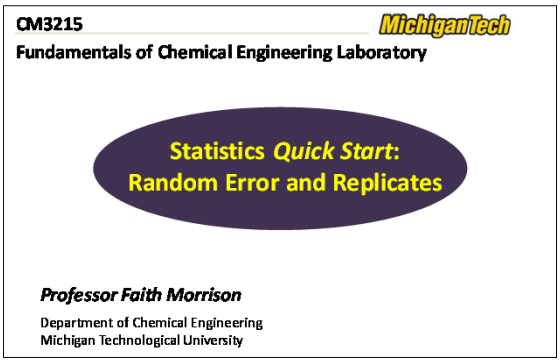
Where are we in our discussion of error analysis?

Let's revisit:



1

From Lecture 1: Quick Start, Replicate Errors:

Measurements are affected by errors

(uncertainty)

There are two general categories of errors (uncertainties) in experimental measurements:

- **Systematic errors**
- **Random errors**

2

From Lecture 1: Quick Start, Replicate Errors:

Measurements are affected by errors

Random errors (uncertainty)

1. Varies in sign and magnitude for identical conditions
2. May be due to the instrument or the process being measured
3. Must be understood and communicated with results

Sources:

Always present (need to minimize):
- Random process, instrument fluctuations
- Randomized systematic trends (e.g. operator identity, thermal drift)
- Rare events

Solutions:

Do:
- Replicate and average

Always an option:
- Improve measurement methods, practices
- Isolate from rare events

3

From Lecture 1: Quick Start, Replicate Errors:

Solution for Random Errors:

$\bar{x}$ **is a Good Estimate of** x

Use $t_{0.025,n-1}$ for $n < 7$)

Sample mean

$$E(x) = \bar{x} \pm 2e_s \text{ with 95\% confidence}$$

Expected value of x

Standard Error. If only random error present:

$$e_s = \frac{s}{\sqrt{n}}$$

Sample standard deviation

1. Minimize whatever is causing random errors
2. Replicate, average, construct 95% CI of mean

DONE…

4

But, more than random errors are present

NOT ~~DONE~~

Systematic Errors →

5

From Lecture 1: Quick Start, Replicate Errors:

Measurements are affected by errors

Systematic errors (uncertainty)

1. Has same sign and magnitude for identical conditions
2. Must be checked for, identified, eliminated, randomized

Sources:

- Calibration of instruments
- Reading error (resolution, coarse scale)
- Consistent operator error
- Failure to produce experimentally conditions assumed in an analysis (e.g. steady state, isothermal, well mixed, pure component, etc.)

Solutions:

- Recalibrate
- Improve instrument resolution
- Apply correction for identified error
- Improve procedures, experimental design
- Shift to other methods
- Take data in random order; rotate operators

6

From Lecture 1: Quick Start, Replicate Errors:

Measurements are affected by errors

Systematic errors

(uncertainty)

1. Has same sign and magnitude for identical conditions
2. Must be checked for, identified, eliminated, randomized

Sources:

- Calibration of instruments
- Reading error (resolution, coarse scale)
- Consistent operator error
- Failure to produce experimentally conditions assumed in an analysis (e.g. steady state, isothermal, well mixed, pure component, etc.)

Mistakes (need to fix)

Solutions:

Always an option:
- Recalibrate
- Improve instrument resolution
- Apply correction for identified error
- Improve procedures, experimental design
- Shift to other methods

Do:
- Take data in random order; rotate operators

7

From Lecture 1: Quick Start, Replicate Errors:

Measurements are affected by errors

Systematic errors

(uncertainty)

1. Has same sign and magnitude for identical conditions
2. Must be checked for, identified, eliminated, randomized

Sources:

Unavoidable:
- Calibration of instruments

Unavoidable:
- Reading error (resolution, coarse scale)

- Consistent operator error
- Failure to produce experimentally conditions assumed in an analysis (e.g. steady state, isothermal, well mixed, pure component, etc.)

Mistakes (need to fix)

Solutions:

Always an option:
- Recalibrate
- Improve instrument resolution
- Apply correction for identified error
- Improve procedures, experimental design
- Shift to other methods

Do:
- Take data in random order; rotate operators

8

Measurements are affected by errors (uncertainty)

We have identified three sources of standard error:

- **Random errors (replicate error)**
- **Reading errors**
- **Calibration errors**

$e_s = \frac{s}{\sqrt{n}}$	Standard error of replicates
$e_s = ?$	Standard reading error
$e_s = ?$	Standard calibration error

The techniques developed to understand and report **replicate error** can be the template that we use to account for the other two sources of uncertainty.

9

Measurements are affected by errors (uncertainty)

We have identified three sources of standard error:

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- **Reading errors**
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The techniques developed to understand and report **replicate error** can be the template that we use to account for the other two sources of uncertainty.

10

Measurements are affected by errors (uncertainty)

We have identified three sources of standard error:

First:

- Random errors (replicate error)
- Reading errors
- Calibration errors

$e_s = \frac{s}{\sqrt{n}}$	Standard error of replicates
$e_s = ?$	Standard reading error
$e_s = ?$	Standard calibration error

The techniques developed to understand and report **replicate error** can be the template that we use to account for the other two sources of uncertainty.

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CM3215

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Fundamentals of Chemical Engineering Laboratory

Statistics Lecture 2: Reading Error

Professor Faith Morrison

Department of Chemical Engineering
Michigan Technological University

1. Quick start—Replicate error
2. **Reading Error**
3. Calibration Error
4. Error Propagation
5. Least Squares Curve Fitting

12

Obtaining a Good Estimate of Precision

What is the Standard Error of a Measurement?

$e_s \equiv$ Standard Error

Part 2: Reading Errors

Three sources:

- **Replicate errors**
- **Reading errors**
- **Calibration errors**

Second source of standard error…

Obtaining a Good Estimate of **Reading Error**

Sometimes a measurement is very reproducible (negligible replicate error) but there is still error/uncertainty inherent in how the reading is taken.

Example: Digital Multimeter reading a 4-20mA instrument signal

Image from: appliancerepairstartup.com/wp-content/uploads/2010/12/220px-Digital_Multimeter_Aka.jpg

Second source of standard error…

Obtaining a Good Estimate of **Reading Error**

Systematic errors due to **Reading errors**, include:

1. Limits of instrument sensitivity (i.e. the magnitude of change required for the instrument to respond)
2. Limits of the degree of subdivision of the scale or display
3. Fluctuations of an instrument reading

We take each in turn→

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Reading Error Worksheet
CM3215 Fundamentals of Chemical Engineering Lab
Prof. Faith Morrison

Michigan Technological University
Department of Chemical Engineering

This worksheet guides the user through the calculation of the standard error and 95% confidence scale or off a digital readout (yielding value X and subject to reading error). The reading-error-related standard error e_s may subsequently be used in propagation of error calculations of derived quantities.

		Reading error		
	Measured Quantity: (give symbol)			
	Representative value:	(include units)		Quantity or Not Applicable
	issue	**contribution to error**		
Reading error, e_R:	Resolution	How much signal does it take to cause the reading to change?	1	
	Limitation on marked scale or digital readout	Half smallest division or decimal place	2	
	Fluctuations with time of observation	(max-min)/2	3	
		Maximum of 1, 2 & 3:	e_R=	(units)
	Standard error based on reading error:	$e_s=e_R/\sqrt{3}$	e_s=	
		95% Confidence Interval on the reading: $\pm 2e_s$		

Handy worksheet for reading error

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www.chem.mtu.edu/~fmorriso/cm3215/ReadingErrorWorksheet.pdf

Second source of standard error…

Obtaining a Good Estimate of **Reading Error**

Systematic errors due to **Reading errors**, include:

1. Limits of instrument sensitivity (i.e. the magnitude of change required for the instrument to respond)

How to determine?

At the low range of operation, test how much signal must be received in order for the reading to change.

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Second source of standard error…

Obtaining a Good Estimate of **Reading Error**

Systematic errors due to **Reading errors**, include:

2. Limits of the degree of subdivision of the scale or display

How to determine?

Estimate as
- *½ the smallest subdivision on the scale or*
- *½ the smallest digit on a digital readout*

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Second source of standard error…

Obtaining a Good Estimate of **Reading Error**

Systematic errors due to **Reading errors**, include:

3. Fluctuations of an instrument reading

How to determine?

Estimate as: $\frac{1}{2}(x_{max} - x_{min})$ *over an interval*

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One source of standard error…

Obtaining a Good Estimate of **Reading Error**

Possible reading errors:

- Determine the limits of instrument sensitivity (magnitude of change required for instrument to respond)
- Determine ½ the smallest subdivision of the scale or display
- Determine $\frac{1}{2}(x_{max} - x_{min})$ for time-fluctuating data
- Designate the reading error:

e_R = maximum of the possible reading errors

- Calculate the Standard error of Reading Error:

$$e_s = \frac{e_R}{\sqrt{3}}$$ Standard error based on Reading Error

20

One source of standard error…

Obtaining a Good Estimate of **Reading Error**

- Determine the limits of instrument sensitivity (magnitude of change required for instrument to respond)
- Determine ½ the smallest subdivision of the scale or display
- Determine $\frac{1}{2}(x_{max} - x_{min})$ for time-fluctuating data
- Designate the reading error:

e_R = maximum of the possible reading errors

- Calculate the Standard error of Reading Error:

$$e_s = \frac{e_R}{\sqrt{3}}$$ Standard error based on Reading Error

21

Where does that come from?

$$e_s = \frac{e_R}{\sqrt{3}}$$ **Where does that come from?**

We have identified three sources of standard error:

$e_s = \frac{s}{\sqrt{n}}$	Standard error of replicates
$e_s = ?$	Standard reading error
$e_s = ?$	Standard calibration error

We seek to write each in an equivalent form, so that we can see which is dominant.

How to proceed?

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$e_s = \frac{e_R}{\sqrt{3}}$ **Where does that come from?**

Consider: $e_s = \frac{s}{\sqrt{n}}$ Standard error of replicates

$e_s^2 = \frac{s^2}{n}$ **Variance** of n samples, each with individual sample variance s^2

The **variance** is a well defined statistic, designed to measure the *spread* of individual outcomes around the *mean* outcome.

$$x_1, x_2, x_3, x_4, x_5 \ldots x_n$$

$$\bar{x} \equiv \frac{1}{n}\sum_{i=1}^{n} x_i$$

$$s^2 \equiv \frac{1}{n-1}\sum_{i=1}^{n} (x_i - \bar{x})^2$$

We can calculate a **variance** associated with reading error by considering the distribution of individual readings around the mean reading.

23

$e_s = \frac{e_R}{\sqrt{3}}$ **Where does that come from?**

Consider the reading on a digital multimeter (DMM):

For a DMM meter reading (mA) of :

6.7 mA

6.65
6.66
6.67
6.68
6.69
6.70
6.71
6.72
6.73
6.74

A reading of 6.7 may correspond to any of these more precise numbers with equal probability.

½ the smallest subdivision of the scale or display:

$e_R = 0.05$

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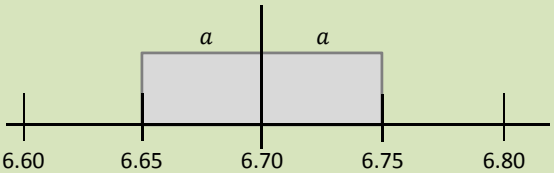
Image from: appliancerepairstartup.com/wp-content/uploads/2010/12/220px-Digital_Multimeter_Aka.jpg

$e_s = \frac{e_R}{\sqrt{3}}$ **Where does that come from?**

6.7 mA

$e_R = 0.05 = a$

Rectangular probability distribution of width a:



A reading of 6.7 may correspond to any of these more precise numbers with equal probability.

The **variance** of the rectangular probability distribution is $\sigma^2 = a^2/3$

(can show mathematically; see literature)

$$e_S = \frac{e_R}{\sqrt{3}}$$

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Measurements are affected by errors (uncertainty)

We have identified three sources of standard error:

- **Random errors (replicate error)**
- **Reading errors**
- **Calibration errors**

Now we know how to compute two out of three measurement steandard errors.

$e_s = \frac{s}{\sqrt{n}}$	Standard error of replicates
$e_s = \frac{e_R}{\sqrt{3}}$	Standard reading error
$e_s = ?$	Standard calibration error

For all three types of errors, we write a **variance**.

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EXAMPLE 1 For a 50ml beaker weighed with the CM3215 laboratory analytical balance, what is the weight and the 95% confidence interval on the weight based on reading error?

You try.

34.4081 g

Image from:
www.coleparmer.com

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Department of Chemical Engineering

This worksheet guides the user through the calculation of the standard error and 95% confidence scale or off a digital readout (yielding value X and subject to reading error). The reading-error-related standard error e_s may subsequently be used in propagation of error calculations of derived quantities.

		Reading error		
	Measured Quantity: (give symbol)	Mass, M		
	Representative value:	(include units) 32.4081 *g*		Quantity or Not Applicable
	issue	contribution to error		
Reading error, e_R:	Resolution	How much signal does it take to cause the reading to change?	1	
	Limitation on marked scale or digital readout	Half smallest division or decimal place	2	
	Fluctuations with time of observation	(max-min)/2	3	
		Maximum of 1, 2 & 3:	e_R=	(units)
	Standard error based on reading error:	$e_s=e_R/\sqrt{3}$	e_s=	
		95% Confidence Interval on the reading: $\pm 2e_s$		

www.chem.mtu.edu/~fmorriso/cm3215/ReadingErrorWorksheet.pdf

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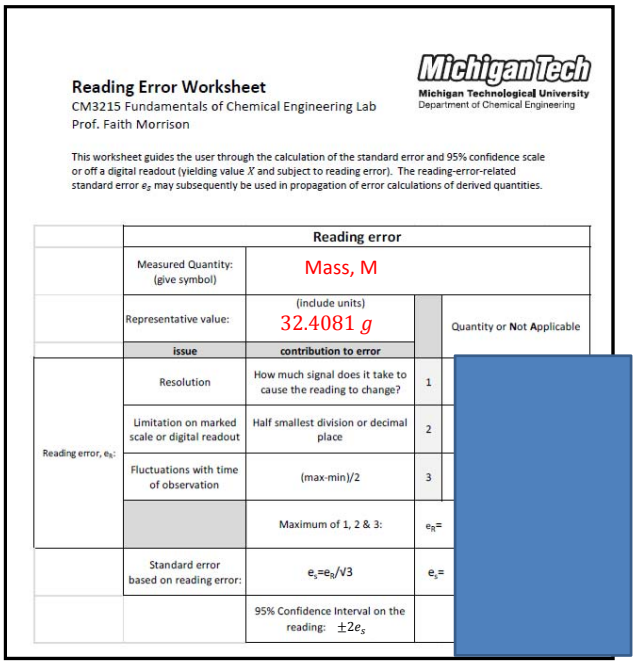

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		Reading error		
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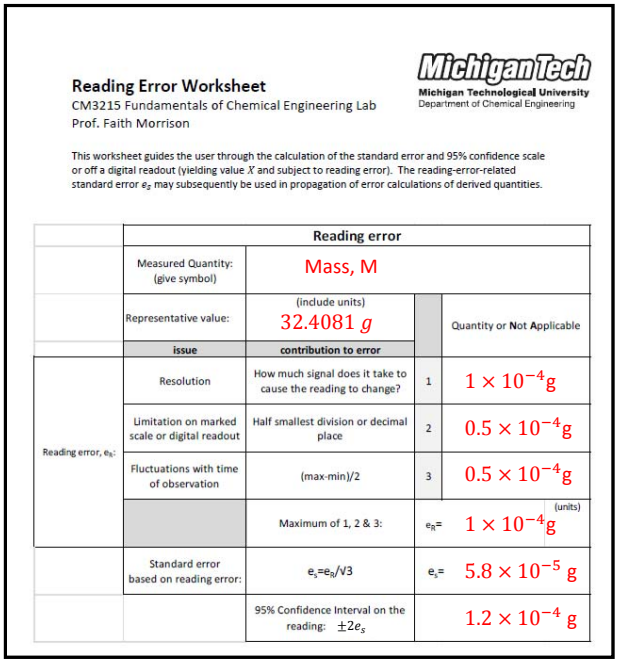
	Representative value:	(include units) 32.4081 g		Quantity or Not Applicable
	issue	contribution to error		
Reading error, e_R:	Resolution	How much signal does it take to cause the reading to change?	1	
	Limitation on marked scale or digital readout	Half smallest division or decimal place	2	
	Fluctuations with time of observation	(max-min)/2	3	
		Maximum of 1, 2 & 3:	e_R=	
	Standard error based on reading error:	$e_s=e_R/\sqrt{3}$	e_s=	
		95% Confidence Interval on the reading: $\pm 2e_s$		

www.chem.mtu.edu/~fmorriso/cm3215/ReadingErrorWorksheet.pdf

29



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		Reading error		
	Measured Quantity: (give symbol)	Mass, M		
	Representative value:	(include units) 32.4081 g		Quantity or Not Applicable
	issue	contribution to error		
Reading error, e_R:	Resolution	How much signal does it take to cause the reading to change?	1	1×10^{-4}g
	Limitation on marked scale or digital readout	Half smallest division or decimal place	2	0.5×10^{-4}g
	Fluctuations with time of observation	(max-min)/2	3	0.5×10^{-4}g
		Maximum of 1, 2 & 3:	e_R=	1×10^{-4}g (units)
	Standard error based on reading error:	$e_s=e_R/\sqrt{3}$	e_s=	5.8×10^{-5} g
		95% Confidence Interval on the reading: $\pm 2e_s$		1.2×10^{-4} g

www.chem.mtu.edu/~fmorriso/cm3215/ReadingErrorWorksheet.pdf

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EXAMPLE 1 For a 50ml beaker weighed with the CM3215 laboratory analytical balance, what is the weight and the 95% confidence interval on the weight based on reading error?

Image from:
www.coleparmer.com

Answer:

$$1.4 \times 10^{-4}\ g$$

EXAMPLE 2 For height of an object measured with a meter stick as shown, what is the value and a 95% confidence interval on the height based on reading error?

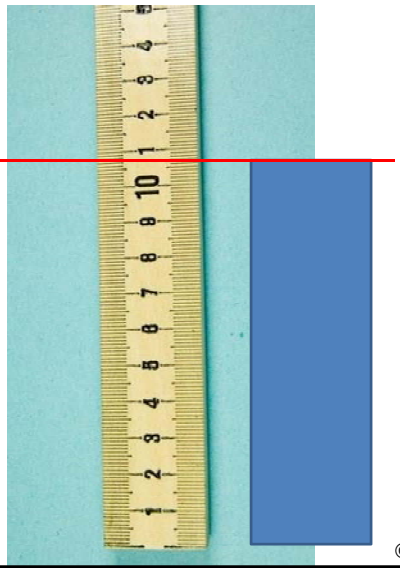
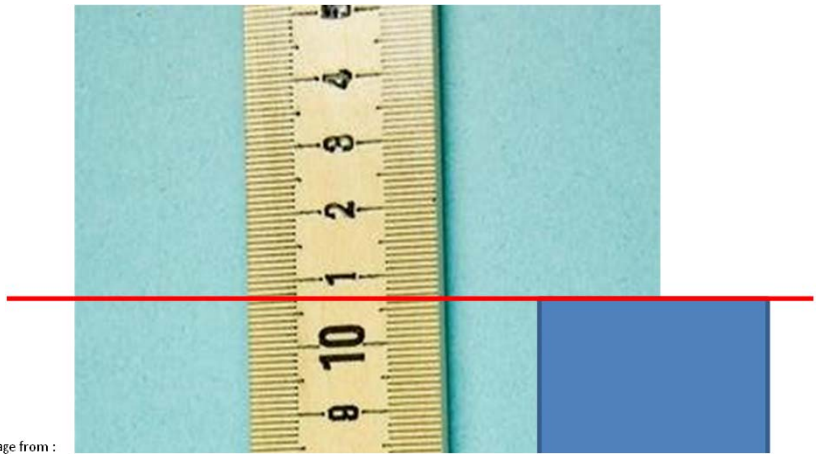


Image from :
www.martinaknezevic.com/events/full-sail-university-course-overview/

You try.

EXAMPLE 2 For height of an object measured with a meter stick as shown, what is the value and a 95% confidence interval on the height based on reading error?



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		Reading error		
	Measured Quantity: (give symbol)	Height, h		
	Representative value:	(include units) 10.63 *cm*		Quantity or Not Applicable
	issue	contribution to error		
Reading error, e_R:	Resolution	How much signal does it take to cause the reading to change?	1	
	Limitation on marked scale or digital readout	Half smallest division or decimal place	2	
	Fluctuations with time of observation	(max-min)/2	3	
		Maximum of 1, 2 & 3:	e_R=	
	Standard error based on reading error:	$e_s=e_R/\sqrt{3}$	e_s=	
		95% Confidence Interval on the reading: $\pm 2e_s$		

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		Reading error		
	Measured Quantity: (give symbol)	Height, h		
	Representative value:	(include units) $10.63\ cm$		Quantity or Not Applicable
	issue	contribution to error		
Reading error, e_R:	Resolution	How much signal does it take to cause the reading to change?	1	$0.03\ cm$
	Limitation on marked scale or digital readout	Half smallest division or decimal place	2	$0.05\ cm$
	Fluctuations with time of observation	(max-min)/2	3	n/a
		Maximum of 1, 2 & 3:	e_R=	$0.05\ mm$ (units)
	Standard error based on reading error:	$e_s=e_R/\sqrt{3}$	e_s=	$0.029\ mm$
		95% Confidence Interval on the reading: $\pm 2e_s$		$0.058\ mm$

(at the most optimistic)

www.chem.mtu.edu/~fmorriso/cm3215/ReadingErrorWorksheet.pdf

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EXAMPLE 2 For height of an object measured with a meter stick as shown, what is the value and a 95% confidence interval on the height based on reading error?

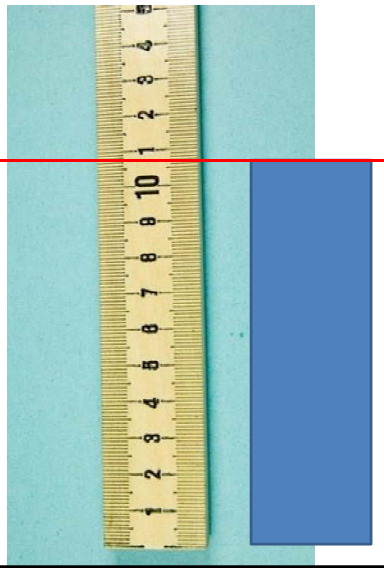

Image from : www.martinaknezevic.com/events/full-sail-university-course-overview/

Answer:

$10.63 \pm 0.06\ cm$

(at the most optimistic)

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Obtaining a Good Estimate of a Quantity

Summary:

Replicate (random) error:

- Measure the quantity several times – replicates
- The average value is a good estimate of the quantity we are measuring if only random errors are present
- The 95% confidence interval comes from $\pm(**)e_s$
- $(**) = 2$ if the number of replicates is 7 or higher
- $(**)$ comes from the Student's t distribution if $N < 7$
- Report one sig fig on error (unless that digit is 1 or 2)

Reading error:

- Determine signal needed to change reading
- Determine half smallest division or decimal place
- Determine average of fluctuations
- Max of those $/\sqrt{3}$ =reading standard error
- use $\pm 2e_s$ for 95% confidence interval

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Measurements are affected by errors (uncertainty)

We have identified three sources of standard error:

- **Random errors (replicate error)**
- **Reading errors**
- **Calibration errors**

Next:

$e_s = \frac{s}{\sqrt{n}}$	Standard error of replicates
$e_s = \frac{e_R}{\sqrt{3}}$	Standard reading error
$e_s = ?$	Standard calibration error

For all three types of errors, we write a **variance**.

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Next: Calibration Errors

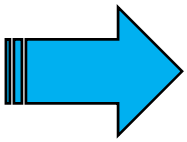
Obtaining a Good Estimate of Precision

What is the Standard Error of a Measurement?

$e_s \equiv$ Standard Error

Part 3: Calibration Errors

Three sources:
- Replicate errors
- Reading errors
- Calibration errors



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