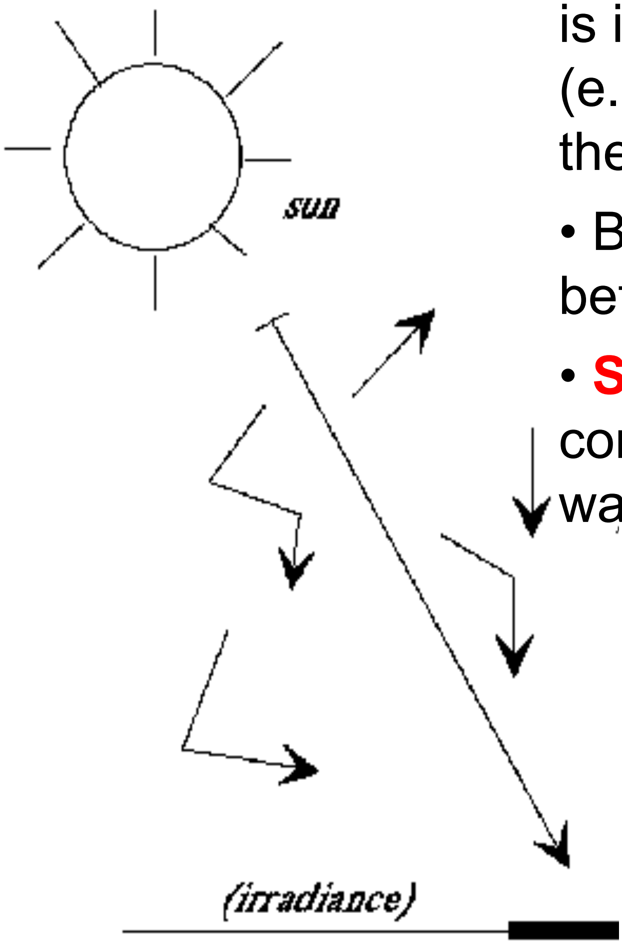
# Properties of Radiation

## Lecture outline

- Flux and intensity
- Solid angle and the steradian
- Inverse square law
- Global insolation
- Interaction of radiation with matter

# Flux or Flux density

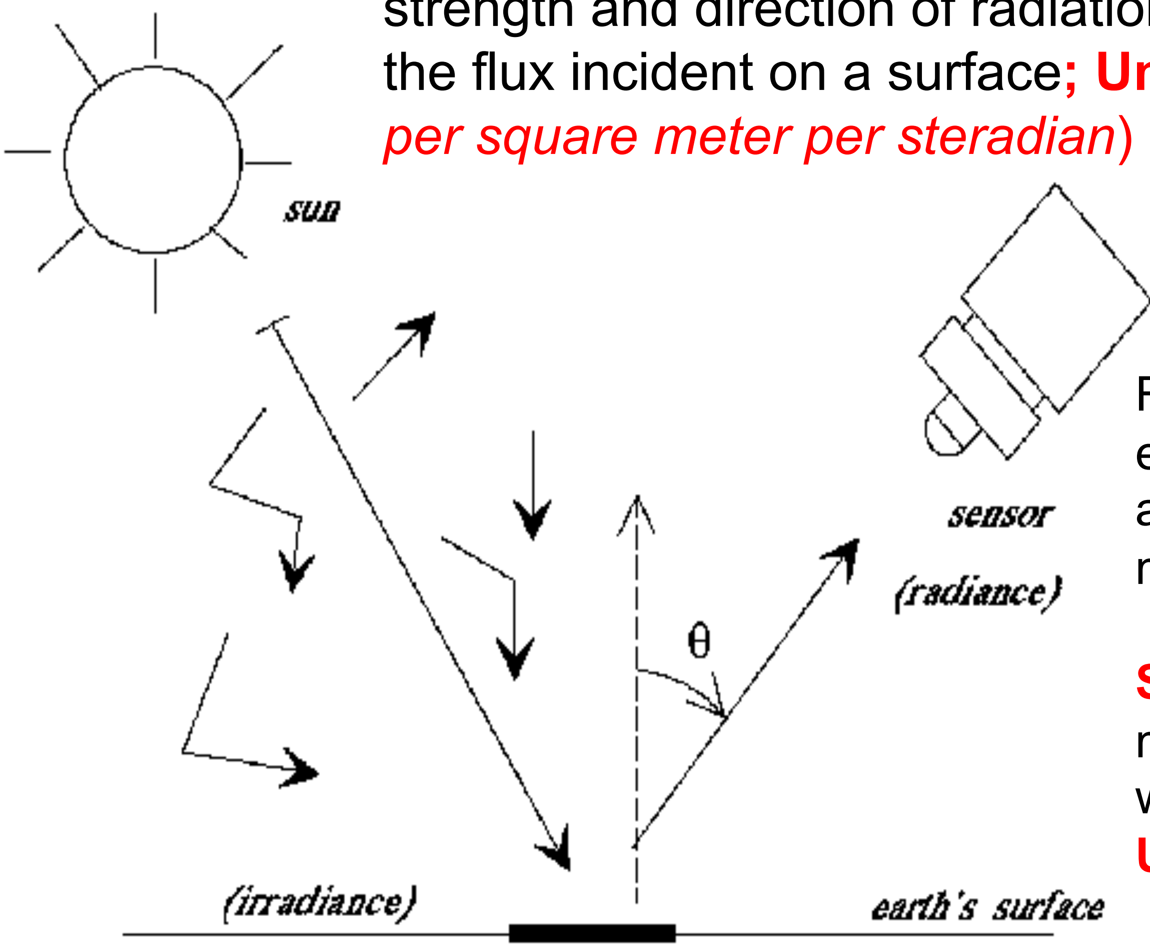



- **Flux (or flux density), F**: rate at which radiation is incident on, or passes through, a flat surface (e.g., the ground, the top of a cloud layer, a level in the atmosphere…)**; Units: W m$^{-2}$**
- By definition, a *broadband* quantity integrated between wavelength limits ($\lambda_1$ and $\lambda_2$)
- **Spectral flux (or monochromatic flux )**: flux contributed by radiation over some narrow wavelength interval**; Units: W m$^{-2}$ μm$^{-1}$**

- e.g., the incident flux of solar radiation on an area of Earth's surface
- No information on direction of origin

# Intensity or radiance

- **Radiant intensity or radiance, I**: describes both the strength and direction of radiation sources contributing to the flux incident on a surface; **Units: W $m^{-2}$ $sr^{-1}$** (*Watts per square meter per steradian*)



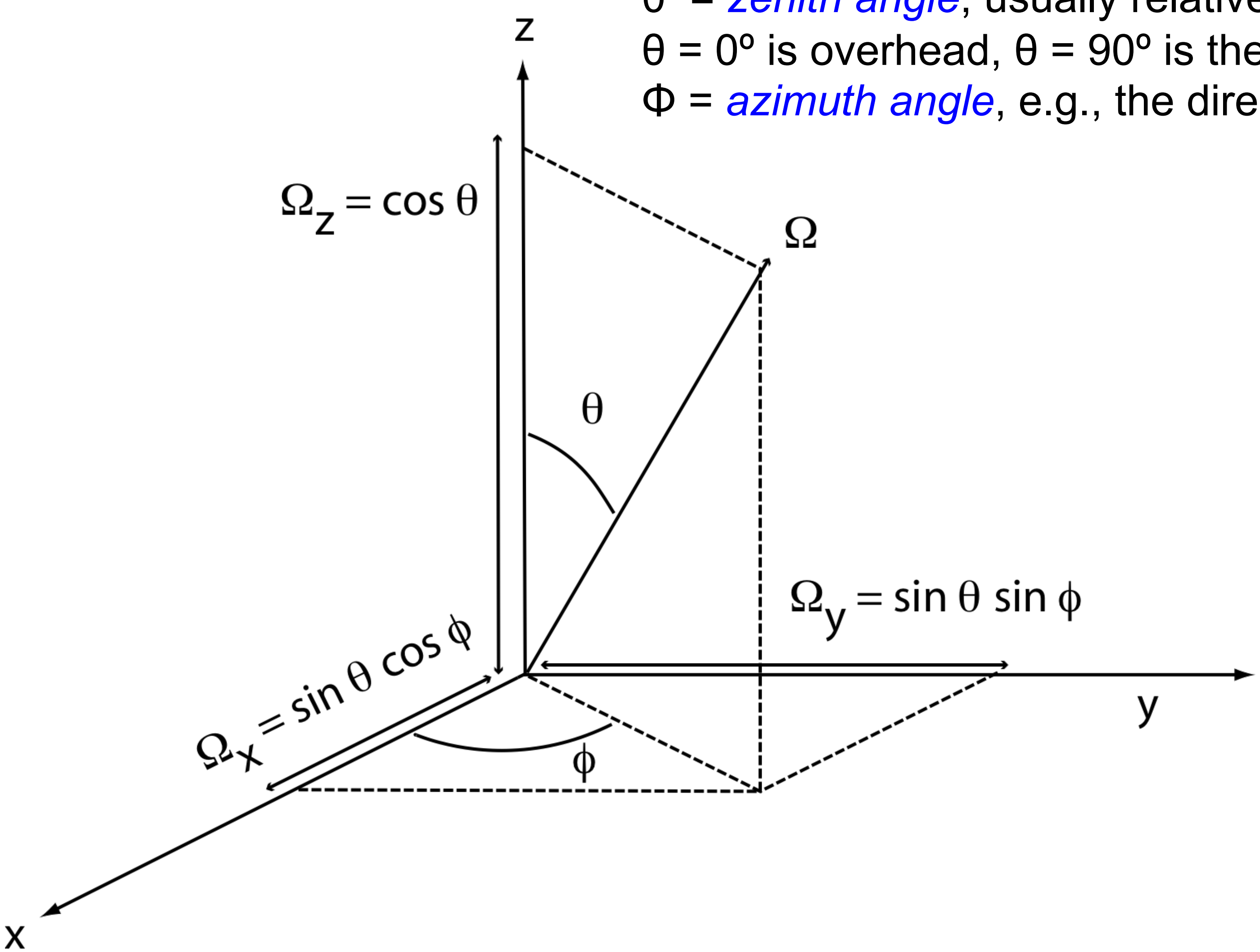

Remote sensing instruments, e.g., a satellite sensor viewing a narrow range of directions, measure *radiance*

**Spectral radiance**: intensity measured over some narrow wavelength interval; **Units: W $m^{-2}$ $sr^{-1}$ $\mu m^{-1}$**

- Roughly corresponds to 'brightness' of a radiation source – e.g., the sky, clouds, the Sun.

# Spherical polar coordinates

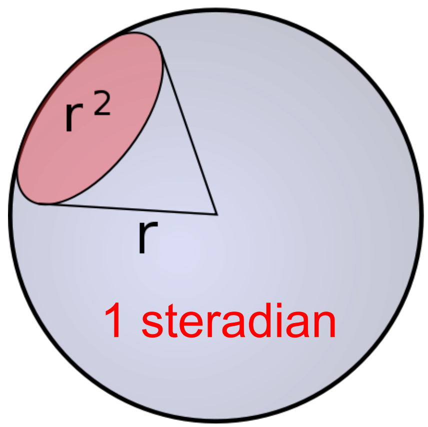
θ = *zenith angle*, usually relative to local vertical
θ = 0º is overhead, θ = 90º is the horizon.
Φ = *azimuth angle*, e.g., the direction of the sun

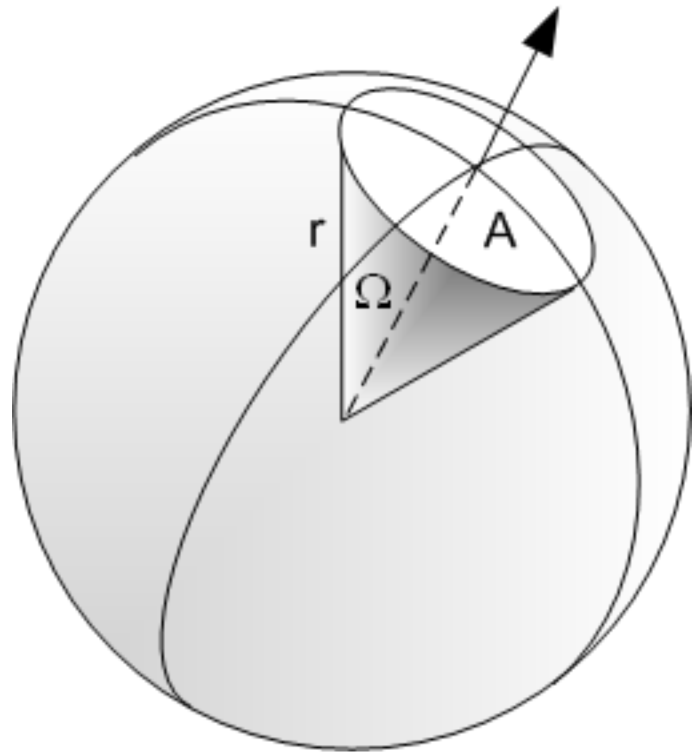



- **Direction plays an important role in any discussion of radiation**

# Solid angle: the steradian

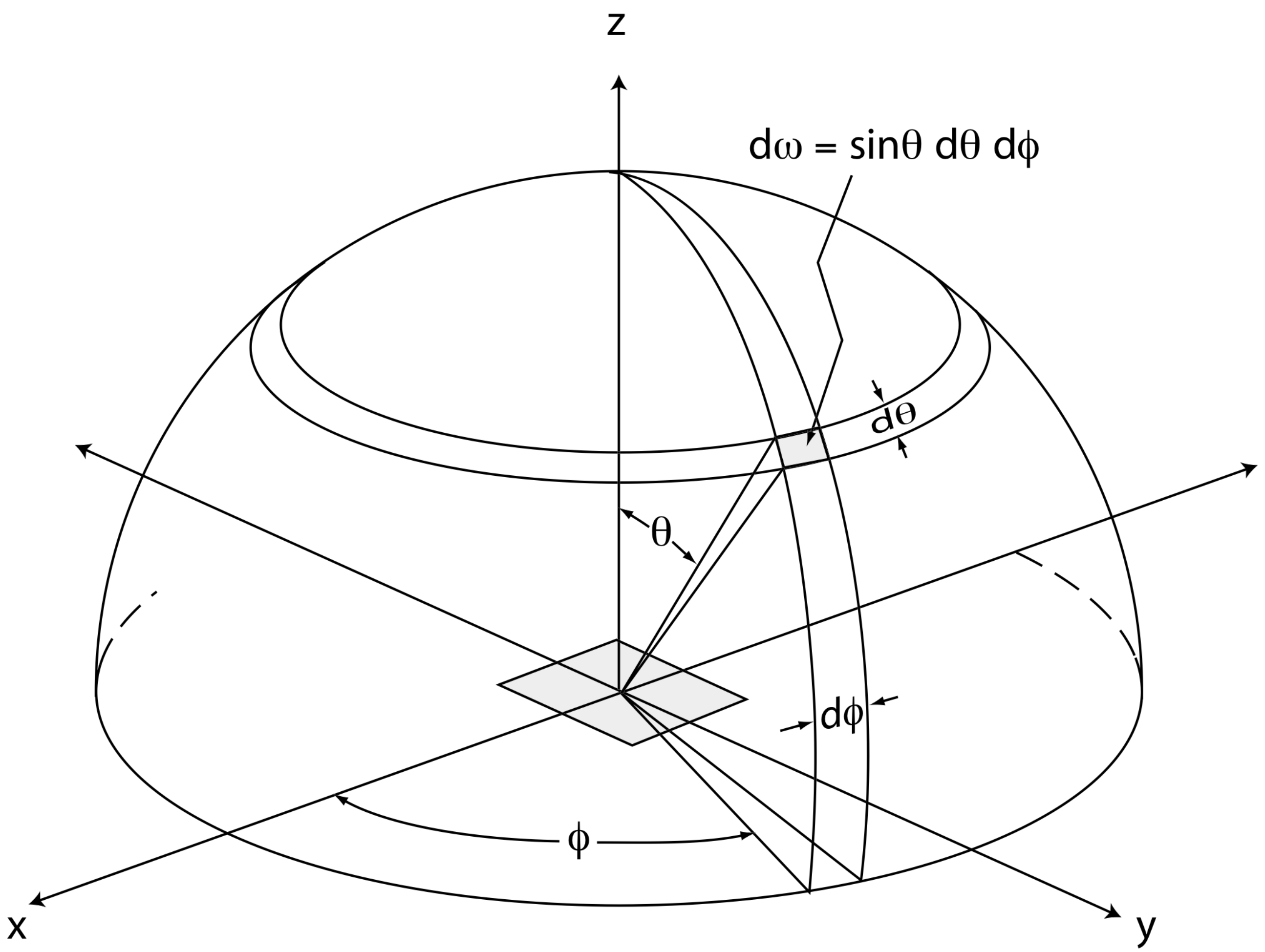
*How much of the visual field of view is occupied by an object?*




- Three-dimensional analog of the planar radian (ratio of arc length:radius)
- Ω = area of surface A / $r^2$ (NB. *Solid angle is dimensionless*)
- Sphere subtends a solid angle of … sr
- The entire sky (hemisphere) above the horizon subtends … sr
- Sun seen from Earth covers ~0.00006 sr

# Solid angle: the steradian

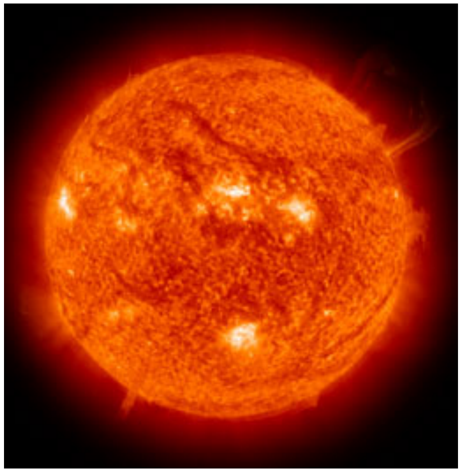



- *sin θ* accounts for the convergence of 'longitude' lines at the 'pole'

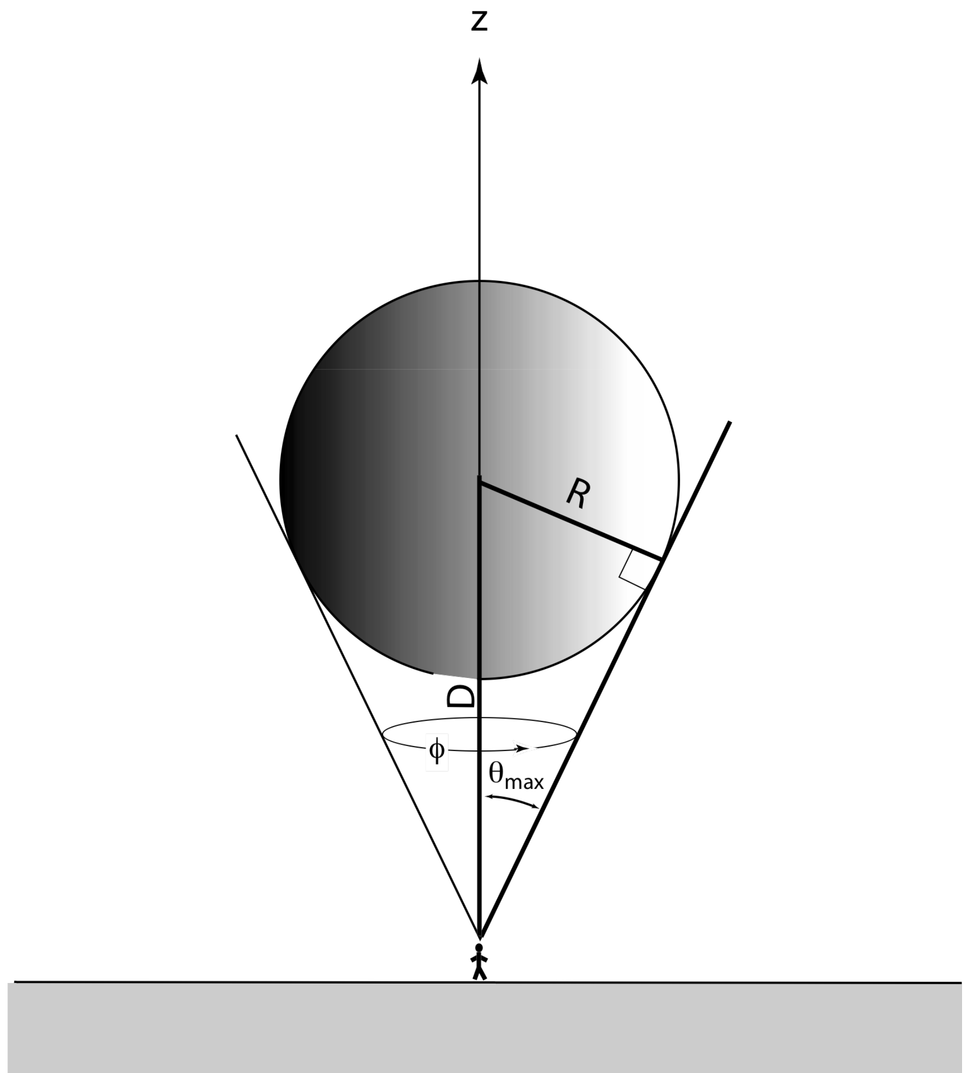
$$\int_{4\pi} d\omega = \int_0^{2\pi}\int_0^{\pi} \sin\theta \, d\theta \, d\phi = 2\pi \int_0^{\pi} \sin\theta \, d\theta = 4\pi$$

# Solid angle problem

- Mean distance of the moon from Earth = $3.84 \times 10^5$ km
- Radius of the moon = $1.74 \times 10^3$ km
- Mean distance of the Sun from Earth = $1.496 \times 10^8$ km
- Radius of the Sun = $6.96 \times 10^5$ km
- What is the angular diameter subtended by the Sun and moon?
- What is the solid angle subtended by the Sun and moon?
- Which appears larger from the Earth?

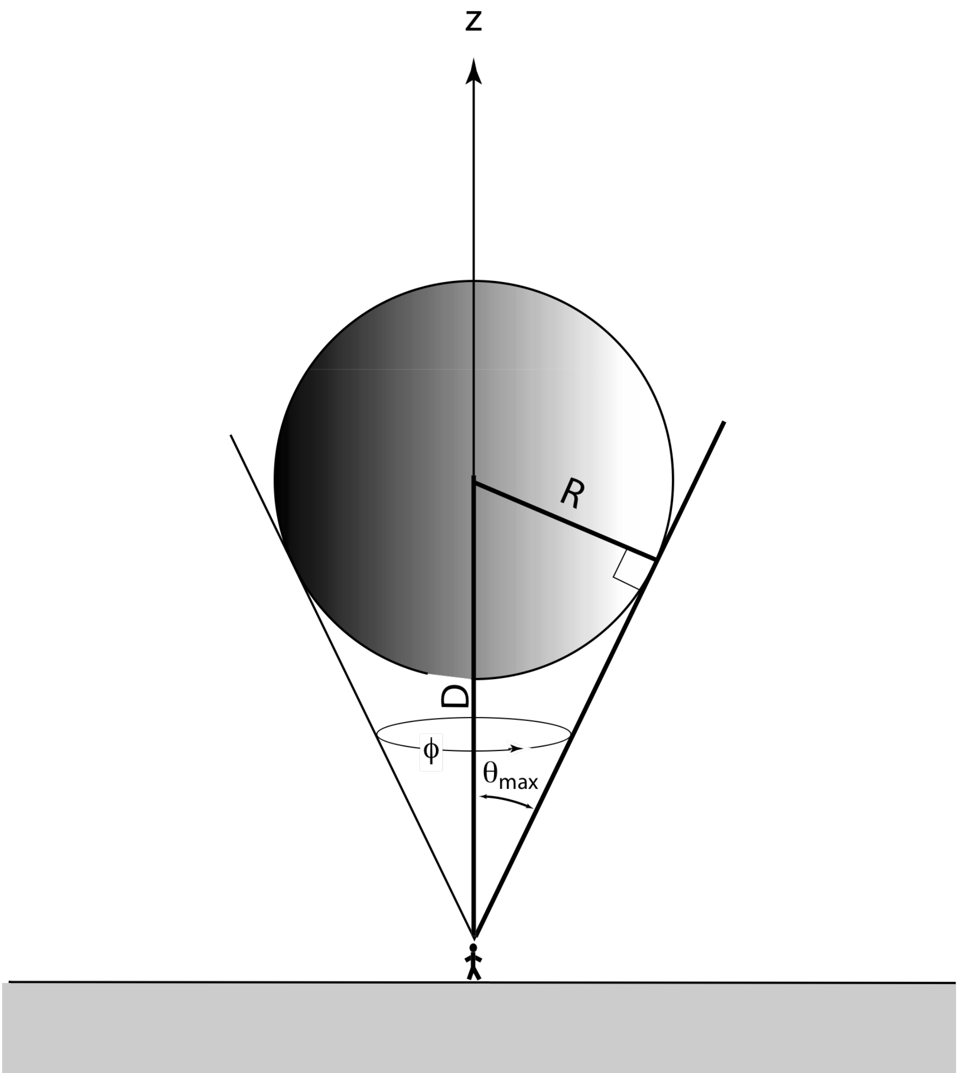


# Solid angle problem




Geometric framework for calculating the solid angle subtended by a sphere of radius *R* whose center is a distance *D* from the observer

# Solid angle problem




(a) $\theta_{max} = 2 \sin^{-1} (R/D)$; Moon = 0.52º

Sun = 0.53º

(b) Solid angle of a cone with apex angle 2θ:

$$\int_0^{2\pi}\int_0^{\theta} \sin\theta \, d\theta \, d\phi = 2\pi \int_0^{\theta} \sin\theta \, d\theta$$

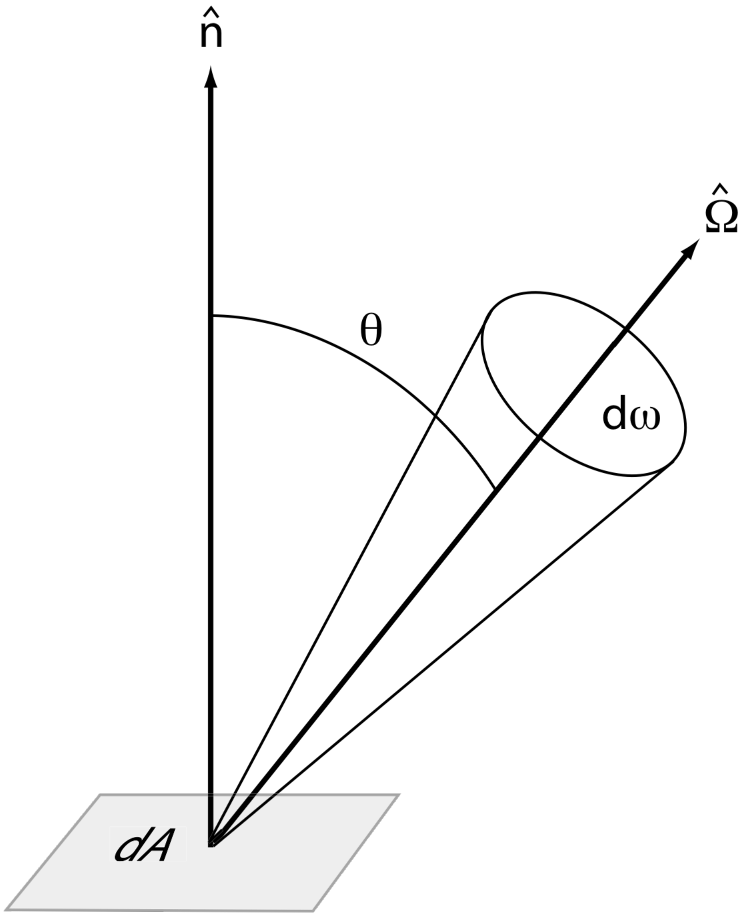
$$= 2\pi\left[-\cos\theta\right]_0^{\theta} = 2\pi(1-\cos\theta)$$

Moon: $6.5\times10^{-5}$ Sun: $6.8\times10^{-5}$

(c) Sun subtends a solid angle 5% larger than the moon

If these values were constant, could total solar eclipses be explained?

# Formal definition of intensity



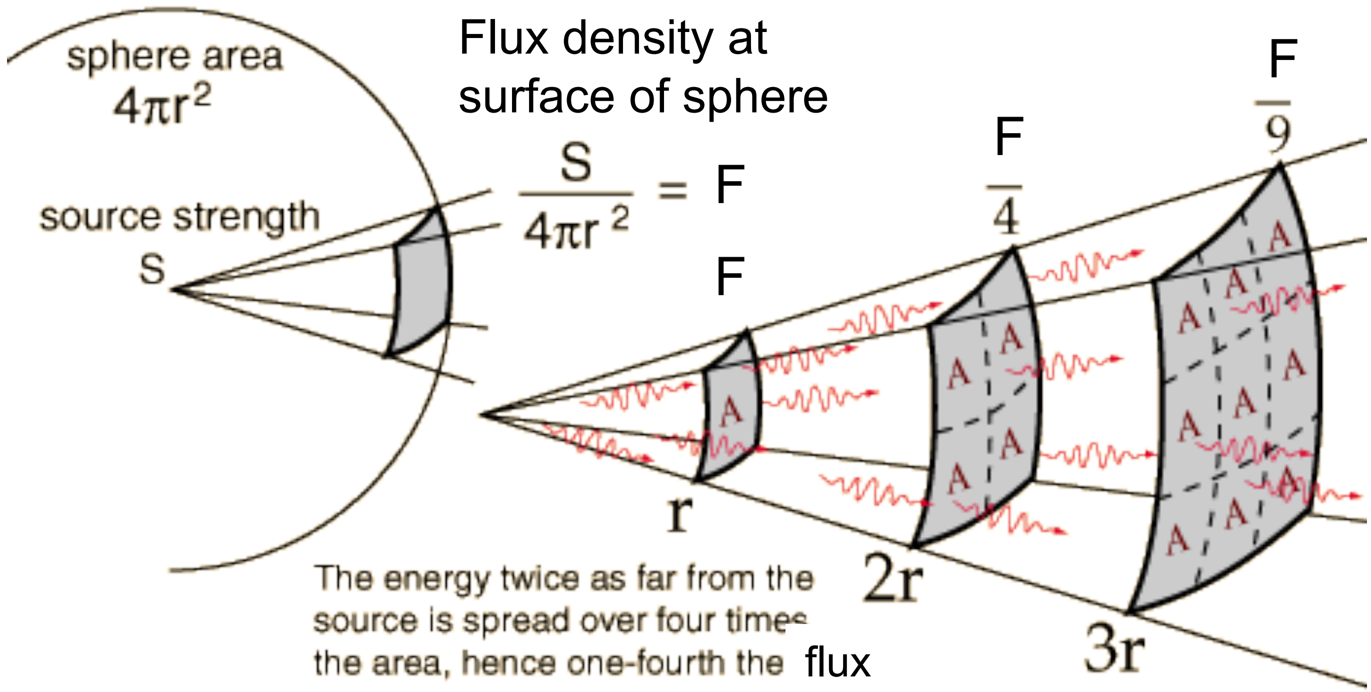

- Flux (F; measured on a surface normal to the beam) per unit solid angle (ω) traveling in a particular direction $\hat{\Omega}$

$$I(\hat{\Omega}) = \frac{\delta F}{\delta \omega}$$

- Typical units: Watts per square meter per steradian (W $m^{-2}$ $sr^{-1}$)

- **Conservation of intensity:** *intensity (radiance) does not decrease with distance from the source* (within a vacuum or other transparent medium)
- Contrast with flux density

# Inverse square law

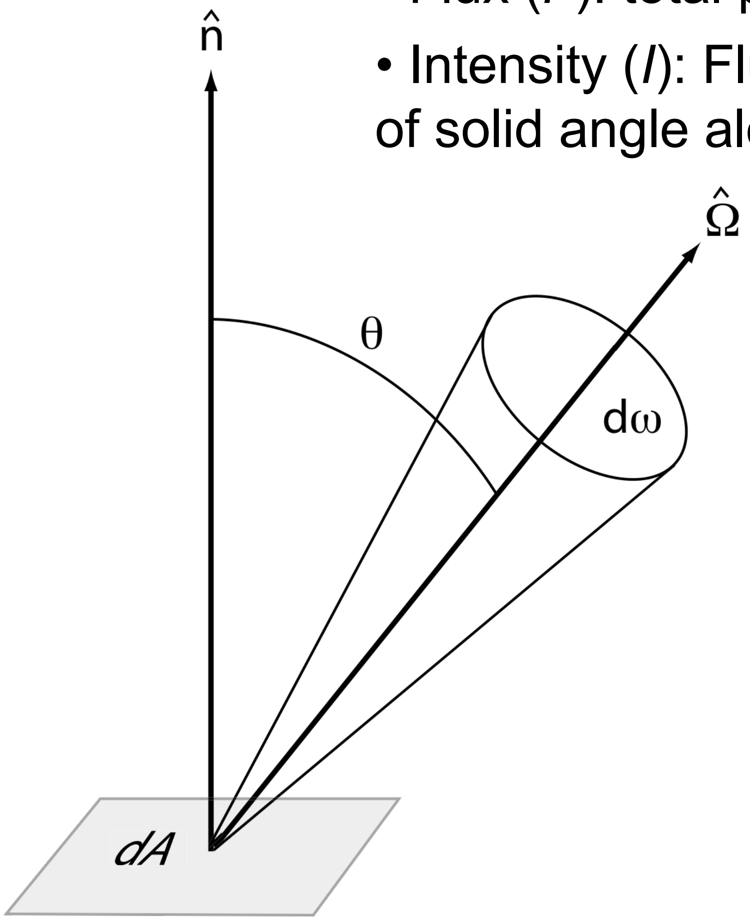



- Irradiance (or flux density) *decreases as the square of distance from the source*
- Radiance *is invariant with distance* (note dependence of solid angle on $r^2$)
- Solar ‘constant’ (irradiance at top of Earth’s atmosphere) is ~1370 W $m^{-2}$

# Relationship between flux and intensity

• Flux (*F*): total power incident on a unit surface area

• Intensity (*I*): Flux contribution arriving from small element of solid angle along a direction $\hat{\Omega}$




• Hence, flux incident on or emerging from an arbitrary surface is found by integrating *I* over all relevant solid angles

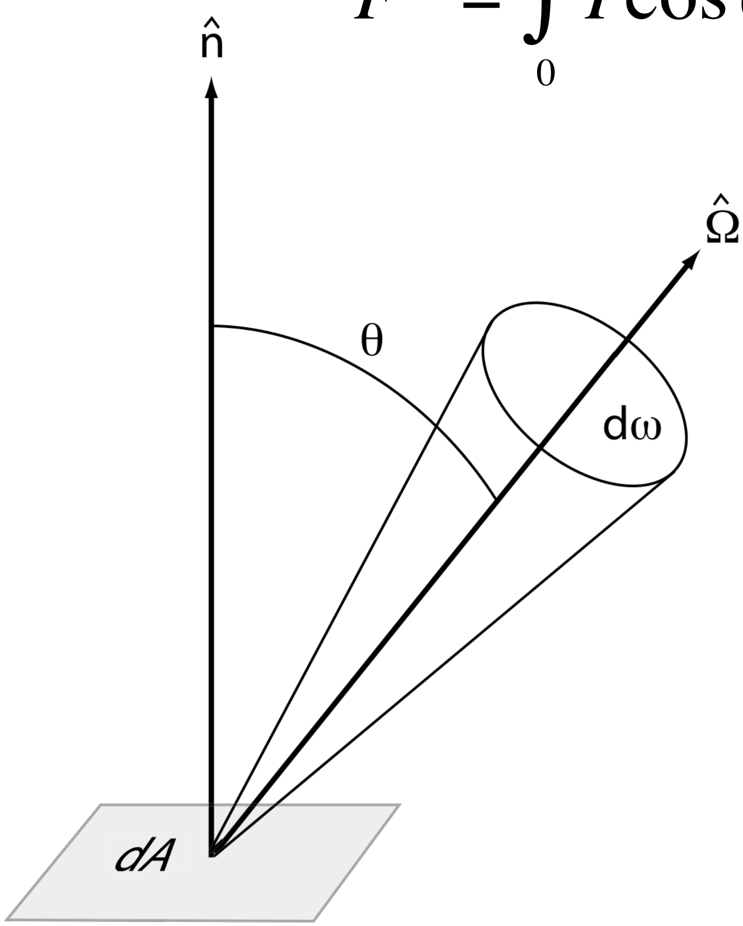
• Since intensity is defined as the flux per unit solid angle *normal to the beam*, the contributions to the flux must be weighted by a factor of cos θ

The flux density of radiation carried by a beam in the direction Ω through a surface element dA is proportional to cos θ = $\hat{n} \cdot \hat{\Omega}$

# Relationship between flux and intensity

- The upward-directed flux from a surface is therefore given by:

$$F^{\uparrow} = \int_0^{2\pi} I\cos\theta\, d\omega = \int_0^{2\pi}\int_0^{\pi/2} I^{\uparrow}(\theta,\phi)\cos\theta\sin\theta\, d\theta\, d\phi$$




- What is the flux density of *isotropic radiation*? (i.e., if intensity is constant)

$$F^{\uparrow} = \int_0^{2\pi}\int_0^{\pi/2} I^{\uparrow}(\theta,\phi)\cos\theta\sin\theta\, d\theta\, d\phi$$

$$= 2\pi I \int_0^{\pi/2} \cos\theta\sin\theta\, d\theta$$

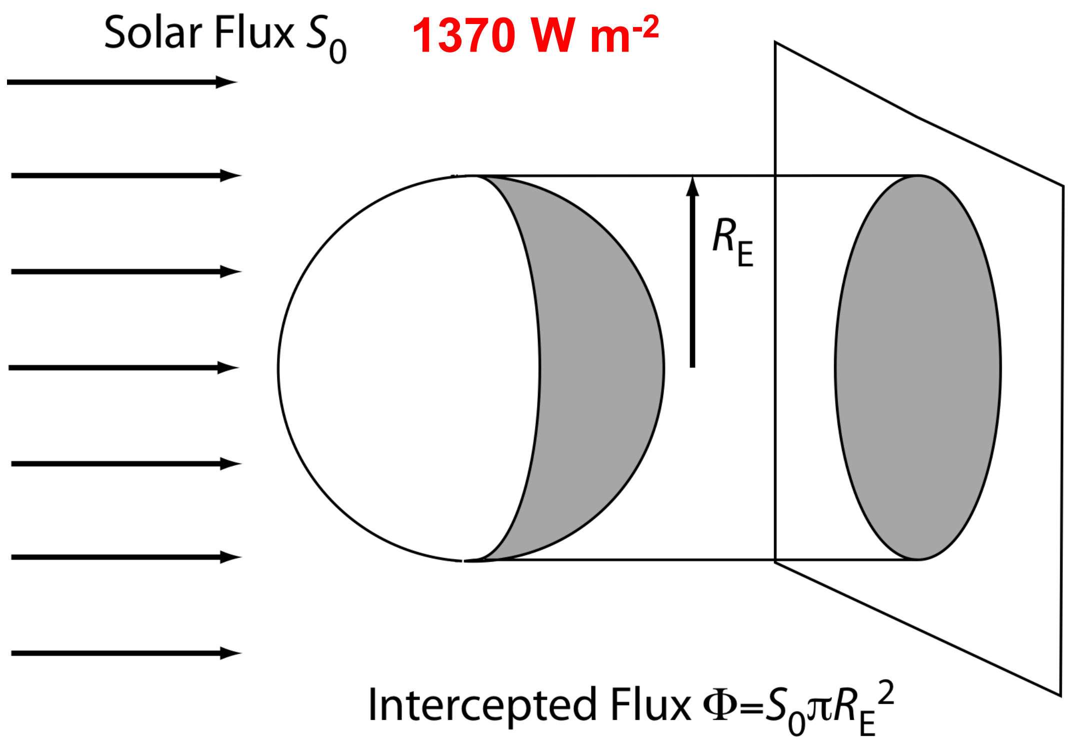
$$= 2\pi I \int_0^{\pi/2} \frac{1}{2}\sin 2\theta\, d\theta$$

$$= \pi I$$

e.g., illumination of a horizontal surface under heavily overcast skies

# Global insolation

- How much total solar radiation Φ is incident on Earth's atmosphere?
- Consider the amount of radiation intercepted by the Earth's disk

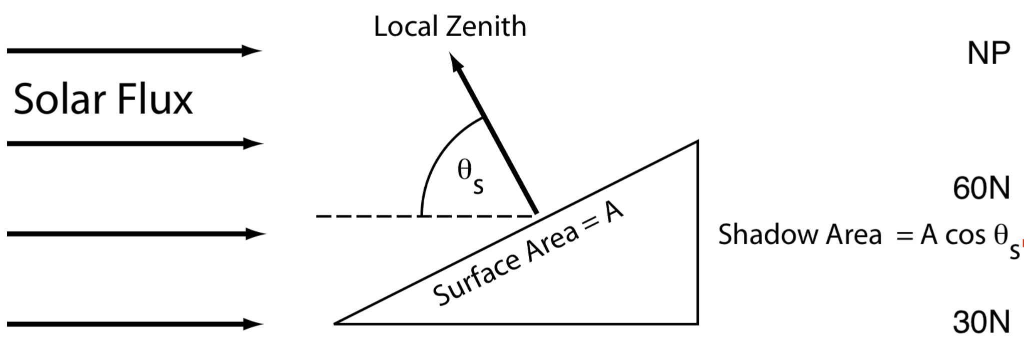


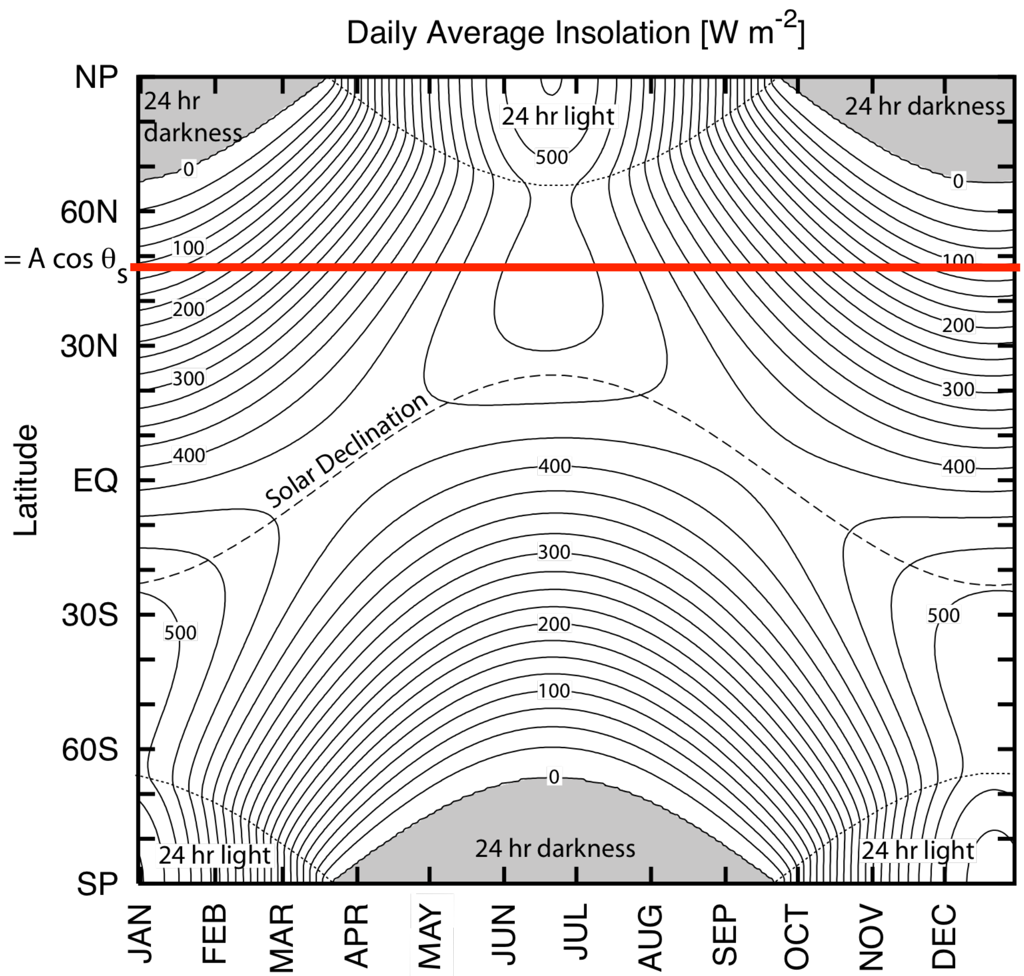

$$\Phi = S_0 \pi R_E^2$$
$$= 1.74 \times 10^{17}\, W$$

- Applies for mean Sun-Earth distance of 1.496 x $10^8$ km
- But Earth's orbit is elliptical, so the *solar flux (S)* actually varies from 1330 W m$^{-2}$ in July to 1420 W m$^{-2}$ in January

# Global insolation

- In addition to Earth's orbit, the power output from the Sun also varies over time (e.g., sunspot cycles)
- Furthermore, radiation is not uniformly incident on the surface but varies with incidence angle of the Sun



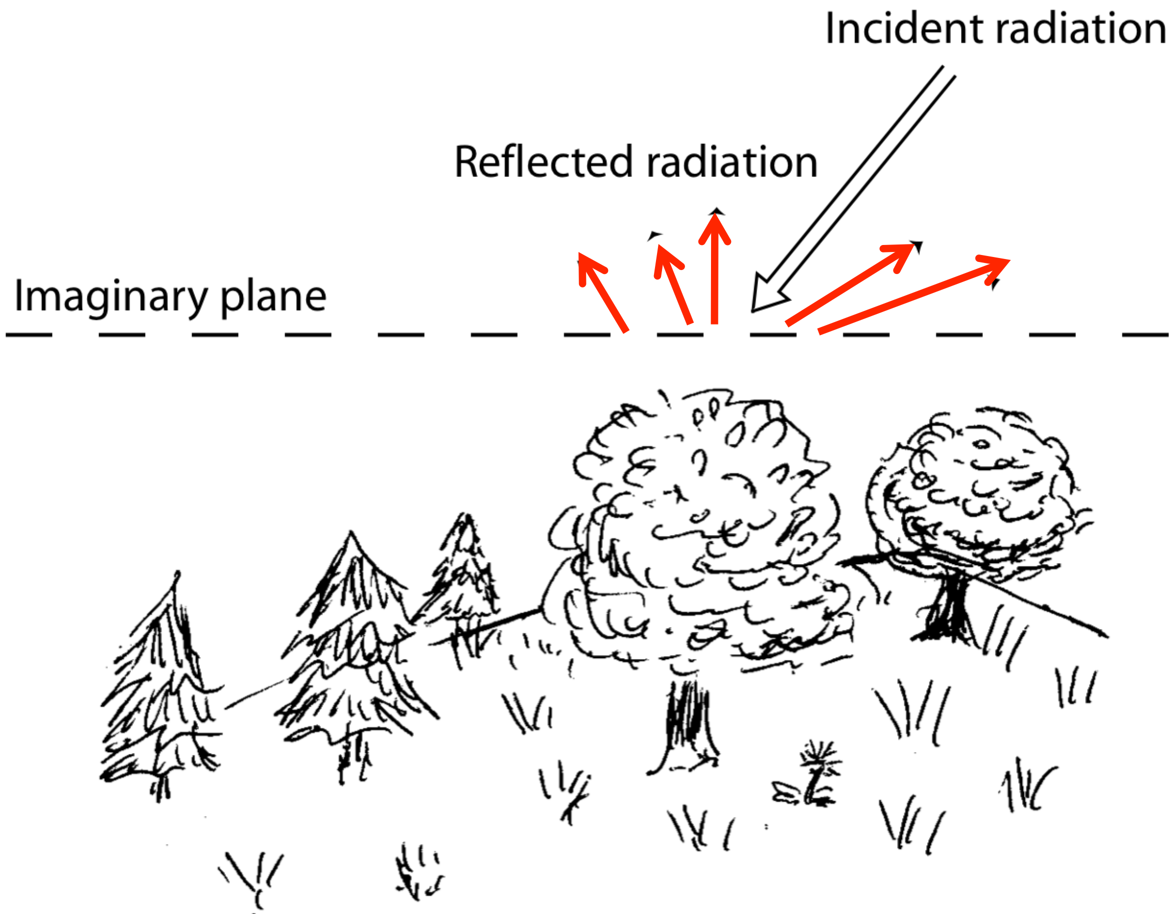




Houghton

Where is the peak?

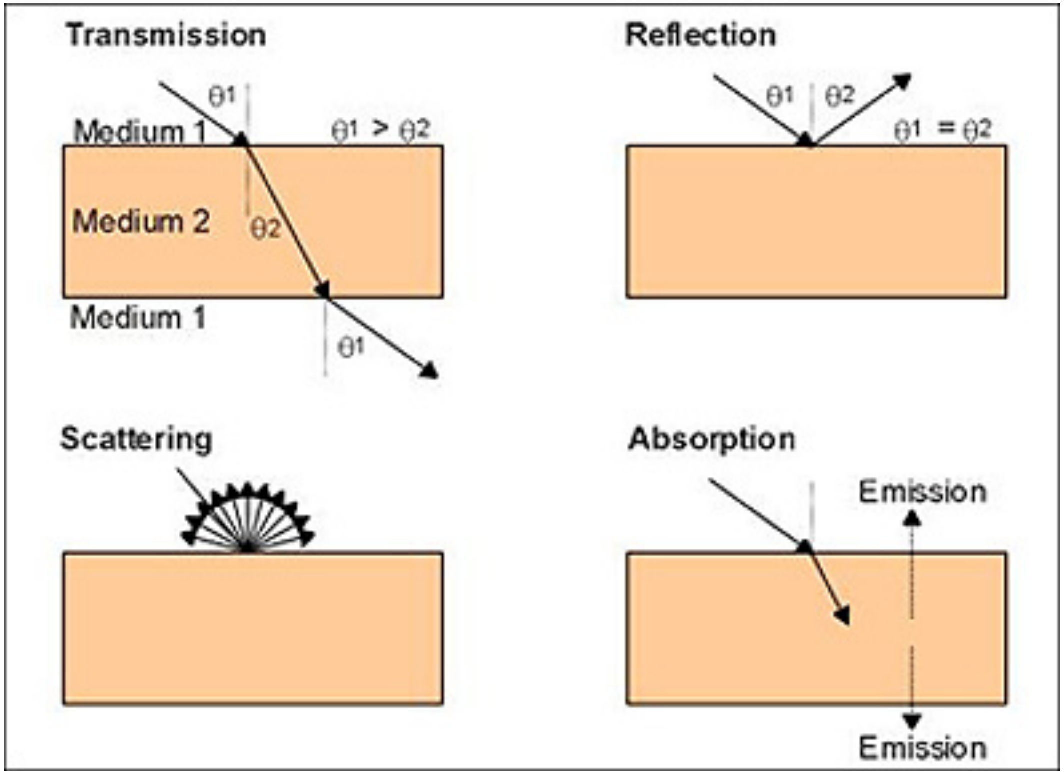
*Note that this shows the amount of radiation incident at the top of the atmosphere only*, not that which is available for absorption by the surface and atmosphere

# Radiative properties of natural surfaces




- Illumination of the atmosphere from below by 'upwelling' radiation

# The fate of incident radiation




Reflectivity, $r_\lambda$
(Albedo)

Absorptivity, $a_\lambda$

Transmissivity, $t_\lambda$
(Transmittance)

$$a_\lambda + r_\lambda + t_\lambda = 1$$

- Radiation incident on a surface is either *reflected, absorbed or transmitted*
- Conservation of energy dictates that the sum of these must be 100% (or 1)
- For opaque objects ➔ negligible transmission ➔ $a_\lambda + r_\lambda = 1$ and $a_\lambda = 1 - r_\lambda$
- If the object is highly reflective, $a_\lambda$ must be low (e.g., aluminum foil)
- Both $a_\lambda$ and $r_\lambda$ depend on the direction of the incident radiation

# Transmission - Beer's Law

• The rate of power attenuation per unit distance is given by the *absorption coefficient* $\beta_a$ (with dimensions of inverse length), related to $n_i$ and wavelength $\lambda$ by:

$$\beta_a = \frac{4\pi n_i}{\lambda}$$

• For an initial intensity $I_0$ at position $x = 0$ in a medium, propagating in the $x$-direction:

$$I(x) = I_0 e^{-\beta_a x}$$

$$\Rightarrow \frac{I(x)}{I_0} = e^{-\beta_a x} = t(x)$$

• Where $t$ is the *transmittance*: the fraction of radiation that survives the trip over the distance $x$ (i.e., that which is not absorbed)

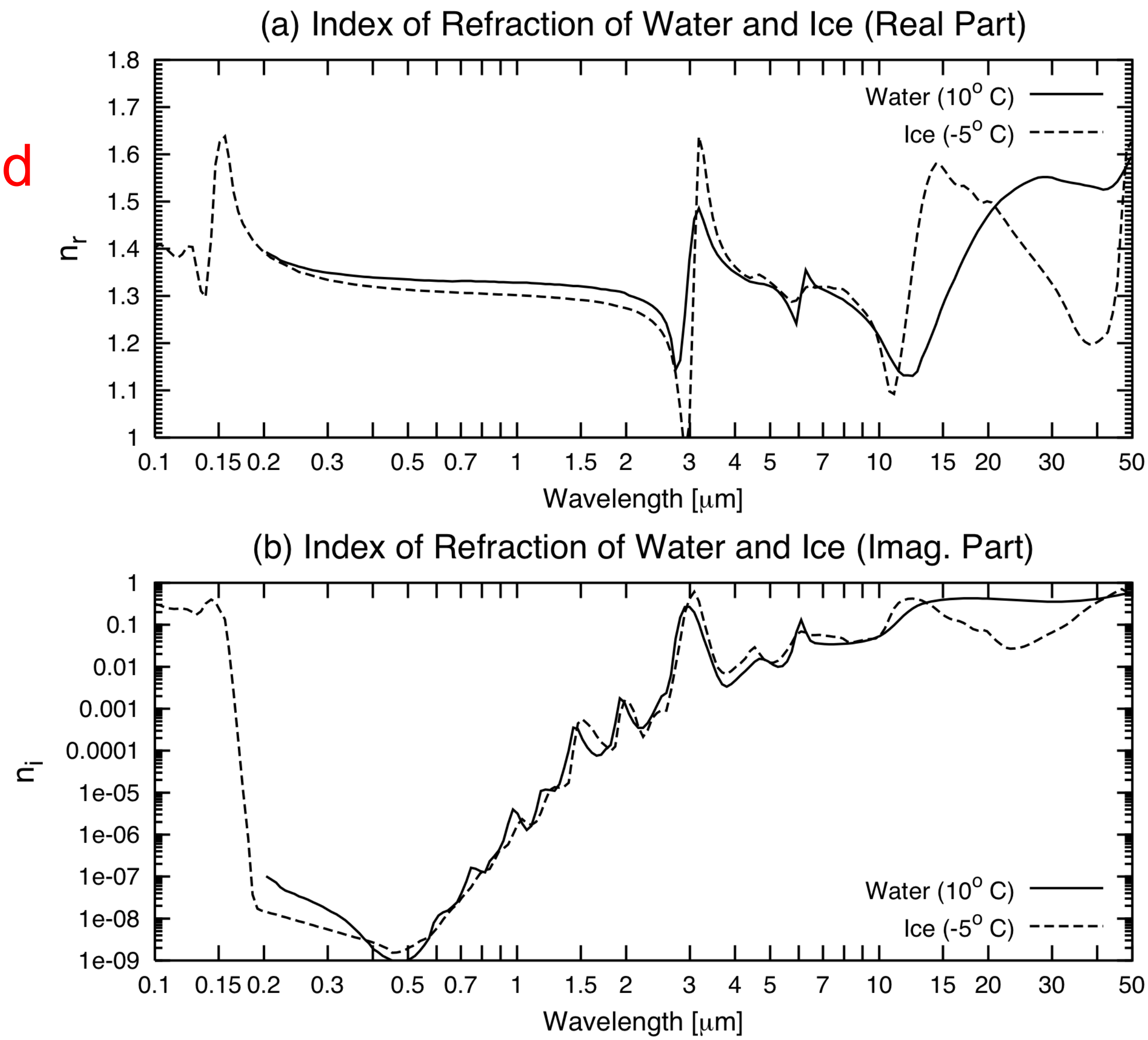
• This equation is a form of a very important relationship known as

**BEER'S LAW** *(also referred to as the Beer-Bouguer-Lambert Law)*

# Interaction of radiation with matter

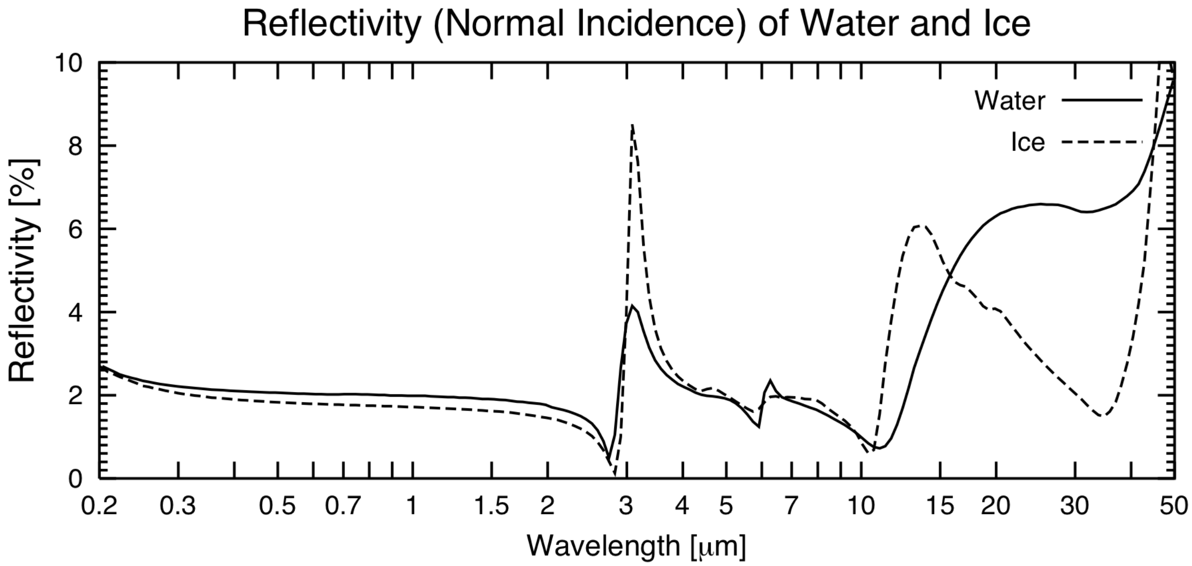
Reflection and refraction

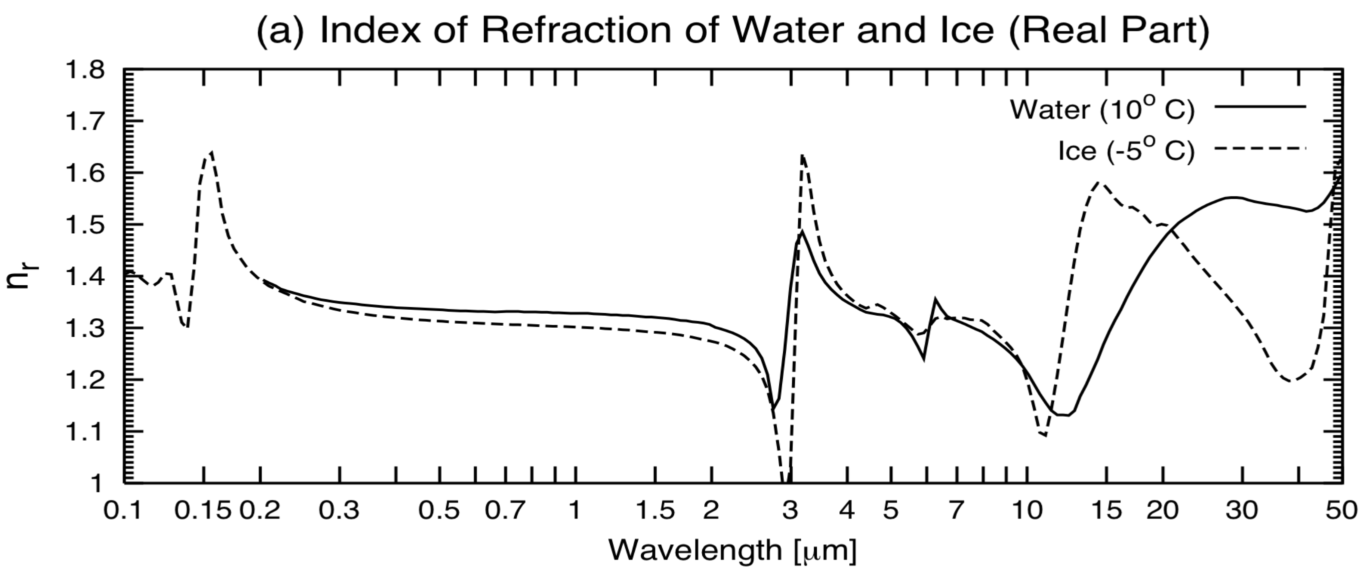
Absorption




- Note very small $n_i$ in the visible band, increasing in the UV and IR bands

# Reflectivity




$$R_p = \left| \frac{\cos\theta_t - m\cos\theta_i}{\cos\theta_t + m\cos\theta_i} \right|^2$$

$$R_s = \left| \frac{\cos\theta_i - m\cos\theta_t}{\cos\theta_i + m\cos\theta_t} \right|^2$$

- Fresnel relations for normal incidence:

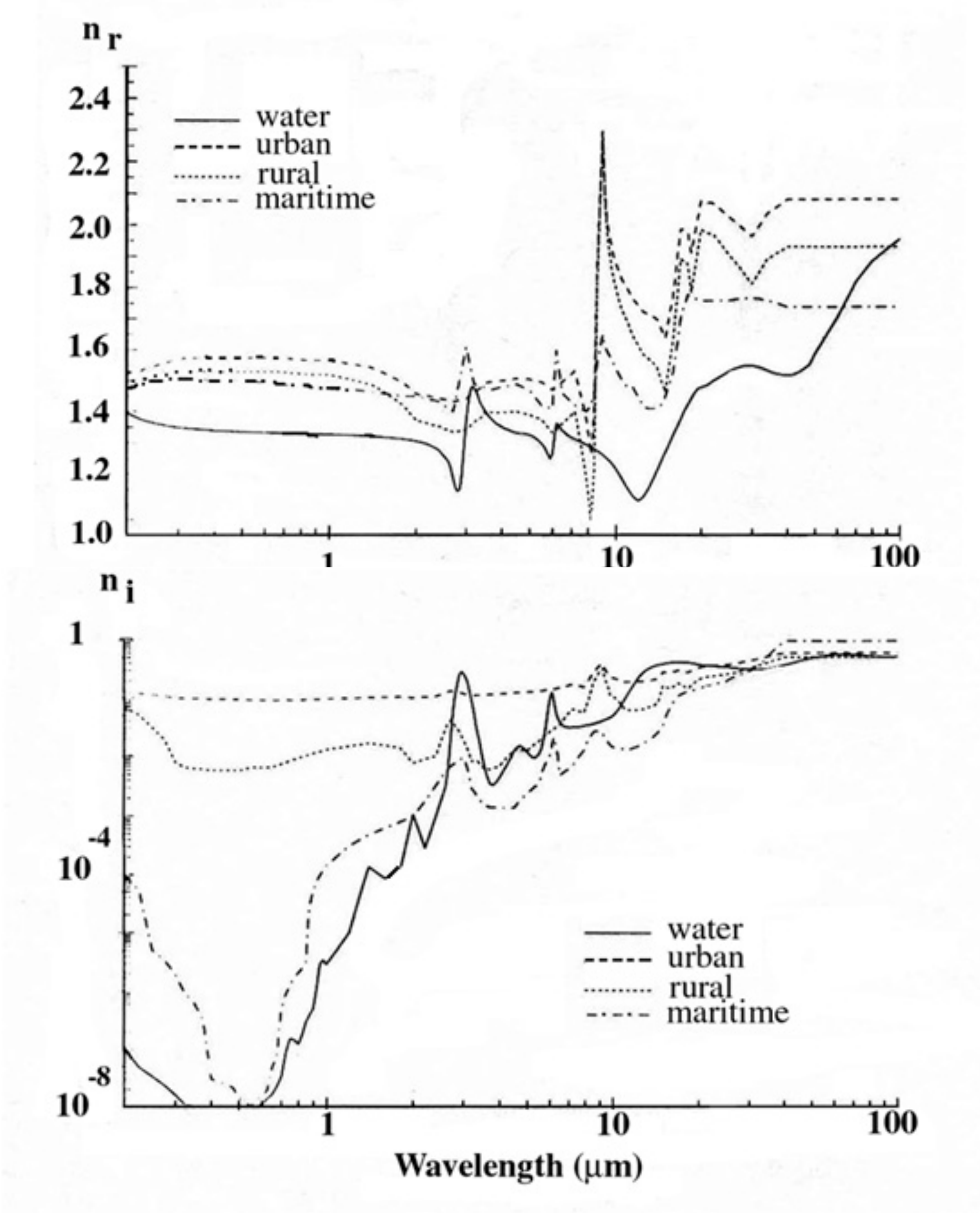
$$R_{normal} = \left| \frac{m-1}{m+1} \right|^2$$

(*m* is the relative refractive index)

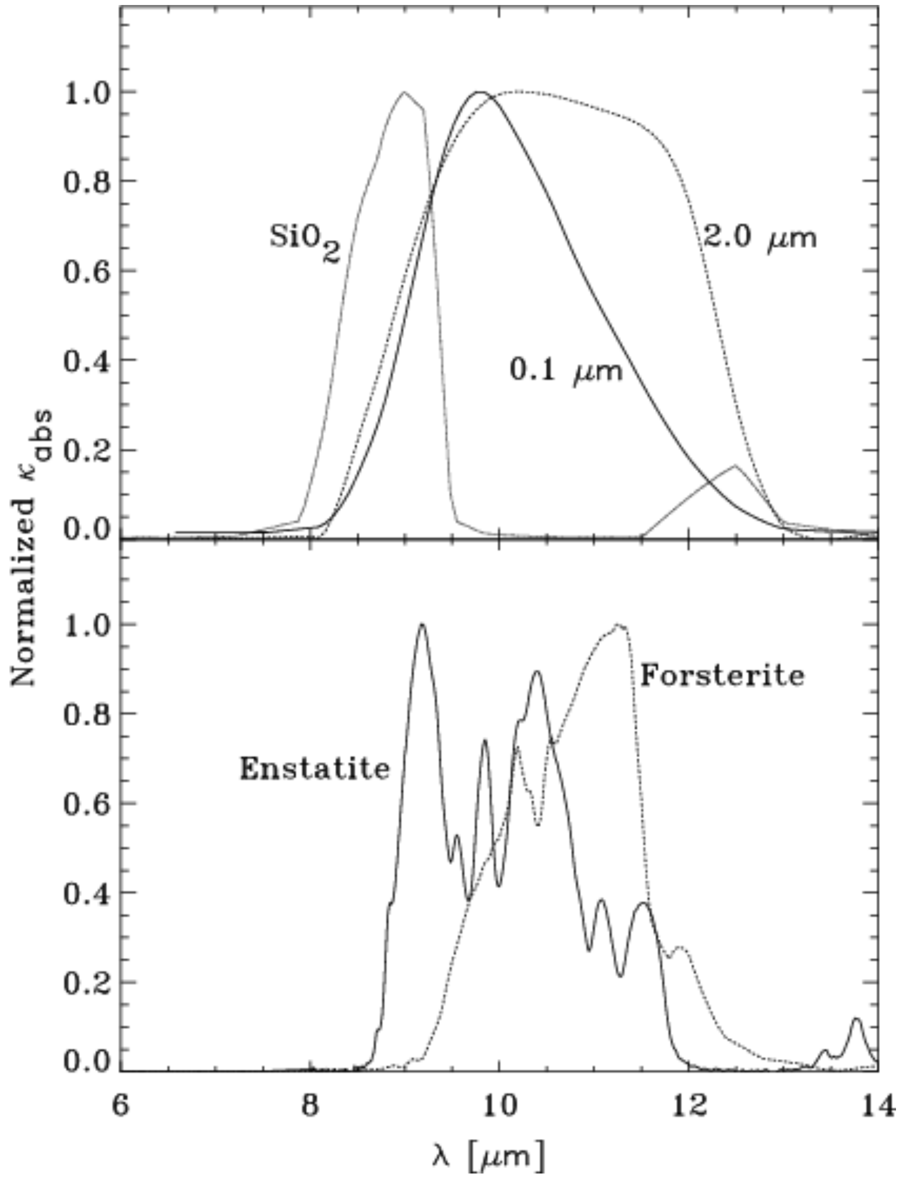
# Refractive index of aerosols

Reflection and refraction

Absorption



# Absorption by silicates

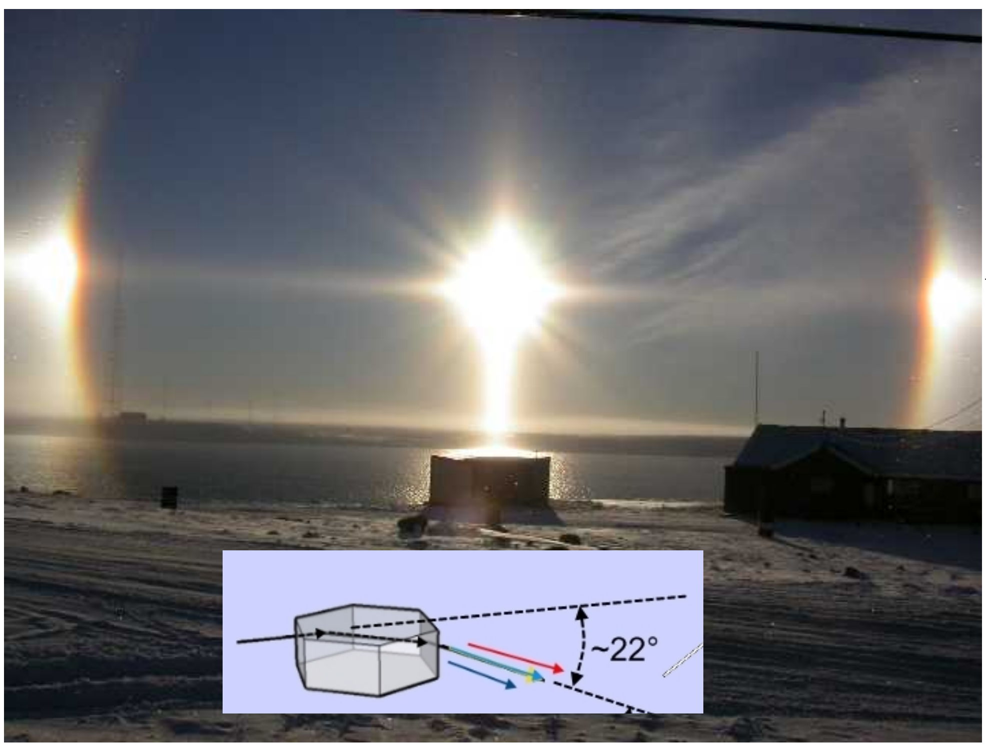


# Geometric optics

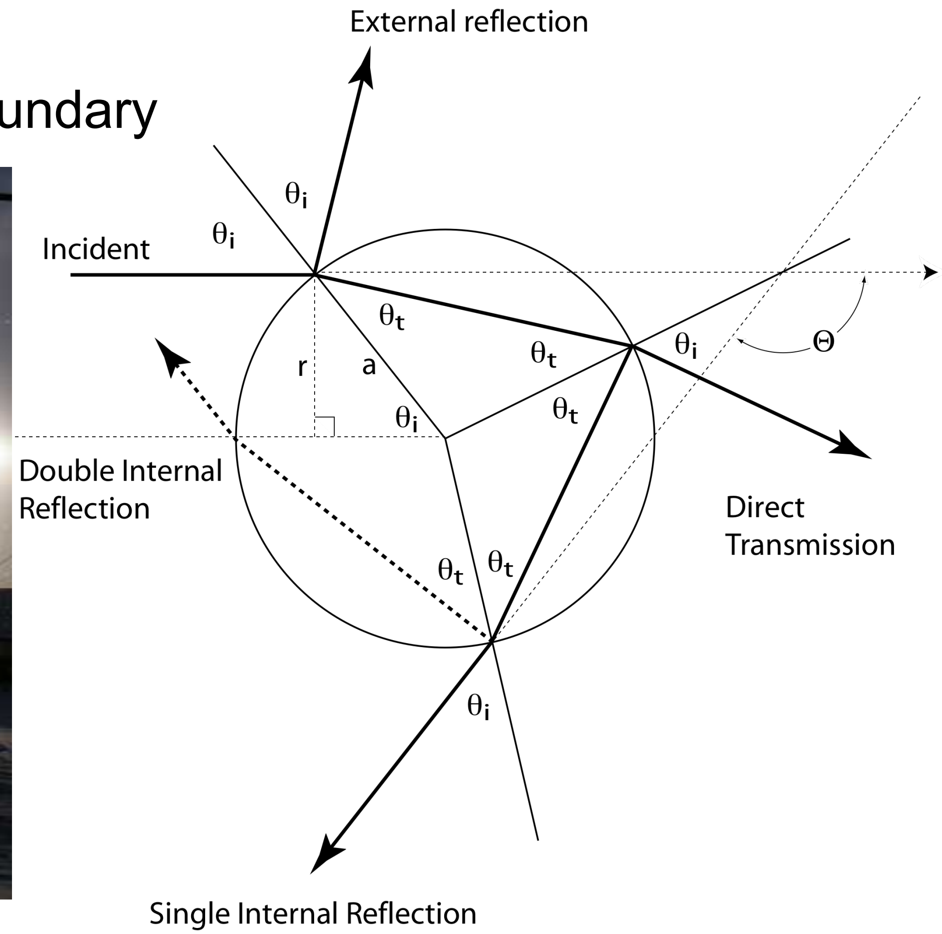
• *When particle size is larger than wavelength*, geometric optics or ray tracing can be used to analyze the interaction of EM waves with matter

• e.g., scattering of visible light by cloud ice particles (>50 μm) and raindrops (100 μm < $r$ < 3 mm)

• Reflection and refraction at each boundary

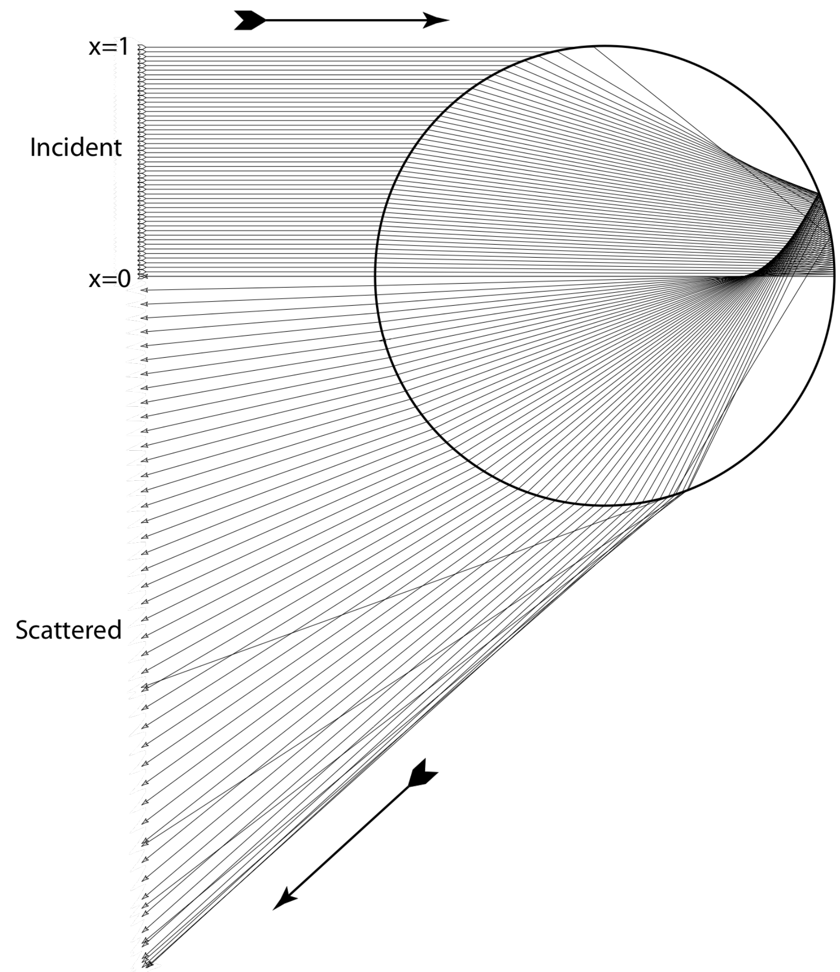



Sundog



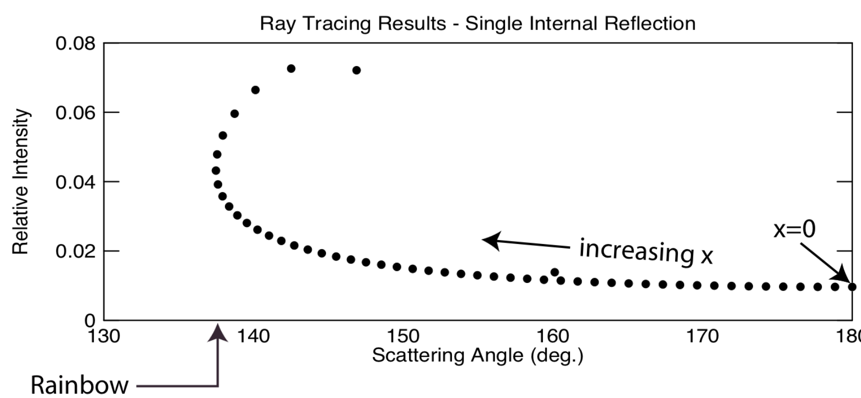
# Geometric optics

• Ray tracing predicts the geometry of the primary and secondary rainbows

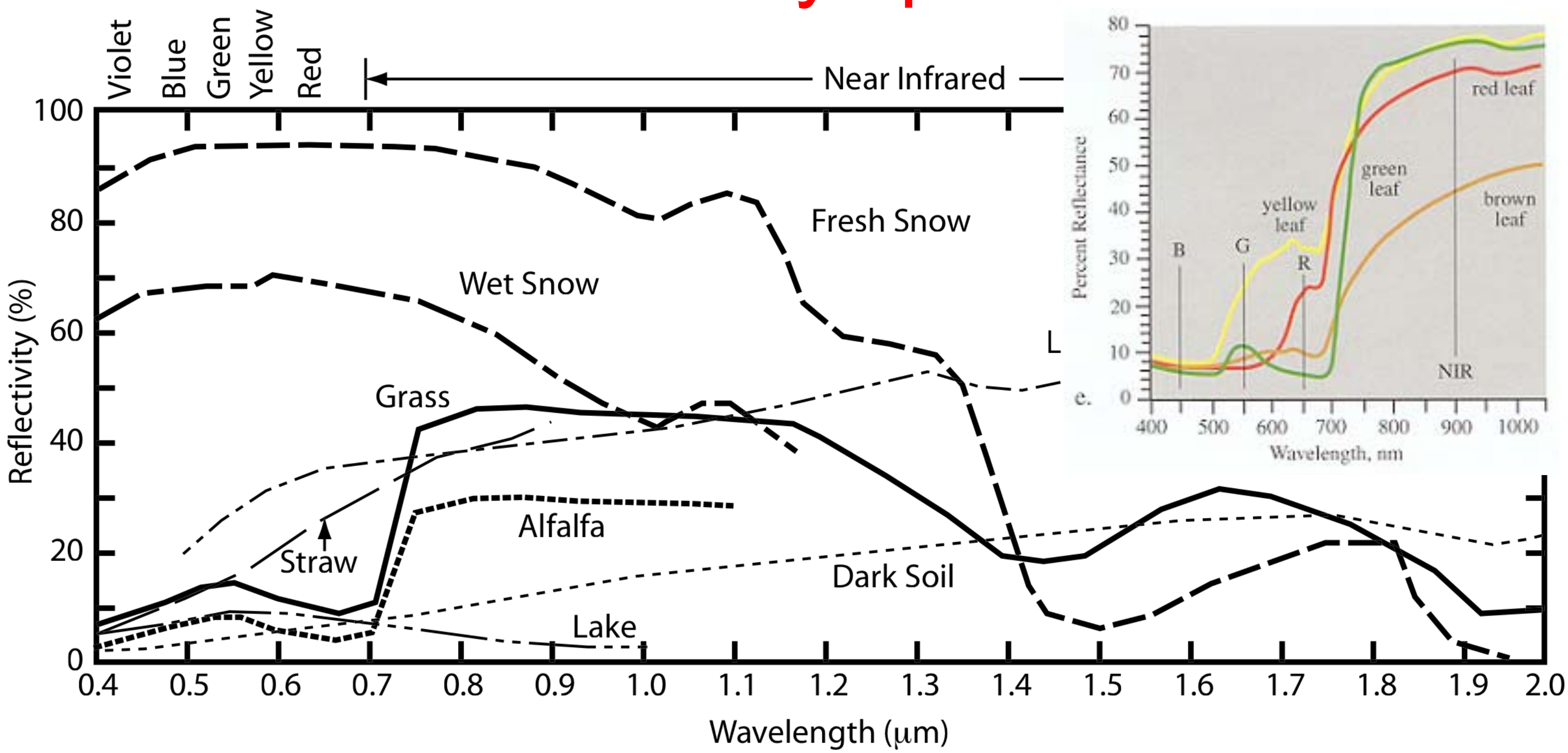



(a)

(b)



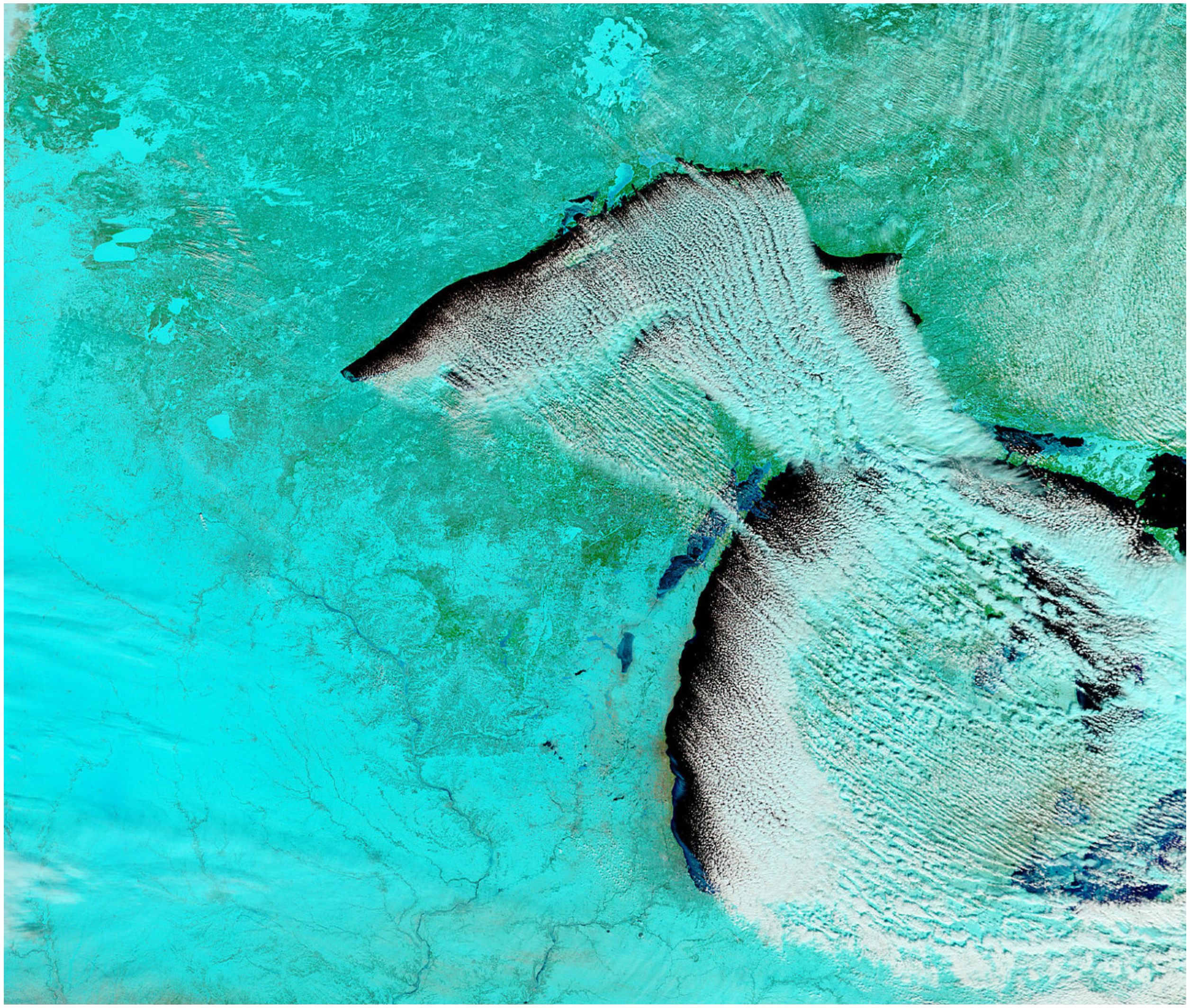
# Reflectivity spectra




- Which surface type has the generally highest albedo?
- Where is the visible reflectivity peak for grass?
- What about a dead plant vs. a live plant? What happens in the Fall?
- Note generally low reflectivity of vegetation, soil, and water, and strong contrast in reflectivity between visible and near-IR bands

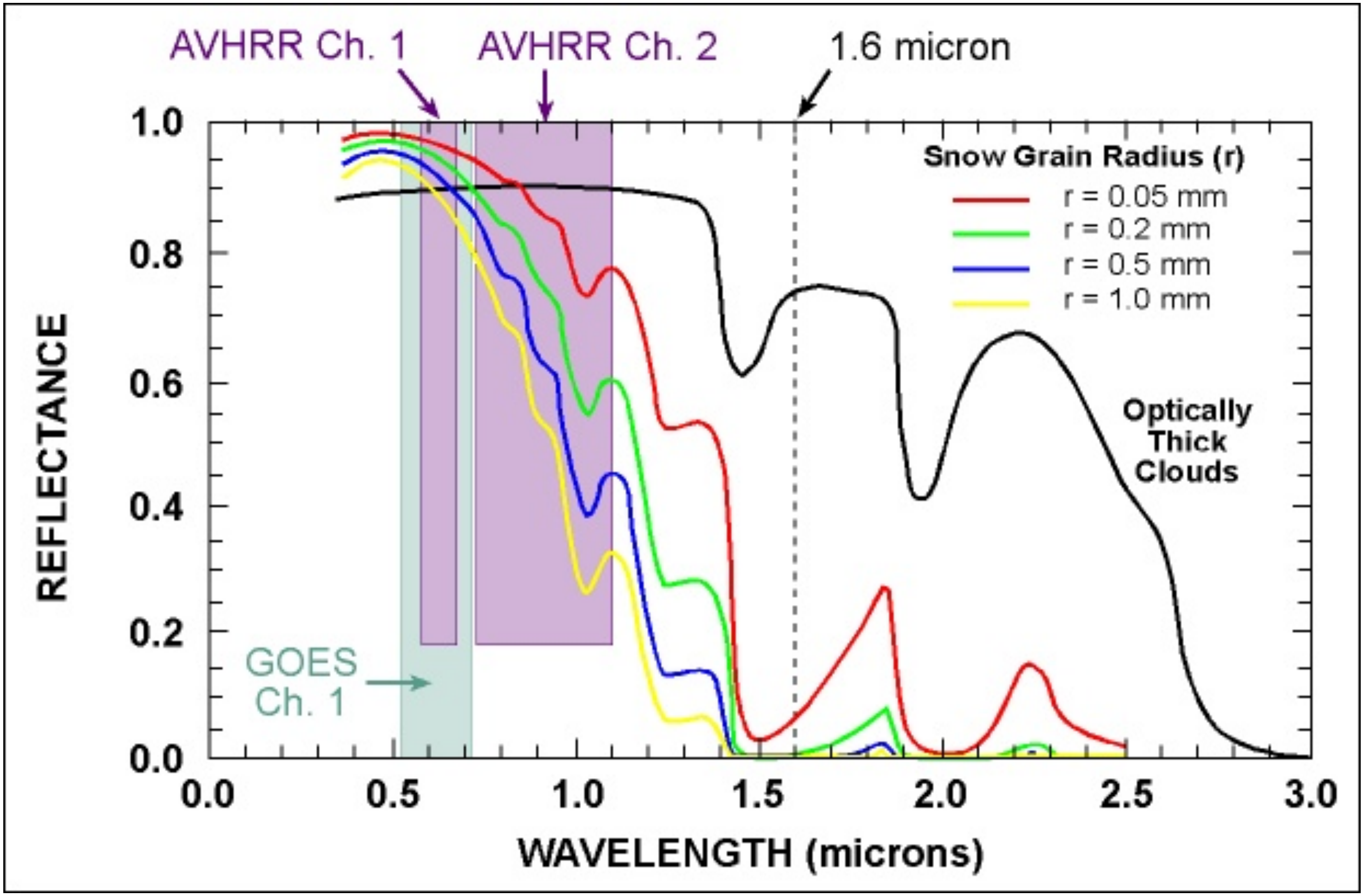
# MODIS images

http://rapidfire.sci.gsfc.nasa.gov/subsets/



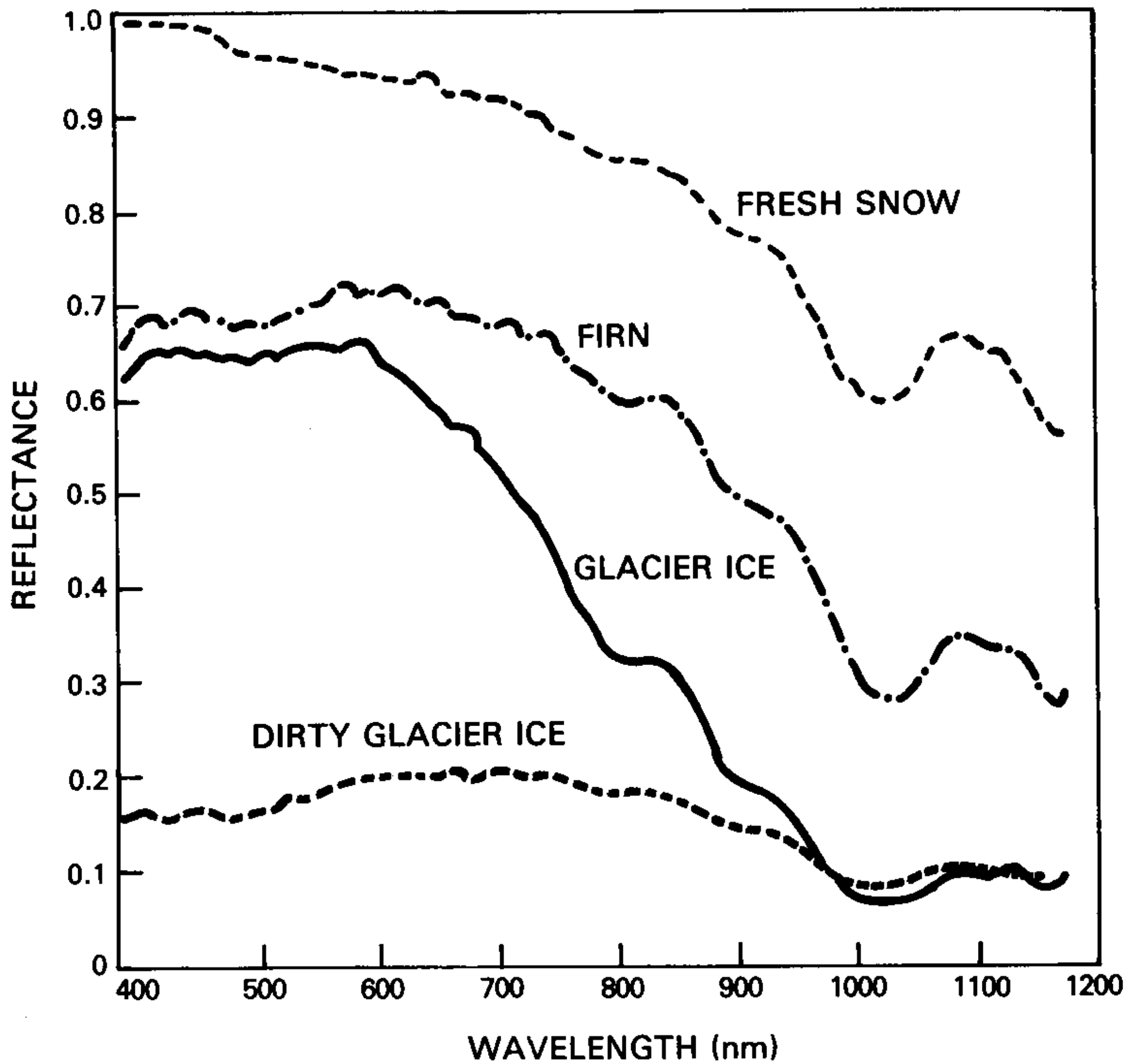
False Color: **R** (Band 7: 2155 nm), **G** (Band 2: 876 nm), **B** (Band 1: 670 nm)

# Snow and cloud reflectance




From: http://www.nohrsc.noaa.gov/technology/avhrr3a/avhrr3a.html

# Dirty snow/ice



# MODerate resolution Imaging Spectroradiometer (MODIS)

- MODIS: a broadband, multispectral satellite instrument
  - Flying on two polar-orbiting NASA satellites: Terra (10:30 am overpass) and Aqua (1:30 pm overpass)
  - http://modis.gsfc.nasa.gov/index.php
  - Swath width: 2330 km
  - 36 spectral bands from VIS to IR, but not contiguous
  - High radiometric resolution: 12-bit (0-4095)
  - Spatial resolution: 250 m (bands 1-2), 500 m (bands 3-7), 1000 m (bands 8-36)
  - Why the difference in spatial resolution between bands?

# MODIS bands

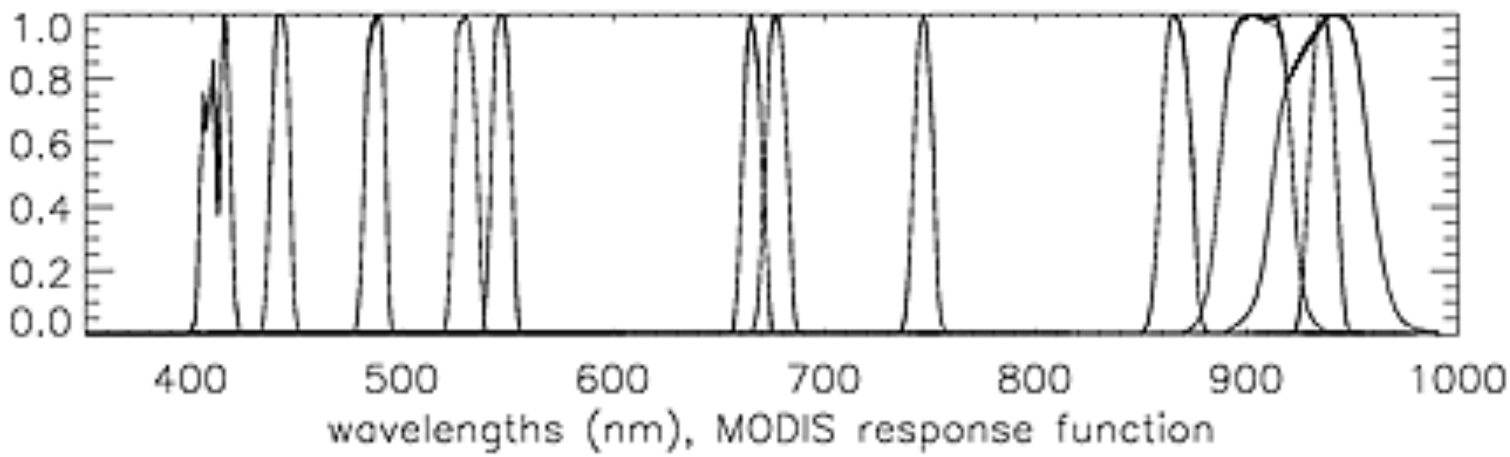
| Primary Use | Band | Bandwidth[1] | Spectral Radiance[2] |
|---|---|---|---|
| Land/Cloud/Aerosols Boundaries | 1 | 620 - 670 | 21.8 |
| | 2 | 841 - 876 | 24.7 |
| Land/Cloud/Aerosols Properties | 3 | 459 - 479 | 35.3 |
| | 4 | 545 - 565 | 29.0 |
| | 5 | 1230 - 1250 | 5.4 |
| | 6 | 1628 - 1652 | 7.3 |
| | 7 | 2105 - 2155 | 1.0 |
| Ocean Color/ Phytoplankton/ Biogeochemistry | 8 | 405 - 420 | 44.9 |
| | 9 | 438 - 448 | 41.9 |
| | 10 | 483 - 493 | 32.1 |
| | 11 | 526 - 536 | 27.9 |
| | 12 | 546 - 556 | 21.0 |
| | 13 | 662 - 672 | 9.5 |
| | 14 | 673 - 683 | 8.7 |
| | 15 | 743 - 753 | 10.2 |
| | 16 | 862 - 877 | 6.2 |
| Atmospheric Water Vapor | 17 | 890 - 920 | 10.0 |
| | 18 | 931 - 941 | 3.6 |
| | 19 | 915 - 965 | 15.0 |

| Primary Use | Band | Bandwidth[1] | Spectral Radiance[2] |
|---|---|---|---|
| Surface/Cloud Temperature | 20 | 3.660 - 3.840 | 0.45(300K) |
| | 21 | 3.929 - 3.989 | 2.38(335K) |
| | 22 | 3.929 - 3.989 | 0.67(300K) |
| | 23 | 4.020 - 4.080 | 0.79(300K) |
| Atmospheric Temperature | 24 | 4.433 - 4.498 | 0.17(250K) |
| | 25 | 4.482 - 4.549 | 0.59(275K) |
| Cirrus Clouds Water Vapor | 26 | 1.360 - 1.390 | 6.00 |
| | 27 | 6.535 - 6.895 | 1.16(240K) |
| | 28 | 7.175 - 7.475 | 2.18(250K) |
| Cloud Properties | 29 | 8.400 - 8.700 | 9.58(300K) |
| Ozone | 30 | 9.580 - 9.880 | 3.69(250K) |
| Surface/Cloud Temperature | 31 | 10.780 - 11.280 | 9.55(300K) |
| | 32 | 11.770 - 12.270 | 8.94(300K) |
| Cloud Top Altitude | 33 | 13.185 - 13.485 | 4.52(260K) |
| | 34 | 13.485 - 13.785 | 3.76(250K) |
| | 35 | 13.785 - 14.085 | 3.11(240K) |
| | 36 | 14.085 - 14.385 | 2.08(220K) |

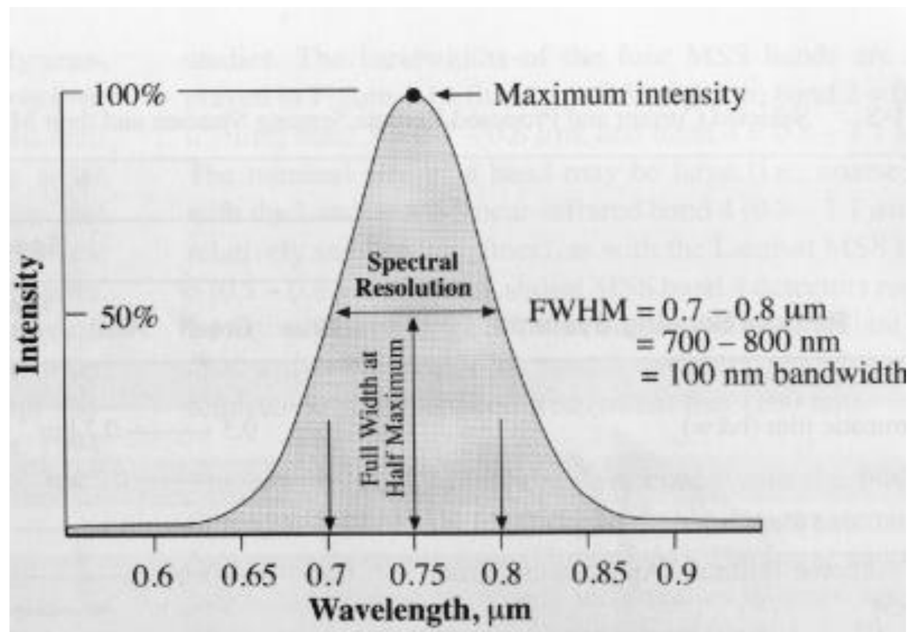
[1] Bands 1 to 19 are in nm; Bands 20 to 36 are in µm
[2] Spectral Radiance values are ($W/m^2$ -µm-sr)

http://modis.gsfc.nasa.gov/about/specifications.php

# MODIS spectral resolution




- Often specified by the full width at half-maximum (FWHM) response
  - ‘bandwidth’ > 100 nm
  - FWHM = 100 nm in this case

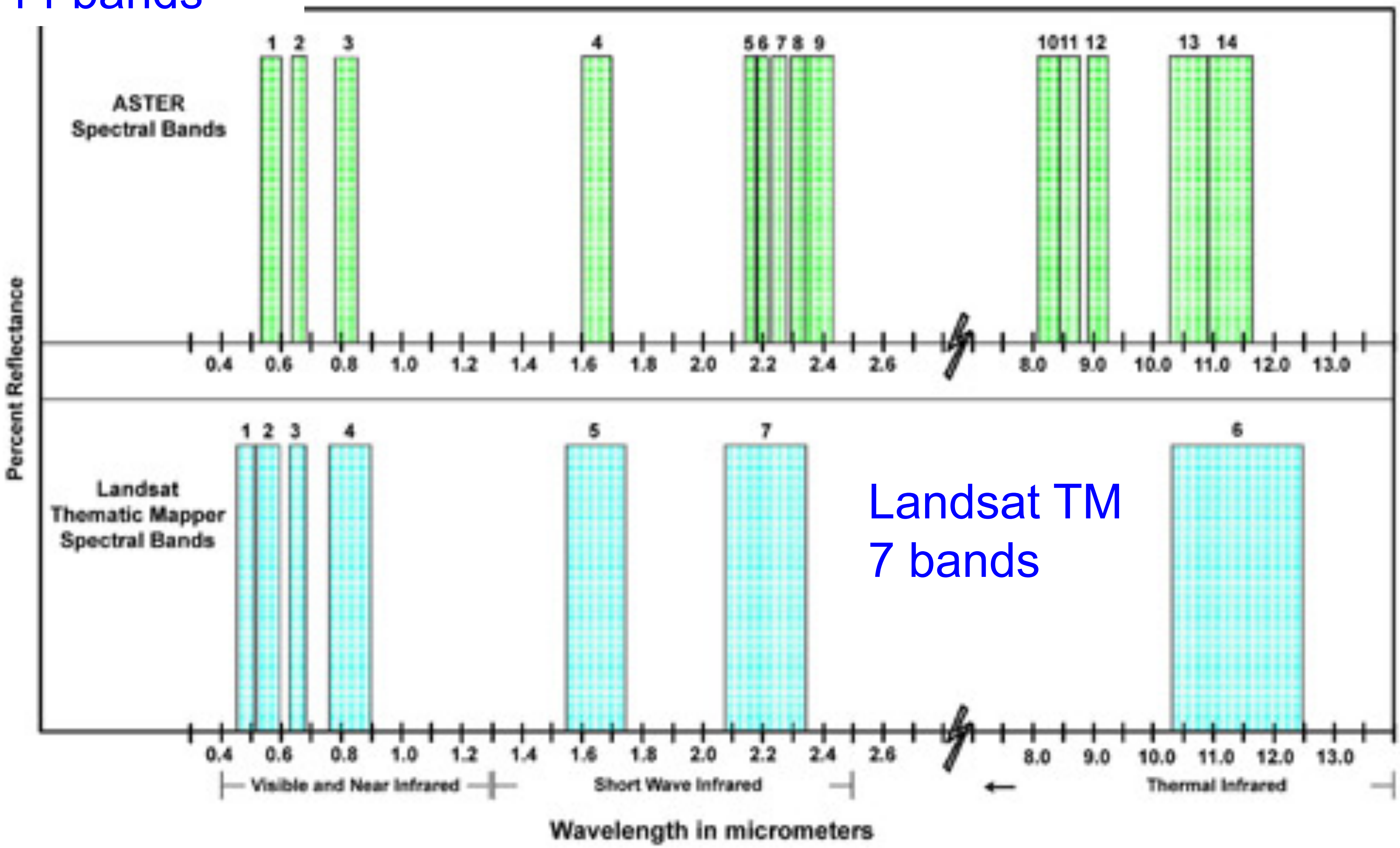



b. Precise bandpass measurement of a detector based on Full Width at Half Maximum (FWHM) criteria

Jensen (2007), Remote sensing: an earth resources perspective, Prentice Hall.

# Multispectral sensors

Terra ASTER
14 bands



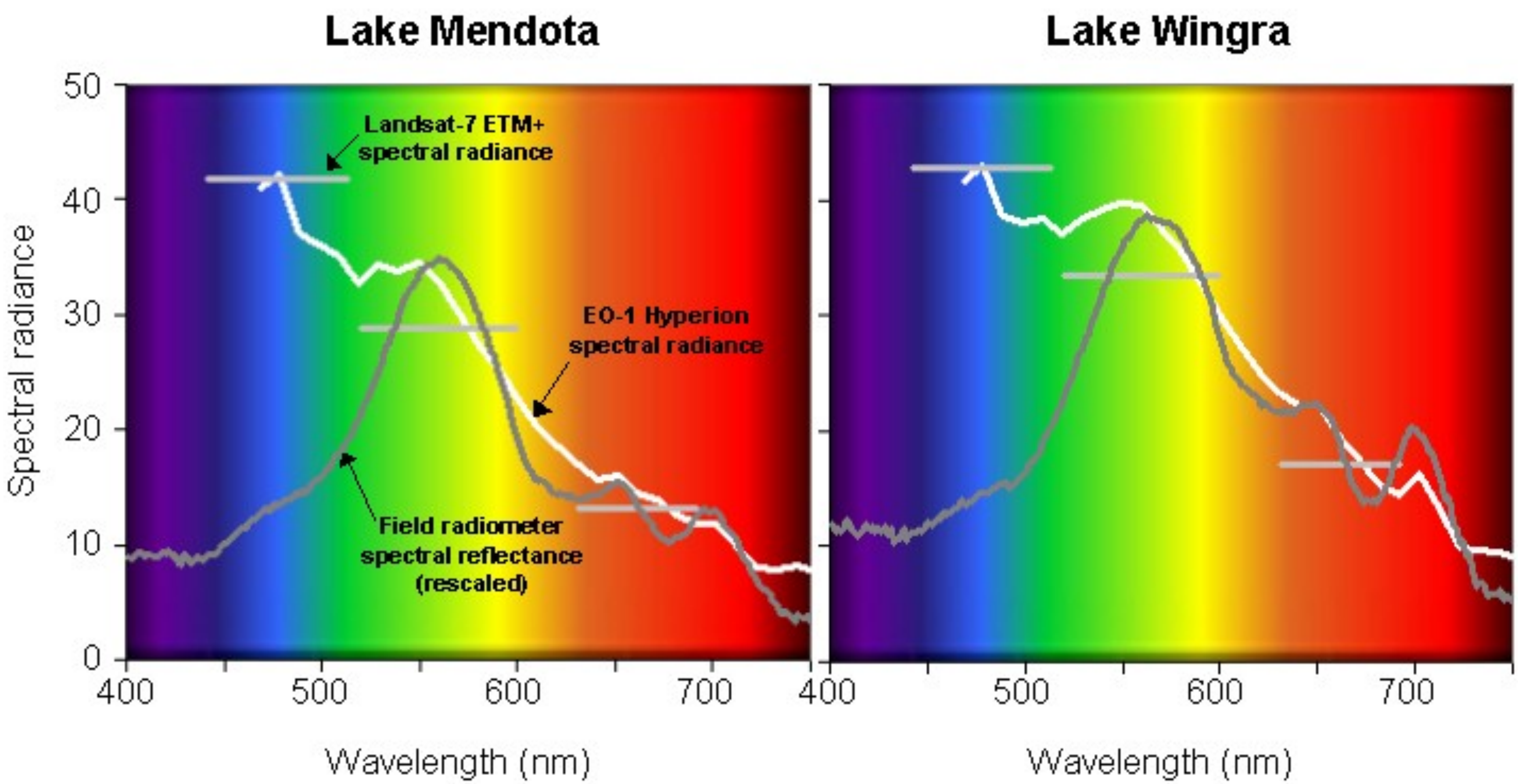

Landsat TM
7 bands

# Hyperspectral sensors

| Parameters | EO-1 | | |
|---|---|---|---|
| | ALI | HYPERION | AC |
| Spectral Range | 0.4 - 2.4 μm | 0.4 - 2.4 μm | 0.9 - 1.6 μm |
| Spatial Resolution | 30 m | 30 m | 250 m |
| Swath Width | 36 Km | 7.6 Km | 185 Km |
| Spectral Resolution | Variable | 10 nm | 6 nm |
| Spectral Coverage | Discrete | Continuous | Continuous |
| Pan Band Resolution | 10 m | N/A | N/A |
| Total Number of Bands | 10 | 220 | 256 |

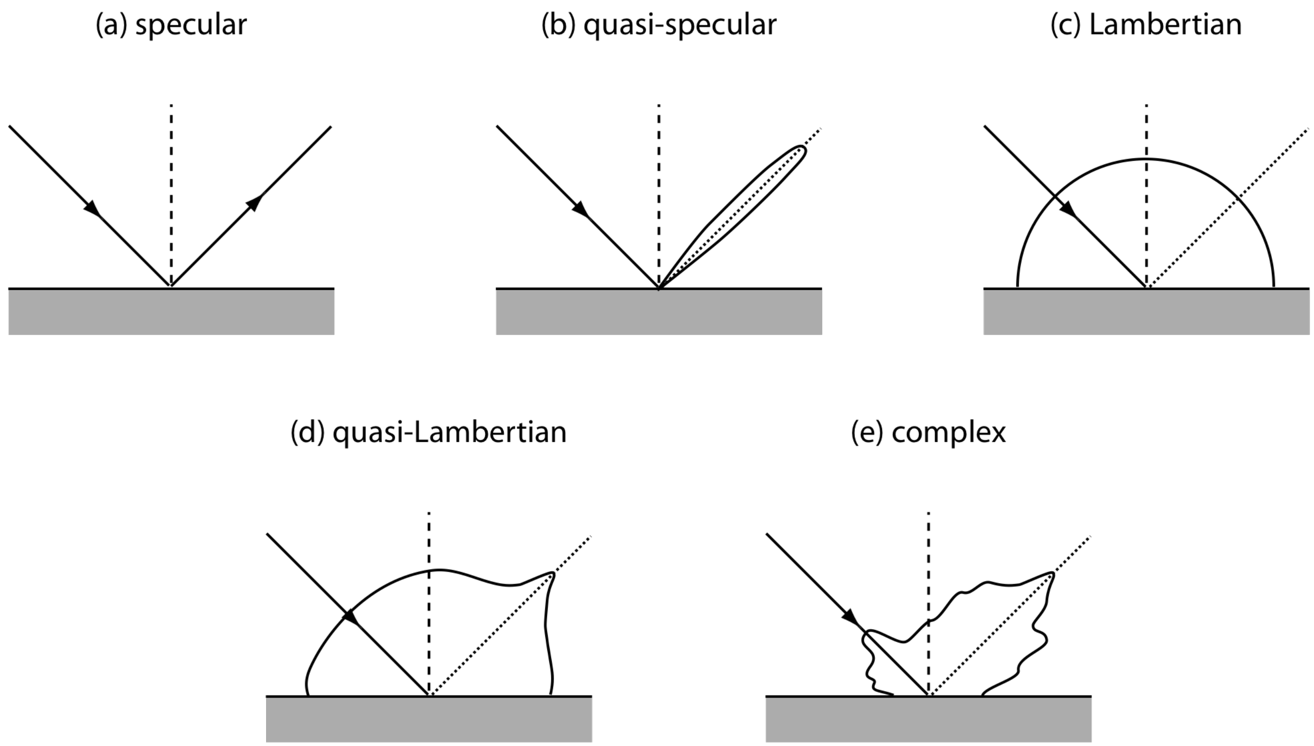
- NASA EO-1 satellite
- Launched in Nov 2000
  - Advanced Land Imager (ALI)
  - Hyperion
  - Atmospheric Corrector (AC)

# Hyperspectral sensors




- Spectral radiance of Madison lakes
- http://www.lakesat.org/madison_compare_hyperion.php

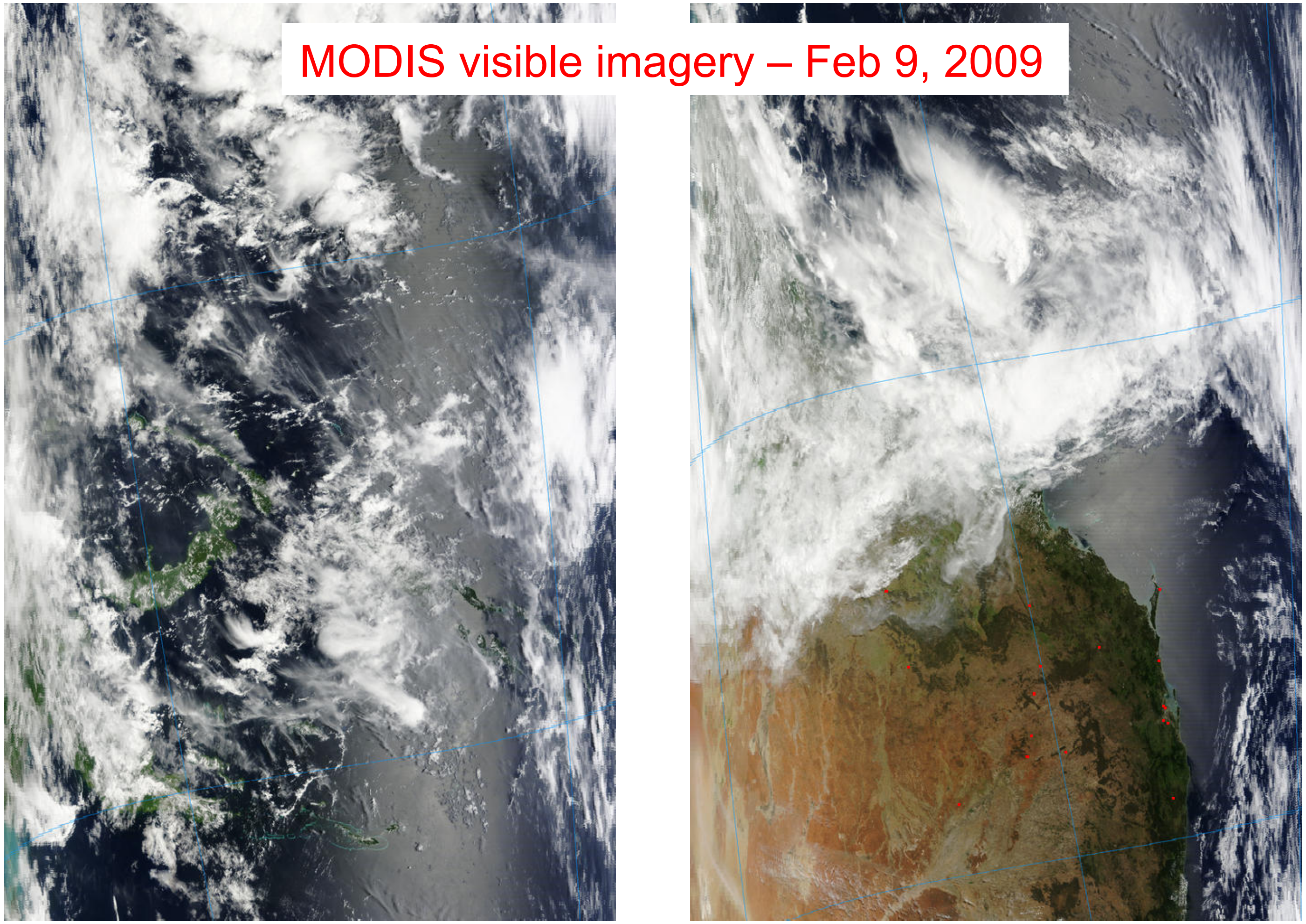
# Angular distribution of reflected radiation




- Specular: radiation incident at angle θ reflected away from surface at angle θ
- Lambertian: radiation incident at angle θ reflected equally at all angles

Example of variable reflectance: glossy, semi gloss and flat paint

# Sun glint




- Observed in direction of reflection of the sun from smooth surfaces
- *Specular reflection* of visible light from smooth water surface

# Lambertian reflectivity

- A Lambertian surface has the same radiance (or brightness) when viewed from any angle
- A common assumption in remote sensing using reflected radiation

$$F_r = rF_i \qquad F_r = \pi I_r$$

← Flux density of isotropic radiation derived earlier

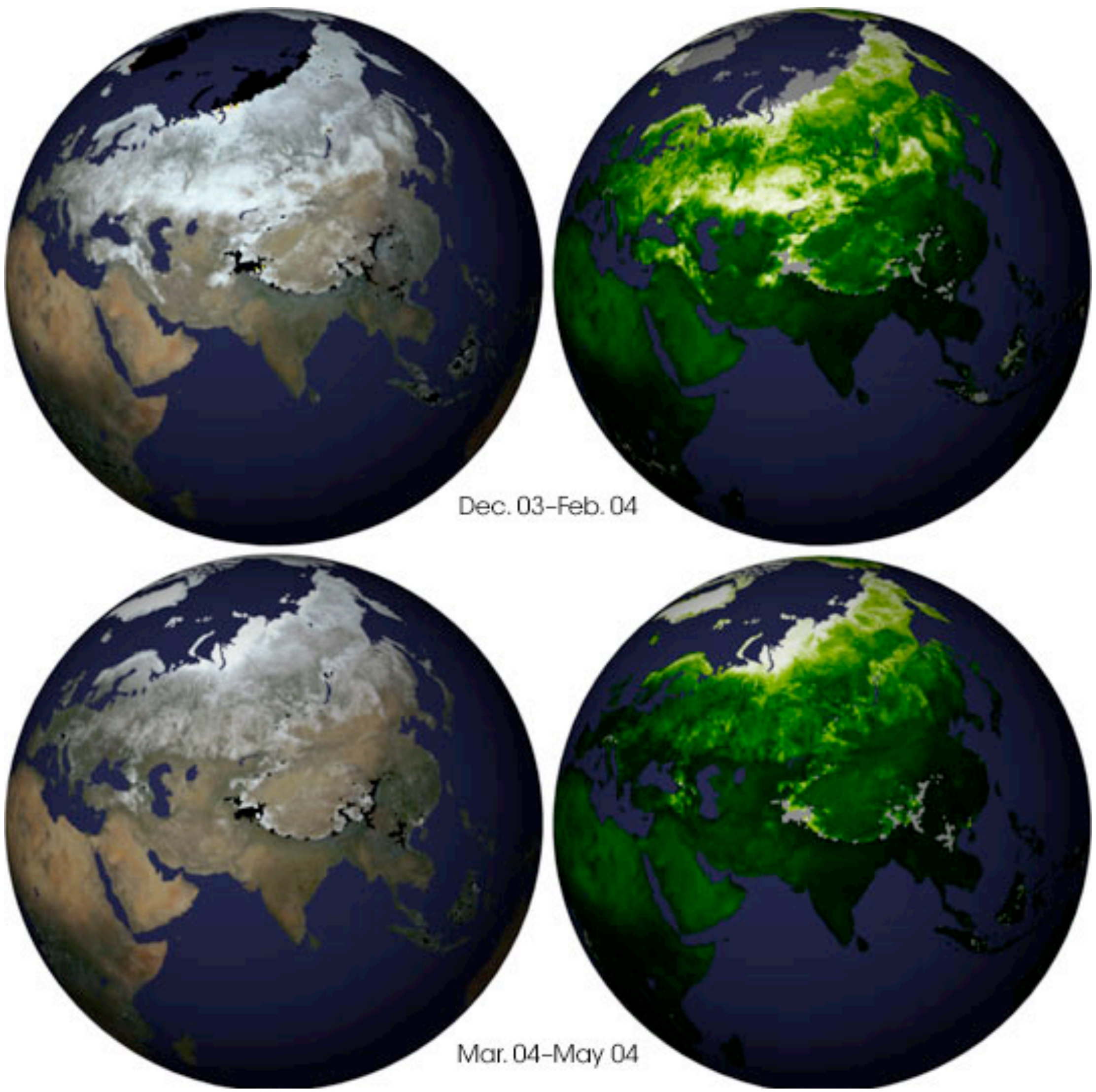
$$I^{\uparrow} = \frac{rF_i}{\pi}$$

$$I^{\uparrow} = \frac{rS_0 \cos\theta_i}{\pi}$$

← Assume incident flux is entirely due to direct solar radiation from a zenith angle $\theta_i$, where $S_0$ is the solar flux

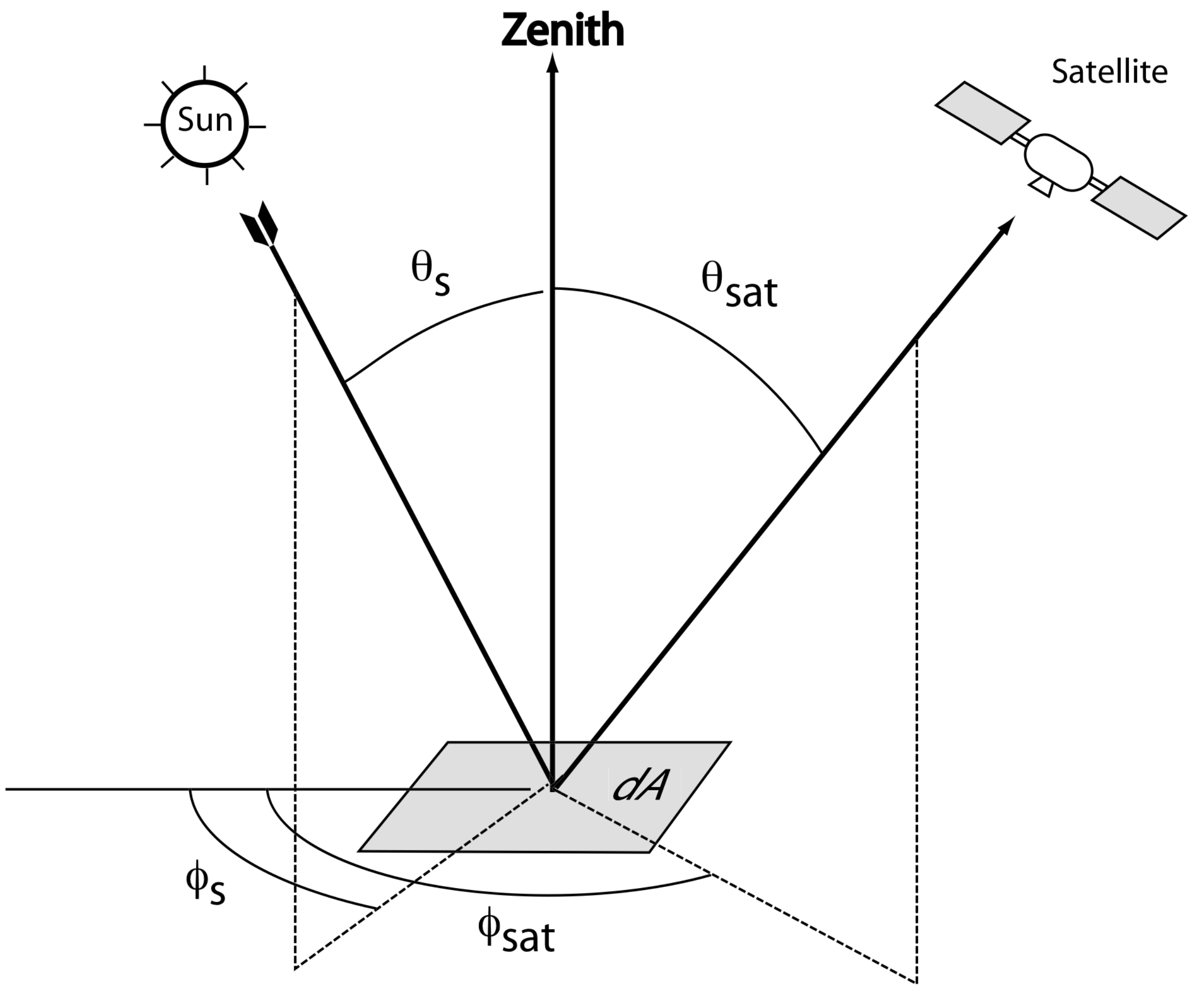
- Note that this is an assumption that simplifies radiative transfer calculations – the actual angular distribution of reflection must be determined from measurements

# Albedo and solar heating

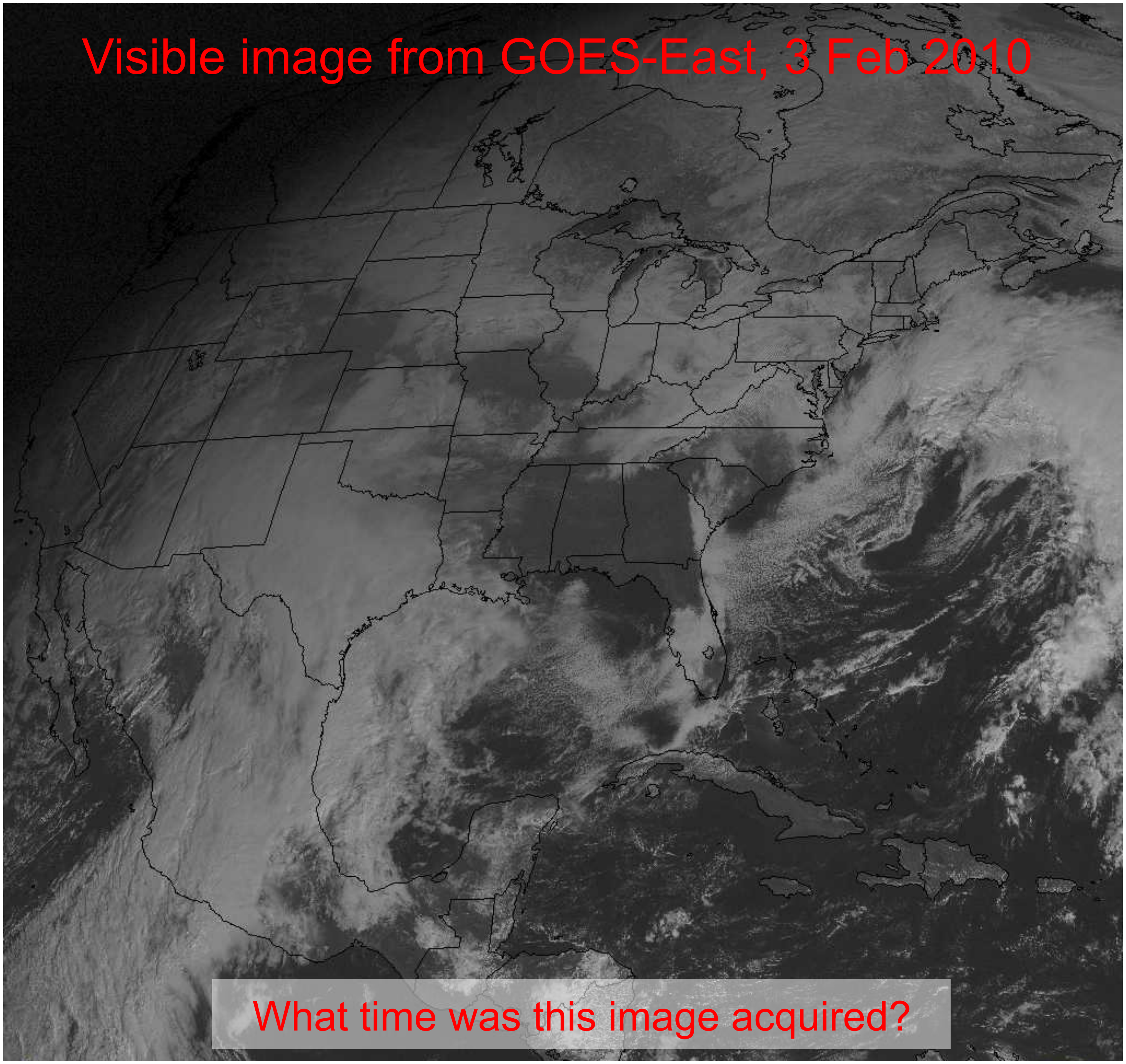



- Surface albedo determines direct heating by sunlight and hence the heating of adjacent air
- Compare bare soil (albedo ~10%) with fresh, dry snow (albedo ~90%)
- Surface type therefore affects the temperature of the overlying air

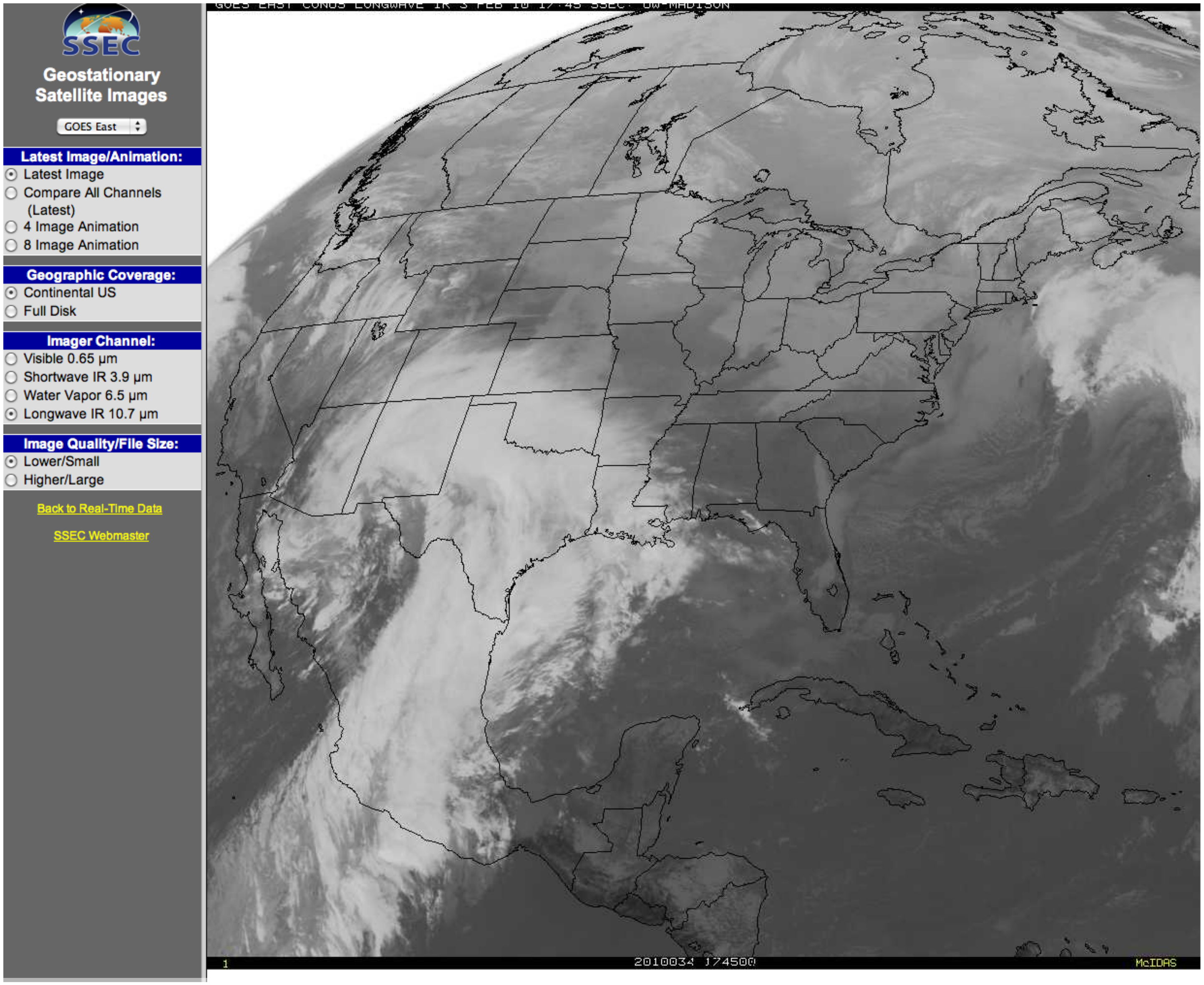
# Satellite imaging in the Visible and Near-IR




- Schematic viewing geometry for satellite remote sensing measurements


Visible image from GOES-East, 3 Feb 2010
What time was this image acquired?

# Geostationary Satellite Imagery from UW-Madison




http://www.ssec.wisc.edu/data/geo/