Error Propagation Worksheet

Faith A. Morrison

Uncertainty Analysis for Engineers and Scientists: A Practical Guide
(Cambridge University Press, 2020)

This worksheet guides the user through the determination of the standard error e_{s_ϕ} of a quantity $\phi(x_1, x_2, x_3, x_4, x_5)$ that is calculated from measured quantities x_1, x_2, x_3, x_4 and x_5. The x_i are subject to uncertainties. The standard error e_{s,x_i} (replicate, reading, calibration; combine in quadrature, if present) for each variable x_i is determined first, and these uncertainties are propagated to determine e_{s_ϕ} using the relationship given below.

$\phi(x_1, x_2, x_3, x_4, x_5)$:	**Formula for ϕ:*** **Note: units must work as written.*	**Representative value of ϕ: (include units)**	**95% level of confidence: $(\phi \pm 2e_{s,\phi})$ (include units)**

Measured quantities, x_i			$\frac{\partial \phi}{\partial x_i}$	e_{s,x_i}	$\left(\frac{\partial \phi}{\partial x_i}\right)^2 e_{s,x_i}^2$
x_i	**Symbol**	**Representative value**			
x_1		units			
x_2		units			
x_3		units			
x_4		units			
x_5		units			

$$e_{s_\phi}^2 = \left(\frac{\partial \phi}{\partial x_1}\right)^2 e_{x_1}^2 + \left(\frac{\partial \phi}{\partial x_2}\right)^2 e_{x_2}^2 + \left(\frac{\partial \phi}{\partial x_3}\right)^2 e_{x_3}^2 + \left(\frac{\partial \phi}{\partial x_4}\right)^2 e_{x_4}^2 + \left(\frac{\partial \phi}{\partial x_5}\right)^2 e_{x_5}^2$$

** All variables x_i must be independent for this equation to hold.*

$e_{s,\phi}^2 =$

$e_{s,\phi} =$ **units**

Standard error of calculated quantity, ϕ

Note: For some quantities, you will look up the uncertainty; for example, the volume of a volumetric flask may be given as $100.00 \pm 0.04ml$. In these circumstances it is reasonable to assume that the reported uncertainty is $\pm 2e_s$. For example, if volume is given as $100.00 \pm 0.04ml$, then $2e_s = 0.04ml$. Ref.: p. 564 of Fritz and Schenk, *Quantitative Analytical Chemistry*, Allyn & Bacon, Boston, 1987.