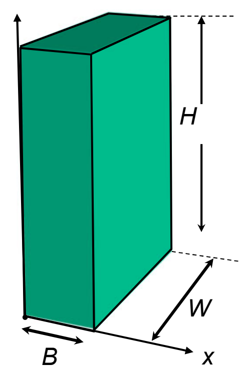
Example 2: Heat flux in a rectangular solid – Newton's law of cooling BC

What is the steady state temperature profile in a rectangular slab if the fluid on one side is held at T_{b1} and the fluid on the other side is held at T_{b2}?

Bulk temperature on left

T_{b1}

HOT SIDE



$T_1 > T_2$

T_{b2}

Bulk temperature on right

COLD SIDE

20

We will need **boundary conditions** on temperature

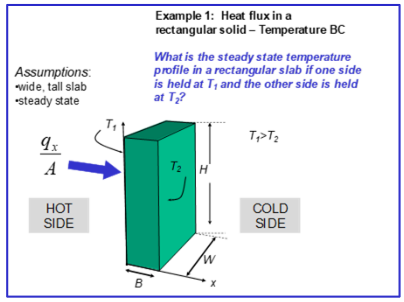
Commonly,

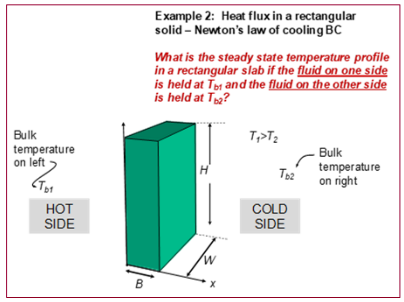
- Temperature boundary conditions
  $x = 0 \quad T = T_1$
  $x = B \quad T = T_2$

- Newton's law of cooling (flux) boundary conditions

$$\left.\frac{q_x}{A}\right|_{x=0} = h_1(T_{b1} - T(0))$$
$$\left.\frac{q_x}{A}\right|_{x=B} = h_1(T(B) - T_{b2})$$

Sometimes, we know a value for the flux at the wall.

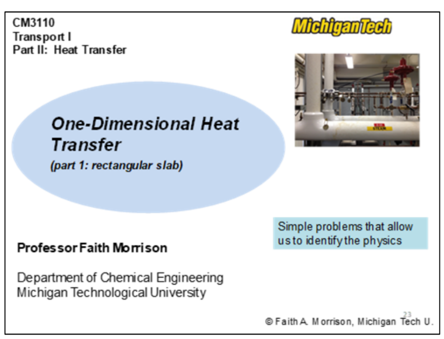




21

CM3110 REVIEW

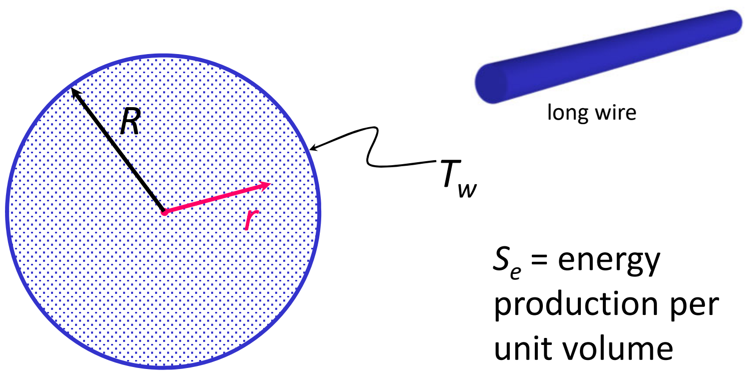
Let's carry out an example of 1D, steady heat transfer



22

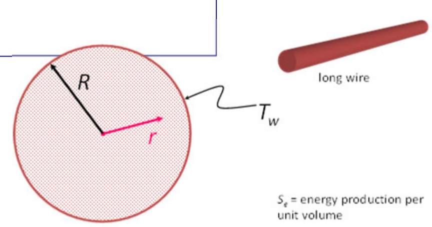
Example 3: Heat Conduction with Generation

What is the steady state temperature profile in a wire if heat is generated uniformly throughout the wire at a rate of S_e W/m^3 and the outer radius is held at T_w? What is the flux?



S_e = energy production per unit volume

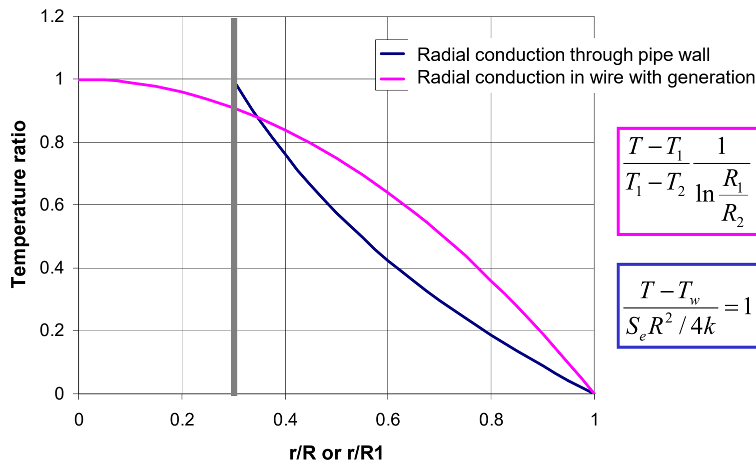
Example: Heat conduction with generation



You try.

24

Compare radial conduction solutions

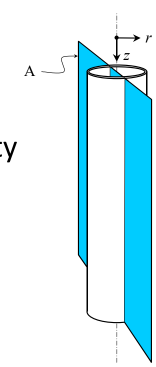


$$\frac{T-T_1}{T_1-T_2}\frac{1}{\ln\frac{R_1}{R_2}}=\ln\frac{r}{R_1}$$

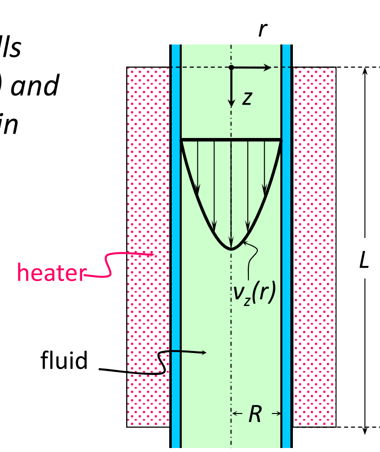
$$\frac{T-T_w}{S_eR^2/4k}=1-\left(\frac{r}{R}\right)^2$$

Example 4: Wall heating of laminar flow. *What is the steady state temperature profile in a flowing fluid in a tube if the walls are heated (constant flux, q_1/A) and if the fluid is a Newtonian fluid in laminar flow?*

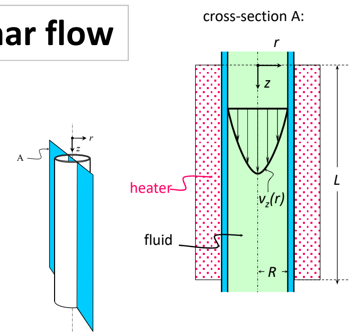
assume:
constant viscosity



cross-section A:



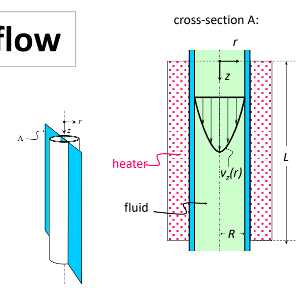
Example 4: Wall heating of laminar flow



You try.

27

Example 4: Wall heating of laminar flow



We need to solve this partial differential equation:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{\partial T}{\partial r}r\right)+\frac{\partial}{\partial z}\left(\frac{\partial T}{\partial z}\right)-\frac{\rho\hat{C}_p}{k}v_z(r)\frac{\partial T}{\partial z}=0$$

with

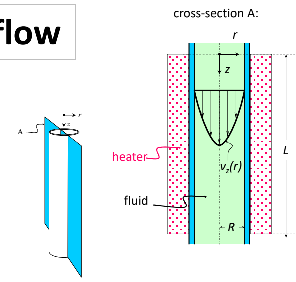
$$v_z=\frac{\Delta p}{4\mu L}(R^2-r^2)$$

and with the appropriate boundary conditions. To see the solution go to:

- R. Siegel, E. M. Sparrow, T. M. Hallman, *Appl. Science Research* A7, 386-392 (1958)
- R. B. Bird, W. Stewart, and E. Lightfoot (BSL), *Transport Phenomena*, Wiley, 1960, p295.

28

Example 4: Wall heating of laminar flow



We need to solve this partial differential equation:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{\partial T}{\partial r} r\right) + \frac{\partial}{\partial z}\left(\frac{\partial T}{\partial z}\right) - \frac{\rho \hat{C}_p}{k} v_z(r) \frac{\partial T}{\partial z} = 0$$

with

$$v_z = \frac{\Delta p}{4 \mu L}(R^2 - r^2)$$

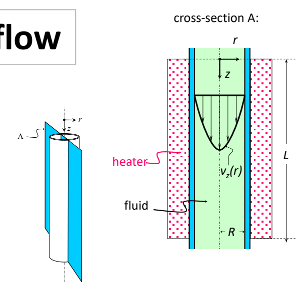
and with the appr ... ne solution go to:

What are the BCs?

- R. Siegel, E. M. ... earch A7, 386-392 (19
- R. B. Bird, W. Stewart, and E. Lightfoot (BSL), *Transport Phenomena*, Wiley, 1960, p295.

29

Example 4: Wall heating of laminar flow



Answer:
Neglect axial conduction

BC:

$$r = 0 \qquad T = \text{finite}$$

$$r = R \qquad \frac{q_r}{A} = \frac{q_1}{A}$$

$$z = 0 \qquad T = T_0$$

See BSL p295

30

1D, steady heat transfer

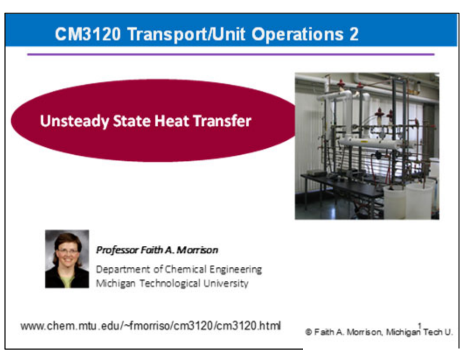
CM3110 REVIEW

SUMMARY

- Microscopic energy balance
- Transport law
- Newton's law of cooling (fluid boundary)
- Know the assumptions that simplify the model of the problem
- Solve with appropriate boundary conditions
- Check that assumptions are valid when using the solution

31

NEXT: Unsteady State Heat Transfer



32