Structural Equation Modeling with lavaan

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2021-04-14

# Confirmatory Factor Analysis, Latent Variable Models, and Structural Equation Modeling

## Additional Sources:

* <http://www.phusewiki.org/docs/Conference%202013%20HE%20Papers/HE06.pdf>
* <https://blogs.baylor.edu/rlatentvariable/sample-page/r-syntax/>
* <https://methodenlehre.github.io/SGSCLM-R-course/cfa-and-sem-with-lavaan.html>
* <https://dominikfroehlich.com/wp-content/uploads/2018/07/Analyses-1.html>

## Paper comparing different packages (within and outside of R):

* <https://psychology.concordia.ca/fac/kline/sem/istql/narayanan.pdf>

## Packages:

* lavaan
* semPlot These install a lot of additional packages for analysis and visualization.

Exploratory factor analysis (EFA) can be used to discover common underlying factors. But sometimes you might have hypotheses about the factor structure, and want to either test or confirm this in your data. The particular factor structure that come out of EFA might be the best bottom-up description, but one associated with a particular theory may be nearly as good, or perhaps even equally good, and it might be better than a description related to an alternate theory. For example, in PCA, we saw a data set created so that factors of visual and verbal intelligence emerged. The solution showed two factors, but one related to general intelligence and one that was a difference between visual and verbal. When we applied exploratory factor analysis, a rotation changed this to map more closely onto our theory, but what if we’d like to identify a particular structure (or set of alternative structures) and determine whether the data are well described by this structure? We can do this with a set of methods known as confirmatory factor analysis, structure equation modeling, and other advanced related methods.

Methods for doing this in the context of factor analysis are often called confirmatory factor analysis. However, these probably only properly apply to independent factor models. That is, you could create multiple specific models that look like regression equations, but the predictor variable is a latent, hidden variable (much like mixture-modeling). But researchers found that their theoretical models were more complex than this, and the same machinery that permits inferring latent factors can infer much more sit models that determine a more complex structure. There are many varieties of these that go under many names; most commonly structure equation models (SEM), latent variable models, path analysis models, and specific applications like latent growth models. Furthermore, advances in hierarchical Bayes models provides substantial ability to infer latent structures. Many of these models (especially the SEM variety) still rely on correlation matrices for the data, and so even those that claim to unearth na causal structure are basing this on correlations, and are really not able to make causal inferences (this relies on experimental design, not statistical inference).

# Confirmatory Factor Analysis Example

Confirmatory factor analysis (CFA) is a way to create a latent structure and test it (usually against another latent structure) in order to compare hypotheses. Generally, you will specify the factors loading onto variables. These variables are independent factors. You can compare different numbers of factors, or different loadings. This is useful if you do a follow-up experiment after a first experiment where you conducted an EFA to identify general factors.

We will use the lavaan library for this, although other free and commercial projects exist. We will start by looking at the built-in confirmatory factor analysis example in lavaan:

library(lavaan) ##needed for holzingerswineford data  
data <- dplyr::select(HolzingerSwineford1939, x1:x9)  
cc <- cor(data)  
pca <- princomp(cc)  
loadings(pca)

Loadings:  
 Comp.1 Comp.2 Comp.3 Comp.4 Comp.5 Comp.6 Comp.7 Comp.8 Comp.9  
x1 0.290 0.607 0.566 0.419 0.169  
x2 0.522 -0.506 0.379 -0.183 0.529  
x3 -0.136 0.436 0.336 -0.549 -0.276 -0.410 0.356  
x4 0.498 -0.145 0.101 0.106 -0.428 -0.715   
x5 0.519 -0.184 0.175 -0.162 -0.313 0.609 0.399  
x6 0.486 -0.238 0.815 -0.142   
x7 -0.273 -0.532 -0.318 0.413 0.169 0.576  
x8 -0.322 -0.320 0.113 0.302 0.461 -0.610 -0.203 0.241  
 [ reached getOption("max.print") -- omitted 1 row ]  
  
 Comp.1 Comp.2 Comp.3 Comp.4 Comp.5 Comp.6 Comp.7 Comp.8 Comp.9  
SS loadings 1.000 1.000 1.000 1.000 1.000 1.000 1.000 1.000 1.000  
Proportion Var 0.111 0.111 0.111 0.111 0.111 0.111 0.111 0.111 0.111  
Cumulative Var 0.111 0.222 0.333 0.444 0.556 0.667 0.778 0.889 1.000

f <- factanal(data, factors = 3)  
  
# f2 <- fa:fa(data,factors=3)

library(lavaan)  
  
library(semPlot) #this won't compile for me  
example(cfa)

cfa> ## The famous Holzinger and Swineford (1939) example  
cfa> HS.model <- ' visual =~ x1 + x2 + x3  
cfa+ textual =~ x4 + x5 + x6  
cfa+ speed =~ x7 + x8 + x9 '  
  
cfa> fit <- cfa(HS.model, data = HolzingerSwineford1939)  
  
cfa> summary(fit, fit.measures = TRUE)  
lavaan 0.6-8 ended normally after 35 iterations  
  
 Estimator ML  
 Optimization method NLMINB  
 Number of model parameters 21  
   
 Number of observations 301  
   
Model Test User Model:  
   
 Test statistic 85.306  
 Degrees of freedom 24  
 P-value (Chi-square) 0.000  
  
Model Test Baseline Model:  
  
 Test statistic 918.852  
 Degrees of freedom 36  
 P-value 0.000  
  
User Model versus Baseline Model:  
  
 Comparative Fit Index (CFI) 0.931  
 Tucker-Lewis Index (TLI) 0.896  
  
Loglikelihood and Information Criteria:  
  
 Loglikelihood user model (H0) -3737.745  
 Loglikelihood unrestricted model (H1) -3695.092  
   
 Akaike (AIC) 7517.490  
 Bayesian (BIC) 7595.339  
 Sample-size adjusted Bayesian (BIC) 7528.739  
  
Root Mean Square Error of Approximation:  
  
 RMSEA 0.092  
 90 Percent confidence interval - lower 0.071  
 90 Percent confidence interval - upper 0.114  
 P-value RMSEA <= 0.05 0.001  
  
Standardized Root Mean Square Residual:  
  
 SRMR 0.065  
  
Parameter Estimates:  
  
 Standard errors Standard  
 Information Expected  
 Information saturated (h1) model Structured  
  
Latent Variables:  
 Estimate Std.Err z-value P(>|z|)  
 visual =~   
 x1 1.000   
 x2 0.554 0.100 5.554 0.000  
 x3 0.729 0.109 6.685 0.000  
 textual =~   
 x4 1.000   
 x5 1.113 0.065 17.014 0.000  
 x6 0.926 0.055 16.703 0.000  
 speed =~   
 x7 1.000   
 x8 1.180 0.165 7.152 0.000  
 x9 1.082 0.151 7.155 0.000  
  
Covariances:  
 Estimate Std.Err z-value P(>|z|)  
 visual ~~   
 textual 0.408 0.074 5.552 0.000  
 speed 0.262 0.056 4.660 0.000  
 textual ~~   
 speed 0.173 0.049 3.518 0.000  
  
Variances:  
 Estimate Std.Err z-value P(>|z|)  
 .x1 0.549 0.114 4.833 0.000  
 .x2 1.134 0.102 11.146 0.000  
 .x3 0.844 0.091 9.317 0.000  
 .x4 0.371 0.048 7.779 0.000  
 .x5 0.446 0.058 7.642 0.000  
 .x6 0.356 0.043 8.277 0.000  
 .x7 0.799 0.081 9.823 0.000  
 .x8 0.488 0.074 6.573 0.000  
 .x9 0.566 0.071 8.003 0.000  
 visual 0.809 0.145 5.564 0.000  
 textual 0.979 0.112 8.737 0.000  
 speed 0.384 0.086 4.451 0.000

let’s break down this. This uses a data set HolzingerSwineford1939. This data set has nine measures that measure a number of abilties:

* x1: visual perception
* x2: cubes test
* x3: lozenges test
* x4: paragraph comprehension
* x5: sentence completion
* x6: word meaning
* x7: speeded addition
* x8: speeded counting of dots
* x9: speeded discrimination of letters

These have often been thought of having three main components: visual (x1.3), text (x4.6) and speed (x7:9).

help(HolzingerSwineford1939)

The name lavaan refers to latent variable analysis, which is the essence of confirmatory factor analysis. This is similar to the latent variables we used in mixture modeling (hidden group membership), as well as latent variables used in item response theory.

For simple confirmatory factor analysis, you can think of it as a set of regressions, but instead of an outcome variable, we relate observable measures to a latent variable. For lavaan, we specify a model using a special text markup that isn’t exactly R code. Enter the latent variable names on the left, the observed names on the right, separated with =~, and with each factor separated by a line break. Then, you can use the cfa function to fit it using a specified data set. The =~ operator indicates a relationship “is manifested by”, which means that the variable on the right need to actually measured. There are other relationships that you might use in more complex models.

HS.model <- " visual =~ x2 + x1 + x3  
 textual =~ x4 + x5 + x6  
 speed =~ x7 + x8 + x9 "  
  
  
fit <- cfa(HS.model, data = HolzingerSwineford1939)  
fit

lavaan 0.6-8 ended normally after 38 iterations  
  
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 Number of observations 301  
   
Model Test User Model:  
   
 Test statistic 85.306  
 Degrees of freedom 24  
 P-value (Chi-square) 0.000

summary(fit, fit.measures = T)

lavaan 0.6-8 ended normally after 38 iterations  
  
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Model Test Baseline Model:  
  
 Test statistic 918.852  
 Degrees of freedom 36  
 P-value 0.000  
  
User Model versus Baseline Model:  
  
 Comparative Fit Index (CFI) 0.931  
 Tucker-Lewis Index (TLI) 0.896  
  
Loglikelihood and Information Criteria:  
  
 Loglikelihood user model (H0) -3737.745  
 Loglikelihood unrestricted model (H1) -3695.092  
   
 Akaike (AIC) 7517.490  
 Bayesian (BIC) 7595.339  
 Sample-size adjusted Bayesian (BIC) 7528.739  
  
Root Mean Square Error of Approximation:  
  
 RMSEA 0.092  
 90 Percent confidence interval - lower 0.071  
 90 Percent confidence interval - upper 0.114  
 P-value RMSEA <= 0.05 0.001  
  
Standardized Root Mean Square Residual:  
  
 SRMR 0.065  
  
Parameter Estimates:  
  
 Standard errors Standard  
 Information Expected  
 Information saturated (h1) model Structured  
  
Latent Variables:  
 Estimate Std.Err z-value P(>|z|)  
 visual =~   
 x2 1.000   
 x1 1.807 0.325 5.554 0.000  
 x3 1.318 0.239 5.509 0.000  
 textual =~   
 x4 1.000   
 x5 1.113 0.065 17.014 0.000  
 x6 0.926 0.055 16.703 0.000  
 speed =~   
 x7 1.000   
 x8 1.180 0.165 7.152 0.000  
 x9 1.082 0.151 7.155 0.000  
  
Covariances:  
 Estimate Std.Err z-value P(>|z|)  
 visual ~~   
 textual 0.226 0.052 4.329 0.000  
 speed 0.145 0.038 3.864 0.000  
 textual ~~   
 speed 0.173 0.049 3.518 0.000  
  
Variances:  
 Estimate Std.Err z-value P(>|z|)  
 .x2 1.134 0.102 11.146 0.000  
 .x1 0.549 0.114 4.833 0.000  
 .x3 0.844 0.091 9.317 0.000  
 .x4 0.371 0.048 7.778 0.000  
 .x5 0.446 0.058 7.642 0.000  
 .x6 0.356 0.043 8.277 0.000  
 .x7 0.799 0.081 9.823 0.000  
 .x8 0.488 0.074 6.573 0.000  
 .x9 0.566 0.071 8.003 0.000  
 visual 0.248 0.077 3.214 0.001  
 textual 0.979 0.112 8.737 0.000  
 speed 0.384 0.086 4.451 0.000

The above model should produce the same model that the ‘example’ did.

HS.model2 <- " visual =~ x1 + x2 + x3 + x4 + x5 + x6  
 speed =~ x7 + x8 + x9 "  
fit2 <- cfa(HS.model2, data = HolzingerSwineford1939)  
fit2

lavaan 0.6-8 ended normally after 35 iterations  
  
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 Optimization method NLMINB  
 Number of model parameters 19  
   
 Number of observations 301  
   
Model Test User Model:  
   
 Test statistic 181.337  
 Degrees of freedom 26  
 P-value (Chi-square) 0.000

summary(fit2, fit.measures = T)

lavaan 0.6-8 ended normally after 35 iterations  
  
 Estimator ML  
 Optimization method NLMINB  
 Number of model parameters 19  
   
 Number of observations 301  
   
Model Test User Model:  
   
 Test statistic 181.337  
 Degrees of freedom 26  
 P-value (Chi-square) 0.000  
  
Model Test Baseline Model:  
  
 Test statistic 918.852  
 Degrees of freedom 36  
 P-value 0.000  
  
User Model versus Baseline Model:  
  
 Comparative Fit Index (CFI) 0.824  
 Tucker-Lewis Index (TLI) 0.756  
  
Loglikelihood and Information Criteria:  
  
 Loglikelihood user model (H0) -3785.760  
 Loglikelihood unrestricted model (H1) -3695.092  
   
 Akaike (AIC) 7609.521  
 Bayesian (BIC) 7679.956  
 Sample-size adjusted Bayesian (BIC) 7619.699  
  
Root Mean Square Error of Approximation:  
  
 RMSEA 0.141  
 90 Percent confidence interval - lower 0.122  
 90 Percent confidence interval - upper 0.161  
 P-value RMSEA <= 0.05 0.000  
  
Standardized Root Mean Square Residual:  
  
 SRMR 0.113  
  
Parameter Estimates:  
  
 Standard errors Standard  
 Information Expected  
 Information saturated (h1) model Structured  
  
Latent Variables:  
 Estimate Std.Err z-value P(>|z|)  
 visual =~   
 x1 1.000   
 x2 0.509 0.156 3.254 0.001  
 x3 0.477 0.150 3.189 0.001  
 x4 1.992 0.272 7.314 0.000  
 x5 2.194 0.300 7.303 0.000  
 x6 1.852 0.254 7.294 0.000  
 speed =~   
 x7 1.000   
 x8 1.148 0.163 7.054 0.000  
 x9 0.891 0.124 7.189 0.000  
  
Covariances:  
 Estimate Std.Err z-value P(>|z|)  
 visual ~~   
 speed 0.095 0.029 3.287 0.001  
  
Variances:  
 Estimate Std.Err z-value P(>|z|)  
 .x1 1.112 0.093 11.922 0.000  
 .x2 1.318 0.108 12.193 0.000  
 .x3 1.219 0.100 12.196 0.000  
 .x4 0.373 0.047 7.853 0.000  
 .x5 0.474 0.059 8.059 0.000  
 .x6 0.351 0.043 8.221 0.000  
 .x7 0.732 0.083 8.821 0.000  
 .x8 0.428 0.082 5.201 0.000  
 .x9 0.658 0.071 9.300 0.000  
 visual 0.246 0.067 3.661 0.000  
 speed 0.451 0.095 4.733 0.000

anova(fit, fit2)

Chi-Squared Difference Test  
  
 Df AIC BIC Chisq Chisq diff Df diff Pr(>Chisq)   
fit 24 7517.5 7595.3 85.305   
fit2 26 7609.5 7680.0 181.337 96.031 2 < 2.2e-16 \*\*\*  
---  
Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

partable(fit)

id lhs op rhs user block group free ustart exo label plabel start est  
1 1 visual =~ x2 1 1 1 0 1 0 .p1. 1.000 1.000  
2 2 visual =~ x1 1 1 1 1 NA 0 .p2. 1.286 1.807  
3 3 visual =~ x3 1 1 1 2 NA 0 .p3. 1.424 1.318  
4 4 textual =~ x4 1 1 1 0 1 0 .p4. 1.000 1.000  
5 5 textual =~ x5 1 1 1 3 NA 0 .p5. 1.133 1.113  
 se  
1 0.000  
2 0.325  
3 0.239  
4 0.000  
5 0.065  
 [ reached 'max' / getOption("max.print") -- omitted 19 rows ]

# install.packages('semPlot',dependencies=T)  
library(psych)  
## library(semPlot) ##this won't install for me! semPaths(fit,  
## 'model','est',curvePivot = FALSE,edge.label.cex = 0.5)  
  
lavaan.diagram(fit, cut = 0.2, digits = 2) ##in psych package  
summary(fit, fit.measures = T, standardize = T)

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 90 Percent confidence interval - lower 0.071  
 90 Percent confidence interval - upper 0.114  
 P-value RMSEA <= 0.05 0.001  
  
Standardized Root Mean Square Residual:  
  
 SRMR 0.065  
  
Parameter Estimates:  
  
 Standard errors Standard  
 Information Expected  
 Information saturated (h1) model Structured  
  
Latent Variables:  
 Estimate Std.Err z-value P(>|z|) Std.lv Std.all  
 visual =~   
 x2 1.000 0.498 0.424  
 x1 1.807 0.325 5.554 0.000 0.900 0.772  
 x3 1.318 0.239 5.509 0.000 0.656 0.581  
 textual =~   
 x4 1.000 0.990 0.852  
 x5 1.113 0.065 17.014 0.000 1.102 0.855  
 x6 0.926 0.055 16.703 0.000 0.917 0.838  
 speed =~   
 x7 1.000 0.619 0.570  
 [ reached getOption("max.print") -- omitted 2 rows ]  
  
Covariances:  
 Estimate Std.Err z-value P(>|z|) Std.lv Std.all  
 visual ~~   
 textual 0.226 0.052 4.329 0.000 0.459 0.459  
 speed 0.145 0.038 3.864 0.000 0.471 0.471  
 textual ~~   
 speed 0.173 0.049 3.518 0.000 0.283 0.283  
  
Variances:  
 Estimate Std.Err z-value P(>|z|) Std.lv Std.all  
 .x2 1.134 0.102 11.146 0.000 1.134 0.821  
 .x1 0.549 0.114 4.833 0.000 0.549 0.404  
 .x3 0.844 0.091 9.317 0.000 0.844 0.662  
 .x4 0.371 0.048 7.778 0.000 0.371 0.275  
 .x5 0.446 0.058 7.642 0.000 0.446 0.269  
 .x6 0.356 0.043 8.277 0.000 0.356 0.298  
 .x7 0.799 0.081 9.823 0.000 0.799 0.676  
 .x8 0.488 0.074 6.573 0.000 0.488 0.477  
 .x9 0.566 0.071 8.003 0.000 0.566 0.558  
 visual 0.248 0.077 3.214 0.001 1.000 1.000  
 [ reached getOption("max.print") -- omitted 2 rows ]

inspect(fit, "rsquare")

x2 x1 x3 x4 x5 x6 x7 x8 x9   
0.179 0.596 0.338 0.725 0.731 0.702 0.324 0.523 0.442

semPaths(fit)

This gives some tests for whether the models are any good. First, let’s look at the diagram.

The circles indicate latent variables. The boxes indicate measured variables. two-way arrows indicate covariances; one-way arrows indicate a scaled covariance between latentt and measured variables (the ~= operation), and this can be interpreted as a factor loading much like exploratory factor analysis.

Notice that for each latent variable, one loading is a dashed line, and ‘fixed’ at 1.0. This is done because there is a lack of identifiability for latent variables–you don’t really know what values they take. So by default, they are pinned to the first measure, and other loadings are expressed relative to that. So, x1 is the strongest contributor to visual performance; x5 the strongest to text, and x8 the strongest to speed. Connection that point in a circle to the same node indicate variances, and can be used to assess reliability of estimates. Finally, two-way arrows indicate covariances between latent variables. Here, we see that the relationships between the three latent variables is small but positive–with scaled covariances of .41, .26, and .17. These numbers appear in the summary() of the fitted model as well.

The summary function provides many statistics that will help interpretation. David Kenny’s page has more details on some of these that were not in standard factor analysis <http://davidakenny.net/cm/fit.htm>

CFI (comparative fit index) is like TLI, but will always be higher as it does not penalize complexity as much. Kenny says CFI/TLI are not meaningful unless RMSEA is below about .15. In this case RMSE is .09, which is OK but not great.

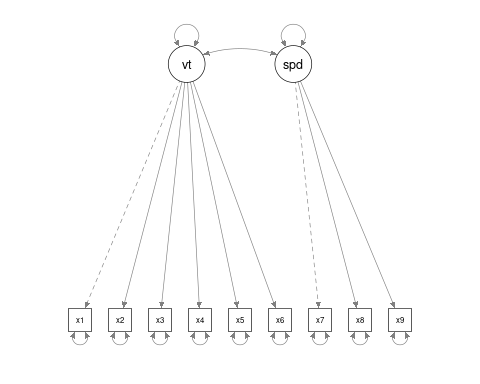
# Testing alternate models

The model is reasonable, but the more powerful argument is to compare it to other models that are also reasonable. Vsl and text are correlated; maybe they really don’t belong as separate latent factors.

HS.model2 <- " vt =~ x1 + x2 + x3 + x4 + x5 + x6  
 speed =~ x7 + x8 + x9 "  
fit2 <- cfa(HS.model2, data = HolzingerSwineford1939)  
fit2

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 P-value (Chi-square) 0.000

semPaths(fit2)

 These are both ‘significant’ but the test statistic differs a lot. We can test whether they differ significantly using the anova function, which also gives us AIC/BIC statistics.

anova(fit, fit2)

Chi-Squared Difference Test  
  
 Df AIC BIC Chisq Chisq diff Df diff Pr(>Chisq)   
fit 24 7517.5 7595.3 85.305   
fit2 26 7609.5 7680.0 181.337 96.031 2 < 2.2e-16 \*\*\*  
---  
Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

summary(fit, fit.measures = T)

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 x1 1.807 0.325 5.554 0.000  
 x3 1.318 0.239 5.509 0.000  
 textual =~   
 x4 1.000   
 x5 1.113 0.065 17.014 0.000  
 x6 0.926 0.055 16.703 0.000  
 speed =~   
 x7 1.000   
 x8 1.180 0.165 7.152 0.000  
 x9 1.082 0.151 7.155 0.000  
  
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 speed 0.145 0.038 3.864 0.000  
 textual ~~   
 speed 0.173 0.049 3.518 0.000  
  
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 .x1 0.549 0.114 4.833 0.000  
 .x3 0.844 0.091 9.317 0.000  
 .x4 0.371 0.048 7.778 0.000  
 .x5 0.446 0.058 7.642 0.000  
 .x6 0.356 0.043 8.277 0.000  
 .x7 0.799 0.081 9.823 0.000  
 .x8 0.488 0.074 6.573 0.000  
 .x9 0.566 0.071 8.003 0.000  
 visual 0.248 0.077 3.214 0.001  
 textual 0.979 0.112 8.737 0.000  
 speed 0.384 0.086 4.451 0.000

summary(fit2, fit.measures = T)

lavaan 0.6-8 ended normally after 35 iterations  
  
 Estimator ML  
 Optimization method NLMINB  
 Number of model parameters 19  
   
 Number of observations 301  
   
Model Test User Model:  
   
 Test statistic 181.337  
 Degrees of freedom 26  
 P-value (Chi-square) 0.000  
  
Model Test Baseline Model:  
  
 Test statistic 918.852  
 Degrees of freedom 36  
 P-value 0.000  
  
User Model versus Baseline Model:  
  
 Comparative Fit Index (CFI) 0.824  
 Tucker-Lewis Index (TLI) 0.756  
  
Loglikelihood and Information Criteria:  
  
 Loglikelihood user model (H0) -3785.760  
 Loglikelihood unrestricted model (H1) -3695.092  
   
 Akaike (AIC) 7609.521  
 Bayesian (BIC) 7679.956  
 Sample-size adjusted Bayesian (BIC) 7619.699  
  
Root Mean Square Error of Approximation:  
  
 RMSEA 0.141  
 90 Percent confidence interval - lower 0.122  
 90 Percent confidence interval - upper 0.161  
 P-value RMSEA <= 0.05 0.000  
  
Standardized Root Mean Square Residual:  
  
 SRMR 0.113  
  
Parameter Estimates:  
  
 Standard errors Standard  
 Information Expected  
 Information saturated (h1) model Structured  
  
Latent Variables:  
 Estimate Std.Err z-value P(>|z|)  
 vt =~   
 x1 1.000   
 x2 0.509 0.156 3.254 0.001  
 x3 0.477 0.150 3.189 0.001  
 x4 1.992 0.272 7.314 0.000  
 x5 2.194 0.300 7.303 0.000  
 x6 1.852 0.254 7.294 0.000  
 speed =~   
 x7 1.000   
 x8 1.148 0.163 7.054 0.000  
 x9 0.891 0.124 7.189 0.000  
  
Covariances:  
 Estimate Std.Err z-value P(>|z|)  
 vt ~~   
 speed 0.095 0.029 3.287 0.001  
  
Variances:  
 Estimate Std.Err z-value P(>|z|)  
 .x1 1.112 0.093 11.922 0.000  
 .x2 1.318 0.108 12.193 0.000  
 .x3 1.219 0.100 12.196 0.000  
 .x4 0.373 0.047 7.853 0.000  
 .x5 0.474 0.059 8.059 0.000  
 .x6 0.351 0.043 8.221 0.000  
 .x7 0.732 0.083 8.821 0.000  
 .x8 0.428 0.082 5.201 0.000  
 .x9 0.658 0.071 9.300 0.000  
 vt 0.246 0.067 3.661 0.000  
 speed 0.451 0.095 4.733 0.000

diagram(fit)  
diagram(fit2)

This suggests that the smaller model is fit more poorly according to RMSEA and TLI measures. Also, BIC is 7620 for the smaller model vs 7628 for the larger model, as are the AIC and related measures. Finally, anova chi-squared test is significantly different, indicating the larger model (fit) accounts for the data better than the smaller model (fit2). Altogether, this justifies the 3-factor model over a 2-factor model.

What if we give it a model we don’t think is correct–here f2 is cobbled together:

HS.model3 <- " f1 =~ x1 + x2  
 f2 =~ x3 + x4  
 f3 =~ x5 + x6  
 f4 =~ x7 + x8 + x9 "  
fit3 <- cfa(HS.model3, data = HolzingerSwineford1939)  
fit3

lavaan 0.6-8 ended normally after 60 iterations  
  
 Estimator ML  
 Optimization method NLMINB  
 Number of model parameters 24  
   
 Number of observations 301  
   
Model Test User Model:  
   
 Test statistic 144.108  
 Degrees of freedom 21  
 P-value (Chi-square) 0.000

There is a warning, but it does produce a result. We can use an anova function to compare the two:

anova(fit, fit3)

Chi-Squared Difference Test  
  
 Df AIC BIC Chisq Chisq diff Df diff Pr(>Chisq)  
fit3 21 7582.3 7671.3 144.108   
fit 24 7517.5 7595.3 85.305 -58.803 3 1

# semPlot::semPaths(fit3,title=F,curvePilot=T)  
lavaan.diagram(fit3, cut = 0.2, digits = 2)  
semPaths(fit3, "par", weighted = TRUE, nCharNodes = 7, shapeMan = "square", sizeMan = 8,   
 sizeMan2 = 5, layout = "tree2")

Now, there new more complex model, which has more parameters and fewer DF,fits more poorly than the smaller model, so we would again prefer the 3-parameter model.

# The Bifactor model

The bifactor model describes a model where everything is related to one general factor (g), but there are several other specific factors other things are related to.

## Compare to the following model, which has the same structure but it does not  
## have the covariance restrictions  
HS.modelG <- "G =~ x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9  
 visual =~ x1 + x2 + x3  
 textual =~ x4 + x5 + x6  
 speed =~ x7 + x8 + x9 "  
  
fitG <- cfa(HS.modelG, data = HolzingerSwineford1939)  
summary(fitG, fit.measures = T)

lavaan 0.6-8 ended normally after 66 iterations  
  
 Estimator ML  
 Optimization method NLMINB  
 Number of model parameters 33  
   
 Number of observations 301  
   
Model Test User Model:  
   
 Test statistic 26.020  
 Degrees of freedom 12  
 P-value (Chi-square) 0.011  
  
Model Test Baseline Model:  
  
 Test statistic 918.852  
 Degrees of freedom 36  
 P-value 0.000  
  
User Model versus Baseline Model:  
  
 Comparative Fit Index (CFI) 0.984  
 Tucker-Lewis Index (TLI) 0.952  
  
Loglikelihood and Information Criteria:  
  
 Loglikelihood user model (H0) -3708.102  
 Loglikelihood unrestricted model (H1) -3695.092  
   
 Akaike (AIC) 7482.204  
 Bayesian (BIC) 7604.539  
 Sample-size adjusted Bayesian (BIC) 7499.882  
  
Root Mean Square Error of Approximation:  
  
 RMSEA 0.062  
 90 Percent confidence interval - lower 0.029  
 90 Percent confidence interval - upper 0.095  
 P-value RMSEA <= 0.05 0.239  
  
Standardized Root Mean Square Residual:  
  
 SRMR 0.028  
  
Parameter Estimates:  
  
 Standard errors Standard  
 Information Expected  
 Information saturated (h1) model Structured  
  
Latent Variables:  
 Estimate Std.Err z-value P(>|z|)  
 G =~   
 x1 1.000   
 x2 0.613 NA   
 x3 0.768 NA   
 x4 0.583 NA   
 x5 0.552 NA   
 x6 0.561 NA   
 x7 0.442 NA   
 x8 0.619 NA   
 x9 0.699 NA   
 visual =~   
 x1 1.000   
 x2 0.152 NA   
 x3 0.127 NA   
 textual =~   
 [ reached getOption("max.print") -- omitted 7 rows ]  
  
Covariances:  
 Estimate Std.Err z-value P(>|z|)  
 G ~~   
 visual -0.061 NA   
 textual -0.334 NA   
 speed -0.435 NA   
 visual ~~   
 textual 0.234 NA   
 speed 0.063 NA   
 textual ~~   
 speed 0.279 NA   
  
Variances:  
 Estimate Std.Err z-value P(>|z|)  
 .x1 1.794 NA   
 .x2 1.025 NA   
 .x3 0.683 NA   
 .x4 0.379 NA   
 .x5 0.398 NA   
 .x6 0.368 NA   
 .x7 0.633 NA   
 .x8 0.469 NA   
 .x9 0.575 NA   
 G 1.062 NA   
 visual -1.377 NA   
 textual 1.000 NA   
 speed 0.727 NA

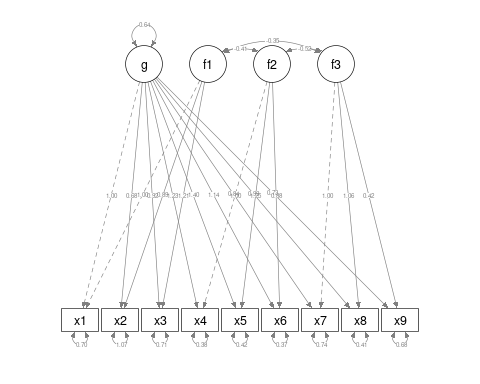
HS.model4 <- "g=~ x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9   
 f1 =~ x1 + x2 + x3  
 f2 =~ x4 + x5 + x6  
 f3 =~ x7 + x8 + x9   
 g ~~ 0\*f1  
 g ~~ 0\*f2  
 g ~~ 0\*f3  
 f1~~ 0\*f1  
 f2~~ 0\*f2  
 f3 ~~ 0\*f3  
"  
  
  
fit4 <- cfa(HS.model4, data = HolzingerSwineford1939)  
# fit4 <- cfa(HS.model4,data=HolzingerSwineford1939,estimator='WLS')  
fit4

lavaan 0.6-8 ended normally after 48 iterations  
  
 Estimator ML  
 Optimization method NLMINB  
 Number of model parameters 27  
   
 Number of observations 301  
   
Model Test User Model:  
   
 Test statistic 56.159  
 Degrees of freedom 18  
 P-value (Chi-square) 0.000

summary(fit4, fit.measures = T)

lavaan 0.6-8 ended normally after 48 iterations  
  
 Estimator ML  
 Optimization method NLMINB  
 Number of model parameters 27  
   
 Number of observations 301  
   
Model Test User Model:  
   
 Test statistic 56.159  
 Degrees of freedom 18  
 P-value (Chi-square) 0.000  
  
Model Test Baseline Model:  
  
 Test statistic 918.852  
 Degrees of freedom 36  
 P-value 0.000  
  
User Model versus Baseline Model:  
  
 Comparative Fit Index (CFI) 0.957  
 Tucker-Lewis Index (TLI) 0.914  
  
Loglikelihood and Information Criteria:  
  
 Loglikelihood user model (H0) -3723.172  
 Loglikelihood unrestricted model (H1) -3695.092  
   
 Akaike (AIC) 7500.344  
 Bayesian (BIC) 7600.435  
 Sample-size adjusted Bayesian (BIC) 7514.807  
  
Root Mean Square Error of Approximation:  
  
 RMSEA 0.084  
 90 Percent confidence interval - lower 0.060  
 90 Percent confidence interval - upper 0.109  
 P-value RMSEA <= 0.05 0.013  
  
Standardized Root Mean Square Residual:  
  
 SRMR 0.051  
  
Parameter Estimates:  
  
 Standard errors Standard  
 Information Expected  
 Information saturated (h1) model Structured  
  
Latent Variables:  
 Estimate Std.Err z-value P(>|z|)  
 g =~   
 x1 1.000   
 x2 0.685 0.121 5.655 0.000  
 x3 0.921 0.151 6.118 0.000  
 x4 1.235 0.144 8.579 0.000  
 x5 1.401 0.163 8.623 0.000  
 x6 1.140 0.134 8.536 0.000  
 x7 0.841 0.125 6.746 0.000  
 x8 0.986 0.133 7.427 0.000  
 x9 0.725 0.111 6.557 0.000  
 f1 =~   
 x1 1.000   
 x2 0.893 0.238 3.758 0.000  
 x3 1.205 0.299 4.033 0.000  
 f2 =~   
 [ reached getOption("max.print") -- omitted 7 rows ]  
  
Covariances:  
 Estimate Std.Err z-value P(>|z|)  
 g ~~   
 f1 0.000   
 f2 0.000   
 f3 0.000   
 f1 ~~   
 f2 -0.411 0.084 -4.897 0.000  
 f3 -0.349 0.081 -4.320 0.000  
 f2 ~~   
 f3 -0.523 0.084 -6.225 0.000  
  
Variances:  
 Estimate Std.Err z-value P(>|z|)  
 f1 0.000   
 f2 0.000   
 f3 0.000   
 .x1 0.703 0.113 6.252 0.000  
 .x2 1.070 0.101 10.545 0.000  
 .x3 0.711 0.102 6.996 0.000  
 .x4 0.383 0.048 7.939 0.000  
 .x5 0.416 0.058 7.143 0.000  
 .x6 0.370 0.044 8.482 0.000  
 .x7 0.737 0.084 8.751 0.000  
 .x8 0.406 0.086 4.704 0.000  
 .x9 0.680 0.071 9.558 0.000  
 g 0.640 0.135 4.748 0.000

semPaths(fit4, "par", weighted = FALSE, nCharNodes = 7, shapeMan = "rectangle", sizeMan = 8,   
 sizeMan2 = 5)



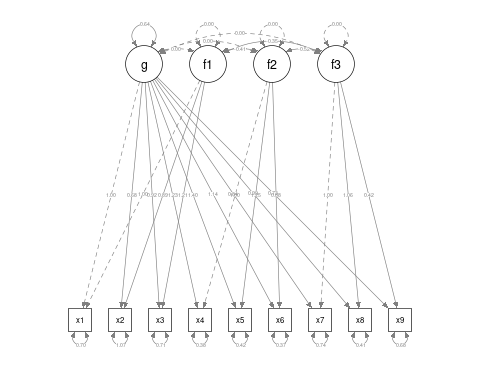
HS.model4b <- "g=~ x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9   
 f1 =~ x1 + x2 + x3  
 f2 =~ x4 + x5 + x6  
 f3 =~ x7 + x8   
 g ~~ 0\*f1  
 g ~~ 0\*f2  
 g ~~ 0\*f3  
 f1~~ 0\*f1  
 f2~~ 0\*f2  
 f3 ~~ 0\*f3  
"  
fit4b <- cfa(HS.model4b, data = HolzingerSwineford1939)  
summary(fit4b, fit.measures = T)

lavaan 0.6-8 ended normally after 44 iterations  
  
 Estimator ML  
 Optimization method NLMINB  
 Number of model parameters 26  
   
 Number of observations 301  
   
Model Test User Model:  
   
 Test statistic 65.101  
 Degrees of freedom 19  
 P-value (Chi-square) 0.000  
  
Model Test Baseline Model:  
  
 Test statistic 918.852  
 Degrees of freedom 36  
 P-value 0.000  
  
User Model versus Baseline Model:  
  
 Comparative Fit Index (CFI) 0.948  
 Tucker-Lewis Index (TLI) 0.901  
  
Loglikelihood and Information Criteria:  
  
 Loglikelihood user model (H0) -3727.643  
 Loglikelihood unrestricted model (H1) -3695.092  
   
 Akaike (AIC) 7507.285  
 Bayesian (BIC) 7603.670  
 Sample-size adjusted Bayesian (BIC) 7521.213  
  
Root Mean Square Error of Approximation:  
  
 RMSEA 0.090  
 90 Percent confidence interval - lower 0.066  
 90 Percent confidence interval - upper 0.114  
 P-value RMSEA <= 0.05 0.003  
  
Standardized Root Mean Square Residual:  
  
 SRMR 0.063  
  
Parameter Estimates:  
  
 Standard errors Standard  
 Information Expected  
 Information saturated (h1) model Structured  
  
Latent Variables:  
 Estimate Std.Err z-value P(>|z|)  
 g =~   
 x1 1.000   
 x2 0.626 0.106 5.902 0.000  
 x3 0.822 0.119 6.913 0.000  
 x4 1.173 0.122 9.591 0.000  
 x5 1.340 0.139 9.632 0.000  
 x6 1.085 0.114 9.538 0.000  
 x7 0.784 0.116 6.742 0.000  
 x8 0.932 0.127 7.363 0.000  
 x9 0.507 0.068 7.429 0.000  
 f1 =~   
 x1 1.000   
 x2 0.747 0.166 4.512 0.000  
 x3 0.986 0.185 5.335 0.000  
 f2 =~   
 [ reached getOption("max.print") -- omitted 6 rows ]  
  
Covariances:  
 Estimate Std.Err z-value P(>|z|)  
 g ~~   
 f1 0.000   
 f2 0.000   
 f3 0.000   
 f1 ~~   
 f2 -0.434 0.075 -5.808 0.000  
 f3 -0.457 0.088 -5.171 0.000  
 f2 ~~   
 f3 -0.533 0.085 -6.237 0.000  
  
Variances:  
 Estimate Std.Err z-value P(>|z|)  
 f1 0.000   
 f2 0.000   
 f3 0.000   
 .x1 0.618 0.105 5.867 0.000  
 .x2 1.091 0.100 10.855 0.000  
 .x3 0.773 0.091 8.450 0.000  
 .x4 0.387 0.048 8.063 0.000  
 .x5 0.419 0.058 7.224 0.000  
 .x6 0.370 0.043 8.546 0.000  
 .x7 0.723 0.096 7.563 0.000  
 .x8 0.375 0.107 3.500 0.000  
 .x9 0.827 0.062 13.266 0.000  
 g 0.734 0.136 5.414 0.000

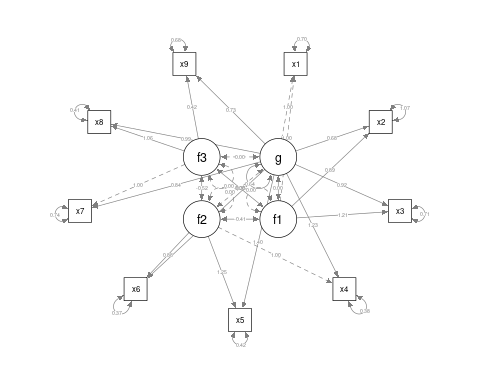
anova(fit4, fit4b)

Chi-Squared Difference Test  
  
 Df AIC BIC Chisq Chisq diff Df diff Pr(>Chisq)   
fit4 18 7500.3 7600.4 56.159   
fit4b 19 7507.3 7603.7 65.101 8.9415 1 0.002788 \*\*  
---  
Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

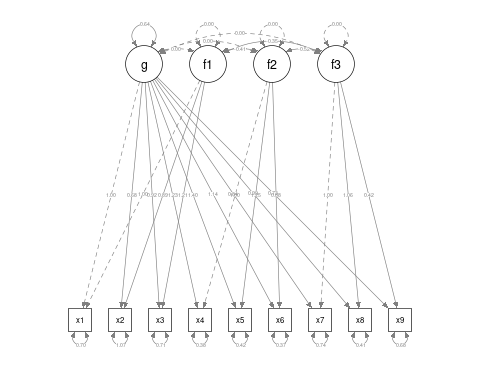
semPaths(fit4, "model", "est", curvePivot = FALSE, edge.label.cex = 0.5)



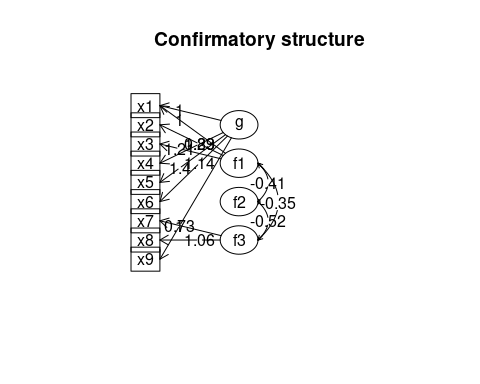
semPaths(fit4, "model", "est", layout = "circle", curvePivot = FALSE, edge.label.cex = 0.5)



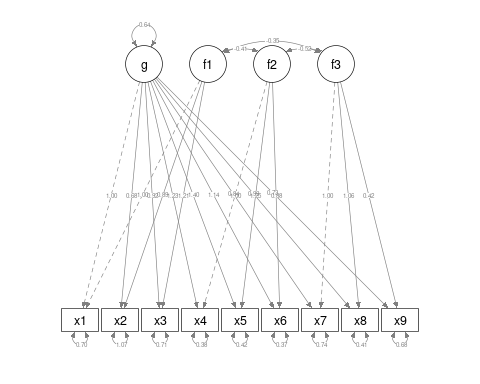
semPaths(fit4, "model", "est", layout = "tree", curvePivot = FALSE, edge.label.cex = 0.5)



lavaan.diagram(fit4, cut = 0.2, digits = 2)



semPaths(fit4, "par", weighted = FALSE, nCharNodes = 7, shapeMan = "rectangle", sizeMan = 8,   
 sizeMan2 = 5)



anova(fit4, fit)

Chi-Squared Difference Test  
  
 Df AIC BIC Chisq Chisq diff Df diff Pr(>Chisq)   
fit4 18 7500.3 7600.4 56.159   
fit 24 7517.5 7595.3 85.305 29.146 6 5.708e-05 \*\*\*  
---  
Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

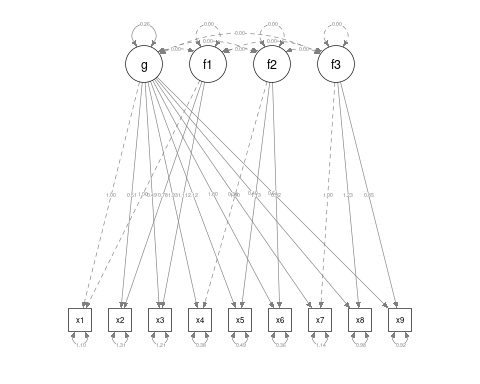
Here, we have mixed results; a Chi-squared test shows the more complex model is better, as does AIC; but BIC is pessimistic, feeling it is too complex. Looking at the parameter estimates, there are some strange things; we no longer have consistent positive loadings to f1, and we get a warning that it is not positive definite, which should make us wary that our data is not sufficiently constraining our model.

We can force the latent variables to be uncorrelated. In this case, the model does not converge, and so there is not much we can take away from it, but it might be useful in some situations. Plus, below we can see code that let’s us see the residuals of the correlation matrix, the basis for RMSE and RMSEA measures.

fit5 <- cfa(HS.model4, data = HolzingerSwineford1939, orthogonal = TRUE)  
summary(fit5, fit.measures = T)

lavaan 0.6-8 ended normally after 31 iterations  
  
 Estimator ML  
 Optimization method NLMINB  
 Number of model parameters 24  
   
 Number of observations 301  
   
Model Test User Model:  
   
 Test statistic 312.264  
 Degrees of freedom 21  
 P-value (Chi-square) 0.000  
  
Model Test Baseline Model:  
  
 Test statistic 918.852  
 Degrees of freedom 36  
 P-value 0.000  
  
User Model versus Baseline Model:  
  
 Comparative Fit Index (CFI) 0.670  
 Tucker-Lewis Index (TLI) 0.434  
  
Loglikelihood and Information Criteria:  
  
 Loglikelihood user model (H0) -3851.224  
 Loglikelihood unrestricted model (H1) -3695.092  
   
 Akaike (AIC) 7750.448  
 Bayesian (BIC) 7839.419  
 Sample-size adjusted Bayesian (BIC) 7763.305  
  
Root Mean Square Error of Approximation:  
  
 RMSEA 0.215  
 90 Percent confidence interval - lower 0.194  
 90 Percent confidence interval - upper 0.236  
 P-value RMSEA <= 0.05 0.000  
  
Standardized Root Mean Square Residual:  
  
 SRMR 0.143  
  
Parameter Estimates:  
  
 Standard errors Standard  
 Information Expected  
 Information saturated (h1) model Structured  
  
Latent Variables:  
 Estimate Std.Err z-value P(>|z|)  
 g =~   
 x1 1.000   
 x2 0.508 NA   
 x3 0.493 NA   
 x4 1.930 NA   
 x5 2.123 NA   
 x6 1.796 NA   
 x7 0.385 NA   
 x8 0.398 NA   
 x9 0.606 NA   
 f1 =~   
 x1 1.000   
 x2 0.778 NA   
 x3 1.107 NA   
 f2 =~   
 [ reached getOption("max.print") -- omitted 7 rows ]  
  
Covariances:  
 Estimate Std.Err z-value P(>|z|)  
 g ~~   
 f1 0.000   
 f2 0.000   
 f3 0.000   
 f1 ~~   
 f2 0.000   
 f3 0.000   
 f2 ~~   
 f3 0.000   
  
Variances:  
 Estimate Std.Err z-value P(>|z|)  
 f1 0.000   
 f2 0.000   
 f3 0.000   
 .x1 1.098 NA   
 .x2 1.315 NA   
 .x3 1.212 NA   
 .x4 0.380 NA   
 .x5 0.486 NA   
 .x6 0.356 NA   
 .x7 1.145 NA   
 .x8 0.981 NA   
 .x9 0.919 NA   
 g 0.261 NA

semPaths(fit5, "model", "est", curvePivot = FALSE, edge.label.cex = 0.5)



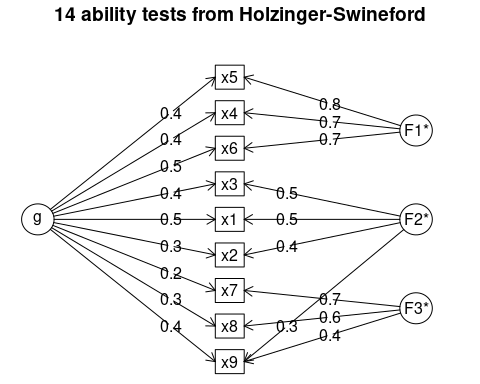
residuals(fit5, type = "cor")

$type  
[1] "cor.bollen"  
  
$cov  
 x1 x2 x3 x4 x5 x6 x7 x8 x9   
x1 0.000   
x2 0.201 0.000   
x3 0.343 0.291 0.000   
x4 0.001 -0.034 -0.030 0.000   
x5 -0.075 -0.046 -0.110 0.020 0.000   
x6 -0.010 0.008 0.011 -0.006 0.015 0.000   
x7 -0.012 -0.115 0.032 0.021 -0.050 -0.030 0.000   
x8 0.136 0.048 0.141 -0.064 -0.031 -0.019 0.450 0.000   
 [ reached getOption("max.print") -- omitted 1 row ]

## Bifactor model with the omega function

A traditional version of the bifactor model is available in the omega function in PSYCH, where the factors are automatically identified. This is more like EFA with a specific structure. Also, we discussed this model when examining psychometrics: this is argued to be the best way to identify whether a single factor exists, and more instructive than measures such as cohen’s or GLB.

library(psych)  
  
holz <- omega(cor(HolzingerSwineford1939[, 7:15]), 3, title = "14 ability tests from Holzinger-Swineford")



holz

14 ability tests from Holzinger-Swineford   
Call: omegah(m = m, nfactors = nfactors, fm = fm, key = key, flip = flip,   
 digits = digits, title = title, sl = sl, labels = labels,   
 plot = plot, n.obs = n.obs, rotate = rotate, Phi = Phi, option = option,   
 covar = covar)  
Alpha: 0.76   
G.6: 0.81   
Omega Hierarchical: 0.45   
Omega H asymptotic: 0.53   
Omega Total 0.85   
  
Schmid Leiman Factor loadings greater than 0.2   
 g F1\* F2\* F3\* h2 u2 p2  
x1 0.49 0.46 0.48 0.52 0.49  
x2 0.29 0.40 0.26 0.74 0.33  
x3 0.41 0.53 0.45 0.55 0.37  
x4 0.45 0.73 0.73 0.27 0.28  
x5 0.42 0.76 0.75 0.25 0.23  
x6 0.46 0.69 0.69 0.31 0.30  
x7 0.23 0.67 0.52 0.48 0.10  
x8 0.35 0.62 0.52 0.48 0.23  
x9 0.45 0.30 0.42 0.46 0.54 0.43  
  
With eigenvalues of:  
 g F1\* F2\* F3\*   
1.44 1.62 0.77 1.03   
  
general/max 0.89 max/min = 2.09  
mean percent general = 0.31 with sd = 0.12 and cv of 0.38   
Explained Common Variance of the general factor = 0.3   
  
The degrees of freedom are 12 and the fit is 0.08   
  
The root mean square of the residuals is 0.02   
The df corrected root mean square of the residuals is 0.03  
  
Compare this with the adequacy of just a general factor and no group factors  
The degrees of freedom for just the general factor are 27 and the fit is 1.75   
  
The root mean square of the residuals is 0.2   
The df corrected root mean square of the residuals is 0.23   
  
Measures of factor score adequacy   
 g F1\* F2\* F3\*  
Correlation of scores with factors 0.68 0.84 0.67 0.79  
Multiple R square of scores with factors 0.46 0.71 0.45 0.62  
Minimum correlation of factor score estimates -0.08 0.42 -0.10 0.23  
  
 Total, General and Subset omega for each subset  
 g F1\* F2\* F3\*  
Omega total for total scores and subscales 0.85 0.89 0.65 0.72  
Omega general for total scores and subscales 0.45 0.24 0.27 0.19  
Omega group for total scores and subscales 0.35 0.65 0.37 0.53

# More on the lavaan syntax and operators

The lavaan syntax lets you describe models with 4 operators:

* ~ Regression (structural model)
* =~ Latent variable definition (also measurement model)
* ~~ Specifying and fixing residual correlations and covariances
* ~1 Specifying intercepts

The third operator ~~ is tricky to understand. The most common use for this is that you are essentially adding the possibility for measured variables to be correlated (vs uncorrelated), or forcing them to be independent, capturing something not otherwise captured by latent variables. You can also use coefficient notation to force covariances to be of some value (helpful because the first covariance is set to 1.0 in a latent model) For example, you might add

L1 ~~ 0\*L2

to force independence of latent variables L1 and L2.

or

L1 =~ f1 + 1\*f2 + f3

The ~1 operator is useful for regression-style modeling, which is used for what are called latent growth models, used for longitidinal, aging, and development research.

# More complex models

Confirmatory factor analysis typically identifies a single set of factors and tries to model the data in that way. But the lavaan library offers more complex structural equation modeling and latent growth curve modeling, and general latent variable regressions, which is also useful in complex situations. For example, you could combine several measures to create a factor, which you then use as a regression predictor or outcome variable to establish relationships between hidden/latent variables. In developmental circles, it is common to use latent growth models to model the development or decline of specific latent cognitive or social behaviors or skills.

A common type of analysis that lavaan permits is looking at the role of mediating variables. There are many tools available for specifically looking at 3-variable problems, but lavaan lets you model arbitrarily complex mediation schemes. The analysis of these is more ad hoc though.

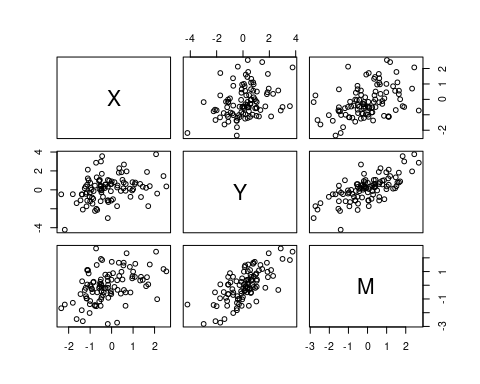
## Mediation Analysis

The notion of doing a mediation analysis is central to many areas of psychology, especially in social domains where you cannot have complete experimental control. In these cases, you will always have potentially confounding variables, and mediation analysis examines a particular kind of causal relationship. Some of the papers describing mediation are among the top 20 scientific research papers cited in any domain.

The notion is that you may observe a correlation between two variables, but also observe that these may be correlated with a third. For example, you might find that as a population ages from teenager to adult, vehicular accidents go down. But, it might not be age per se that causes this. Instead, it might be practice…as you get older, you get more experience driving, and the experience reduces accidents. Experience may be a mediating variable here. Can all the improvement in driver safety be attributed to experience? Maybe, but to find out, you need to study people who started driving at different ages. This would be a classic mediation test.

Below is an example of how we would model this with lavaan. Here, using standard terminology, y is the outcome (accident rate), X is the predictor (age), and M is the mediator variable (experience).

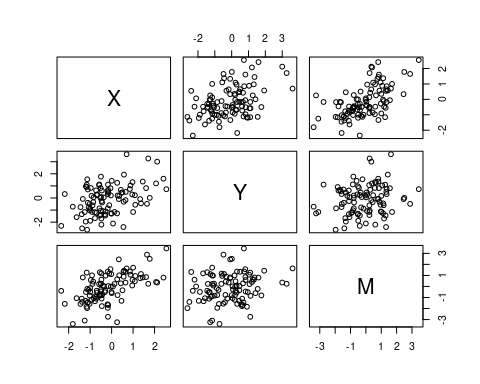
set.seed(1234)  
  
X <- rnorm(100)  
M <- 0.5 \* X + rnorm(100)  
Y <- 0.7 \* M + rnorm(100)  
  
M2 <- 0.6 \* X + rnorm(100)  
Y2 <- 0.6 \* X + rnorm(100)  
  
  
Data <- data.frame(X = X, Y = Y, M = M)  
Data2 <- data.frame(X = X, Y = Y2, M = M2)  
  
pairs(Data)



cor(Data)

X Y M  
X 1.0000000 0.3122748 0.4188856  
Y 0.3122748 1.0000000 0.6909791  
M 0.4188856 0.6909791 1.0000000

pairs(Data2)



cor(Data2)

X Y M  
X 1.0000000 0.4255321 0.5857204  
Y 0.4255321 1.0000000 0.1827901  
M 0.5857204 0.1827901 1.0000000

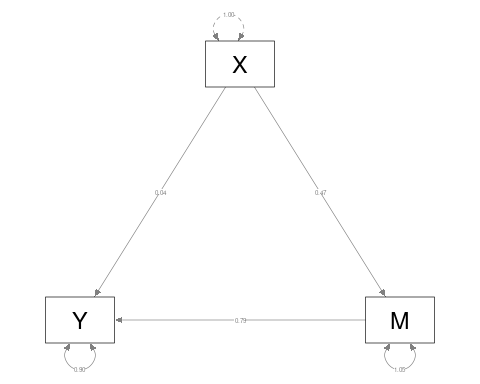
So, for both Data and Data2, we have three variables that are all correlated. In truth, for Data, Y is related only to M, but M is related to X. For Data2 both M and Y are related to X directly.

We use ~ to model direct relationships between observed variables. Here, we will fit a model with a direct relationship between X and Y, and also another link through M. We need to specify a variable to represent the interaction of a and b, using :=, and a final variance total that captures everthing.

library(lavaan)  
  
library(semPlot)  
  
model <- " # direct effect  
Y ~ c\*X  
# mediator  
M ~ a\*X  
Y ~ b\*M  
# indirect effect (a\*b)  
ab := a\*b  
# total effect  
total := c + (a\*b)  
"  
fitmed <- sem(model, data = Data)  
summary(fitmed)

lavaan 0.6-8 ended normally after 12 iterations  
  
 Estimator ML  
 Optimization method NLMINB  
 Number of model parameters 5  
   
 Number of observations 100  
   
Model Test User Model:  
   
 Test statistic 0.000  
 Degrees of freedom 0  
  
Parameter Estimates:  
  
 Standard errors Standard  
 Information Expected  
 Information saturated (h1) model Structured  
  
Regressions:  
 Estimate Std.Err z-value P(>|z|)  
 Y ~   
 X (c) 0.036 0.104 0.348 0.728  
 M ~   
 X (a) 0.474 0.103 4.613 0.000  
 Y ~   
 M (b) 0.788 0.092 8.539 0.000  
  
Variances:  
 Estimate Std.Err z-value P(>|z|)  
 .Y 0.898 0.127 7.071 0.000  
 .M 1.054 0.149 7.071 0.000  
  
Defined Parameters:  
 Estimate Std.Err z-value P(>|z|)  
 ab 0.374 0.092 4.059 0.000  
 total 0.410 0.125 3.287 0.001

semPaths(fitmed, title = F, curvePilot = T, "par", weighted = FALSE, nCharNodes = 7,   
 shapeMan = "rectangle", sizeMan = 15, sizeMan2 = 10)

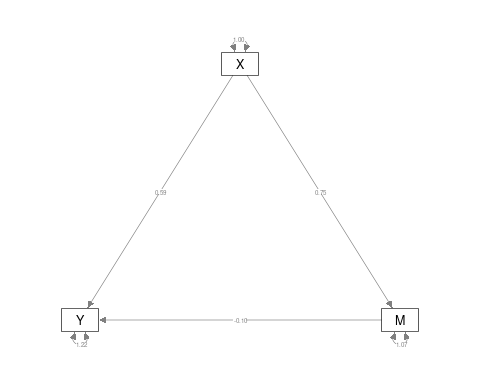


Looking at this first model (where M really mediates X-Y), we see that the edges X-M and M-Y are strong, but X->Y is weak, indicating M mediates the relationship between X and Y.

fitmed2 <- sem(model, data = Data2)  
summary(fitmed2)

lavaan 0.6-8 ended normally after 15 iterations  
  
 Estimator ML  
 Optimization method NLMINB  
 Number of model parameters 5  
   
 Number of observations 100  
   
Model Test User Model:  
   
 Test statistic 0.000  
 Degrees of freedom 0  
  
Parameter Estimates:  
  
 Standard errors Standard  
 Information Expected  
 Information saturated (h1) model Structured  
  
Regressions:  
 Estimate Std.Err z-value P(>|z|)  
 Y ~   
 X (c) 0.594 0.136 4.360 0.000  
 M ~   
 X (a) 0.748 0.104 7.227 0.000  
 Y ~   
 M (b) -0.097 0.107 -0.910 0.363  
  
Variances:  
 Estimate Std.Err z-value P(>|z|)  
 .Y 1.218 0.172 7.071 0.000  
 .M 1.070 0.151 7.071 0.000  
  
Defined Parameters:  
 Estimate Std.Err z-value P(>|z|)  
 ab -0.073 0.080 -0.903 0.367  
 total 0.521 0.111 4.702 0.000

# lavaan.diagram(fit2)  
semPaths(fitmed2, title = F, curvePilot = T, "par", weighted = FALSE, nCharNodes = 7,   
 shapeMan = "rectangle", sizeMan = 8, sizeMan2 = 5)

 Her, Y-M is not significant, but Y-X and M-X are, indicating that M does not mediate the relationship. Note that the model is framed to test whether a particular value is significant–we don’t make two models and compare theme via ANOVA.

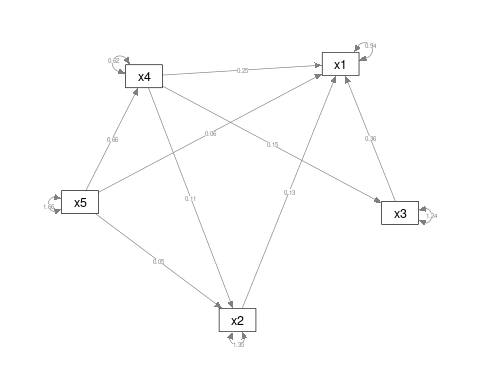
## Path analysis and more advanced structural equation modeling

We can use the ~ operator for characterizing a covariance structure among variables, and compare models. The ~ is directional, so we can make complex chains and directional networks. These are really sort of sets of linear regressions. As an example, suppose we want to characterize x1 and being related to 2/3/4, 2 to 4, 3 to 4, but then x1/2/4 all being related to 5:

pathAn1 <- "x1 ~ x2 + x3 + x4  
 x2 ~ x4  
 x3 ~ x4  
 x1 + x2 +x4 ~ x5  
x5 ~~ x5"  
  
paModel <- sem(pathAn1, data = HolzingerSwineford1939)  
summary(paModel, fit.measures = T)

lavaan 0.6-8 ended normally after 18 iterations  
  
 Estimator ML  
 Optimization method NLMINB  
 Number of model parameters 13  
   
 Number of observations 301  
   
Model Test User Model:  
   
 Test statistic 34.981  
 Degrees of freedom 2  
 P-value (Chi-square) 0.000  
  
Model Test Baseline Model:  
  
 Test statistic 392.278  
 Degrees of freedom 10  
 P-value 0.000  
  
User Model versus Baseline Model:  
  
 Comparative Fit Index (CFI) 0.914  
 Tucker-Lewis Index (TLI) 0.569  
  
Loglikelihood and Information Criteria:  
  
 Loglikelihood user model (H0) -2209.661  
 Loglikelihood unrestricted model (H1) -2192.171  
   
 Akaike (AIC) 4445.323  
 Bayesian (BIC) 4493.515  
 Sample-size adjusted Bayesian (BIC) 4452.287  
  
Root Mean Square Error of Approximation:  
  
 RMSEA 0.234  
 90 Percent confidence interval - lower 0.170  
 90 Percent confidence interval - upper 0.305  
 P-value RMSEA <= 0.05 0.000  
  
Standardized Root Mean Square Residual:  
  
 SRMR 0.088  
  
Parameter Estimates:  
  
 Standard errors Standard  
 Information Expected  
 Information saturated (h1) model Structured  
  
Regressions:  
 Estimate Std.Err z-value P(>|z|)  
 x1 ~   
 x2 0.129 0.048 2.685 0.007  
 x3 0.363 0.050 7.232 0.000  
 x4 0.249 0.071 3.492 0.000  
 x2 ~   
 x4 0.111 0.085 1.311 0.190  
 x3 ~   
 x4 0.154 0.055 2.788 0.005  
 x1 ~   
 x5 0.059 0.064 0.930 0.353  
 x2 ~   
 x5 0.054 0.076 0.704 0.481  
 x4 ~   
 x5 0.661 0.035 18.705 0.000  
  
Variances:  
 Estimate Std.Err z-value P(>|z|)  
 x5 1.660 0.135 12.268 0.000  
 .x1 0.943 0.077 12.268 0.000  
 .x2 1.347 0.110 12.268 0.000  
 .x3 1.243 0.101 12.268 0.000  
 .x4 0.625 0.051 12.268 0.000

semPaths(paModel, "par", weighted = FALSE, nCharNodes = 7, layout = "spring", sizeMan = 8,   
 sizeMan2 = 5)

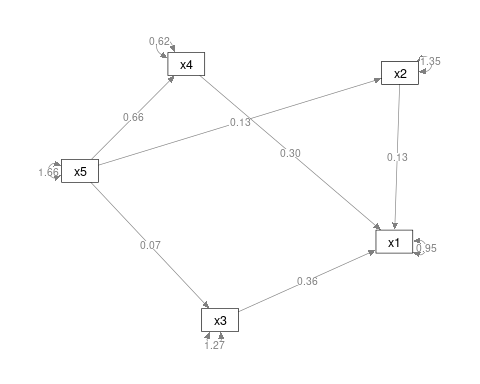


This model is not very good (but that is because we are just using it for example data). However, maybe we can try an alternative. This suggests that x1 depends on 2/3/4, and 2/3/4 depend on 5.

pathAn2 <- "x1 ~ x2 + x3 + x4  
 x2 + x3 + x4 ~ x5  
 x5 ~~ x5"  
  
paModel2 <- sem(pathAn2, data = HolzingerSwineford1939)  
summary(paModel2, fit.measures = T)

lavaan 0.6-8 ended normally after 18 iterations  
  
 Estimator ML  
 Optimization method NLMINB  
 Number of model parameters 11  
   
 Number of observations 301  
   
Model Test User Model:  
   
 Test statistic 43.425  
 Degrees of freedom 4  
 P-value (Chi-square) 0.000  
  
Model Test Baseline Model:  
  
 Test statistic 392.278  
 Degrees of freedom 10  
 P-value 0.000  
  
User Model versus Baseline Model:  
  
 Comparative Fit Index (CFI) 0.897  
 Tucker-Lewis Index (TLI) 0.742  
  
Loglikelihood and Information Criteria:  
  
 Loglikelihood user model (H0) -2213.884  
 Loglikelihood unrestricted model (H1) -2192.171  
   
 Akaike (AIC) 4449.767  
 Bayesian (BIC) 4490.545  
 Sample-size adjusted Bayesian (BIC) 4455.660  
  
Root Mean Square Error of Approximation:  
  
 RMSEA 0.181  
 90 Percent confidence interval - lower 0.135  
 90 Percent confidence interval - upper 0.231  
 P-value RMSEA <= 0.05 0.000  
  
Standardized Root Mean Square Residual:  
  
 SRMR 0.100  
  
Parameter Estimates:  
  
 Standard errors Standard  
 Information Expected  
 Information saturated (h1) model Structured  
  
Regressions:  
 Estimate Std.Err z-value P(>|z|)  
 x1 ~   
 x2 0.132 0.048 2.764 0.006  
 x3 0.359 0.050 7.227 0.000  
 x4 0.298 0.049 6.135 0.000  
 x2 ~   
 x5 0.127 0.052 2.442 0.015  
 x3 ~   
 x5 0.068 0.050 1.343 0.179  
 x4 ~   
 x5 0.661 0.035 18.705 0.000  
  
Variances:  
 Estimate Std.Err z-value P(>|z|)  
 x5 1.660 0.135 12.268 0.000  
 .x1 0.946 0.077 12.268 0.000  
 .x2 1.355 0.110 12.268 0.000  
 .x3 1.267 0.103 12.268 0.000  
 .x4 0.625 0.051 12.268 0.000

semPaths(paModel2, "par", weighted = FALSE, nCharNodes = 7, layout = "spring", sizeMan = 8,   
 sizeMan2 = 5, edge.label.cex = 1)



anova(paModel, paModel2)

Chi-Squared Difference Test  
  
 Df AIC BIC Chisq Chisq diff Df diff Pr(>Chisq)   
paModel 2 4445.3 4493.5 34.981   
paModel2 4 4449.8 4490.5 43.425 8.4443 2 0.01467 \*  
---  
Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

There is a significant difference between these, with model2 fitting significantly worse (but being simpler), so we’d prefer to more complex model. Note that that more complex model is not that great.

This is sort of like a simple factor analysis, and let’s you try to find a structural model that describes the relationship between variables, and compare it to other models. However, these models have no latent nodes–they just try to account for the covariance.

We can combine these path-style models with the latent variable models as well. This is a method generally referred to as ‘structural equation modeling’ or SEM.

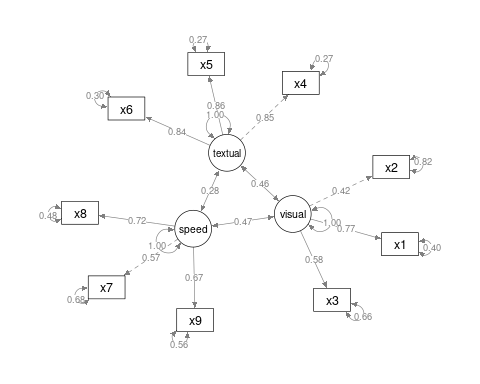
# Structural Equation Modeling

Structural Equation Modeling (SEM) attempts to divide the model into two parts: the measurement model and an underlying causal model relating the latent variables. The measurement model is sort of like a factor analysis: we have measured variables we want to organize into a single latent construct. The causal model is like path analysis–it models relationships between these latent variables. We will start with the CFA model earlier, but this time fit it with sem:

HS.sem <- " visual =~ x2 + x1 + x3  
 textual =~ x4 + x5 + x6  
 speed =~ x7 + x8 + x9 "  
  
fit\_sem <- sem(HS.sem, data = HolzingerSwineford1939)  
summary(fit\_sem, fit.measures = T, standardized = T)

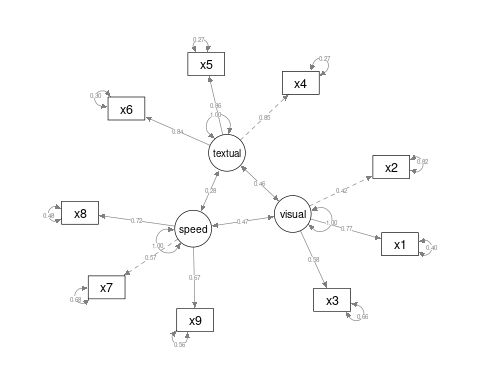
lavaan 0.6-8 ended normally after 38 iterations  
  
 Estimator ML  
 Optimization method NLMINB  
 Number of model parameters 21  
   
 Number of observations 301  
   
Model Test User Model:  
   
 Test statistic 85.306  
 Degrees of freedom 24  
 P-value (Chi-square) 0.000  
  
Model Test Baseline Model:  
  
 Test statistic 918.852  
 Degrees of freedom 36  
 P-value 0.000  
  
User Model versus Baseline Model:  
  
 Comparative Fit Index (CFI) 0.931  
 Tucker-Lewis Index (TLI) 0.896  
  
Loglikelihood and Information Criteria:  
  
 Loglikelihood user model (H0) -3737.745  
 Loglikelihood unrestricted model (H1) -3695.092  
   
 Akaike (AIC) 7517.490  
 Bayesian (BIC) 7595.339  
 Sample-size adjusted Bayesian (BIC) 7528.739  
  
Root Mean Square Error of Approximation:  
  
 RMSEA 0.092  
 90 Percent confidence interval - lower 0.071  
 90 Percent confidence interval - upper 0.114  
 P-value RMSEA <= 0.05 0.001  
  
Standardized Root Mean Square Residual:  
  
 SRMR 0.065  
  
Parameter Estimates:  
  
 Standard errors Standard  
 Information Expected  
 Information saturated (h1) model Structured  
  
Latent Variables:  
 Estimate Std.Err z-value P(>|z|) Std.lv Std.all  
 visual =~   
 x2 1.000 0.498 0.424  
 x1 1.807 0.325 5.554 0.000 0.900 0.772  
 x3 1.318 0.239 5.509 0.000 0.656 0.581  
 textual =~   
 x4 1.000 0.990 0.852  
 x5 1.113 0.065 17.014 0.000 1.102 0.855  
 x6 0.926 0.055 16.703 0.000 0.917 0.838  
 speed =~   
 x7 1.000 0.619 0.570  
 [ reached getOption("max.print") -- omitted 2 rows ]  
  
Covariances:  
 Estimate Std.Err z-value P(>|z|) Std.lv Std.all  
 visual ~~   
 textual 0.226 0.052 4.329 0.000 0.459 0.459  
 speed 0.145 0.038 3.864 0.000 0.471 0.471  
 textual ~~   
 speed 0.173 0.049 3.518 0.000 0.283 0.283  
  
Variances:  
 Estimate Std.Err z-value P(>|z|) Std.lv Std.all  
 .x2 1.134 0.102 11.146 0.000 1.134 0.821  
 .x1 0.549 0.114 4.833 0.000 0.549 0.404  
 .x3 0.844 0.091 9.317 0.000 0.844 0.662  
 .x4 0.371 0.048 7.778 0.000 0.371 0.275  
 .x5 0.446 0.058 7.642 0.000 0.446 0.269  
 .x6 0.356 0.043 8.277 0.000 0.356 0.298  
 .x7 0.799 0.081 9.823 0.000 0.799 0.676  
 .x8 0.488 0.074 6.573 0.000 0.488 0.477  
 .x9 0.566 0.071 8.003 0.000 0.566 0.558  
 visual 0.248 0.077 3.214 0.001 1.000 1.000  
 [ reached getOption("max.print") -- omitted 2 rows ]

semPaths(fit\_sem, what = "col", whatLabels = "std", style = "mx", rotation = 1, layout = "spring",   
 nCharNodes = 7, edge.label.cex = 0.9, shapeMan = "rectangle", sizeMan = 8, sizeMan2 = 5)



With the sem fit, we permit the different factors to have correlations created as bottom-up estimates. This ends up being identical to the cfa solution.

semPaths(fit, what = "col", whatLabels = "std", style = "mx", rotation = 1, layout = "spring",   
 nCharNodes = 7, shapeMan = "rectangle", sizeMan = 8, sizeMan2 = 5)

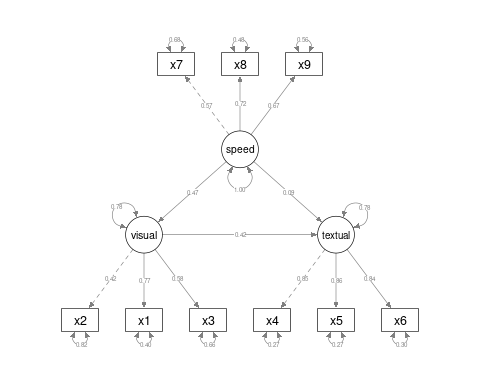


We can also specifiy a structural model with ~ notation. Suppose we think that textual is just visual + speed. I’ll force speed and visual to have a fixed variance and allow these to predict textual.

HS.sem2 <- " visual =~ x2 + x1 + x3  
 textual =~ x4 + x5 + x6  
 speed =~ x7 + x8 + x9   
   
 textual ~ visual + speed  
 visual ~ speed  
   
"  
  
fit\_sem2 <- sem(HS.sem2, data = HolzingerSwineford1939)  
summary(fit\_sem2, fit.measures = T, standardized = T)

lavaan 0.6-8 ended normally after 34 iterations  
  
 Estimator ML  
 Optimization method NLMINB  
 Number of model parameters 21  
   
 Number of observations 301  
   
Model Test User Model:  
   
 Test statistic 85.306  
 Degrees of freedom 24  
 P-value (Chi-square) 0.000  
  
Model Test Baseline Model:  
  
 Test statistic 918.852  
 Degrees of freedom 36  
 P-value 0.000  
  
User Model versus Baseline Model:  
  
 Comparative Fit Index (CFI) 0.931  
 Tucker-Lewis Index (TLI) 0.896  
  
Loglikelihood and Information Criteria:  
  
 Loglikelihood user model (H0) -3737.745  
 Loglikelihood unrestricted model (H1) -3695.092  
   
 Akaike (AIC) 7517.490  
 Bayesian (BIC) 7595.339  
 Sample-size adjusted Bayesian (BIC) 7528.739  
  
Root Mean Square Error of Approximation:  
  
 RMSEA 0.092  
 90 Percent confidence interval - lower 0.071  
 90 Percent confidence interval - upper 0.114  
 P-value RMSEA <= 0.05 0.001  
  
Standardized Root Mean Square Residual:  
  
 SRMR 0.065  
  
Parameter Estimates:  
  
 Standard errors Standard  
 Information Expected  
 Information saturated (h1) model Structured  
  
Latent Variables:  
 Estimate Std.Err z-value P(>|z|) Std.lv Std.all  
 visual =~   
 x2 1.000 0.498 0.424  
 x1 1.807 0.325 5.554 0.000 0.900 0.772  
 x3 1.318 0.239 5.509 0.000 0.656 0.581  
 textual =~   
 x4 1.000 0.990 0.852  
 x5 1.113 0.065 17.014 0.000 1.102 0.855  
 x6 0.926 0.055 16.703 0.000 0.917 0.838  
 speed =~   
 x7 1.000 0.619 0.570  
 [ reached getOption("max.print") -- omitted 2 rows ]  
  
Regressions:  
 Estimate Std.Err z-value P(>|z|) Std.lv Std.all  
 textual ~   
 visual 0.831 0.211 3.944 0.000 0.418 0.418  
 speed 0.138 0.138 1.002 0.316 0.086 0.086  
 visual ~   
 speed 0.378 0.093 4.046 0.000 0.471 0.471  
  
Variances:  
 Estimate Std.Err z-value P(>|z|) Std.lv Std.all  
 .x2 1.134 0.102 11.146 0.000 1.134 0.821  
 .x1 0.549 0.114 4.833 0.000 0.549 0.404  
 .x3 0.844 0.091 9.317 0.000 0.844 0.662  
 .x4 0.371 0.048 7.779 0.000 0.371 0.275  
 .x5 0.446 0.058 7.642 0.000 0.446 0.269  
 .x6 0.356 0.043 8.277 0.000 0.356 0.298  
 .x7 0.799 0.081 9.823 0.000 0.799 0.676  
 .x8 0.488 0.074 6.573 0.000 0.488 0.477  
 .x9 0.566 0.071 8.003 0.000 0.566 0.558  
 .visual 0.193 0.061 3.150 0.002 0.779 0.779  
 [ reached getOption("max.print") -- omitted 2 rows ]

semPaths(fit\_sem2, what = "col", whatLabels = "std", style = "mx", rotation = 1,   
 layout = "tree2", nCharNodes = 7, shapeMan = "rectangle", sizeMan = 8, sizeMan2 = 5)



We can compare the unconstrained model to the specific causal structure we have specified. Here, we see that the unconstrained model differs significantly via the anova, so we would prefer the more complex model.

anova(fit\_sem, fit\_sem2)

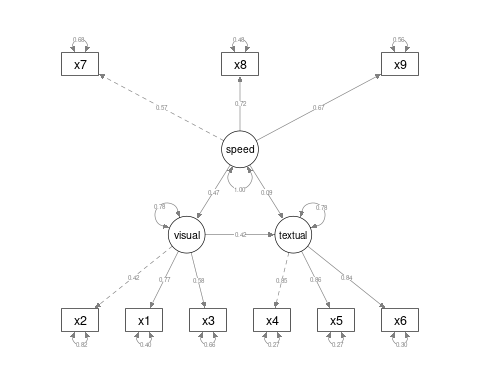
Chi-Squared Difference Test  
  
 Df AIC BIC Chisq Chisq diff Df diff Pr(>Chisq)   
fit\_sem 24 7517.5 7595.3 85.305   
fit\_sem2 25 7544.3 7618.5 114.128 28.823 1 7.931e-08 \*\*\*  
---  
Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

## Here, we can either include the link between speed/text, or eliminate it. this becomes like the mediation analysis we did earlier.

HS.sem3a <- " visual =~ x2 + x1 + x3  
 textual =~ x4 + x5 + x6  
 speed =~ x7 + x8 + x9   
   
 textual ~ a\*visual + b\*speed   
 visual ~ c\*speed  
   
 indirect := a\*b  
 total := indirect + c  
"  
  
fit\_sem3a <- sem(HS.sem3a, data = HolzingerSwineford1939)  
summary(fit\_sem3a, fit.measures = T, standardized = T)

lavaan 0.6-8 ended normally after 34 iterations  
  
 Estimator ML  
 Optimization method NLMINB  
 Number of model parameters 21  
   
 Number of observations 301  
   
Model Test User Model:  
   
 Test statistic 85.306  
 Degrees of freedom 24  
 P-value (Chi-square) 0.000  
  
Model Test Baseline Model:  
  
 Test statistic 918.852  
 Degrees of freedom 36  
 P-value 0.000  
  
User Model versus Baseline Model:  
  
 Comparative Fit Index (CFI) 0.931  
 Tucker-Lewis Index (TLI) 0.896  
  
Loglikelihood and Information Criteria:  
  
 Loglikelihood user model (H0) -3737.745  
 Loglikelihood unrestricted model (H1) -3695.092  
   
 Akaike (AIC) 7517.490  
 Bayesian (BIC) 7595.339  
 Sample-size adjusted Bayesian (BIC) 7528.739  
  
Root Mean Square Error of Approximation:  
  
 RMSEA 0.092  
 90 Percent confidence interval - lower 0.071  
 90 Percent confidence interval - upper 0.114  
 P-value RMSEA <= 0.05 0.001  
  
Standardized Root Mean Square Residual:  
  
 SRMR 0.065  
  
Parameter Estimates:  
  
 Standard errors Standard  
 Information Expected  
 Information saturated (h1) model Structured  
  
Latent Variables:  
 Estimate Std.Err z-value P(>|z|) Std.lv Std.all  
 visual =~   
 x2 1.000 0.498 0.424  
 x1 1.807 0.325 5.554 0.000 0.900 0.772  
 x3 1.318 0.239 5.509 0.000 0.656 0.581  
 textual =~   
 x4 1.000 0.990 0.852  
 x5 1.113 0.065 17.014 0.000 1.102 0.855  
 x6 0.926 0.055 16.703 0.000 0.917 0.838  
 speed =~   
 x7 1.000 0.619 0.570  
 [ reached getOption("max.print") -- omitted 2 rows ]  
  
Regressions:  
 Estimate Std.Err z-value P(>|z|) Std.lv Std.all  
 textual ~   
 visual (a) 0.831 0.211 3.944 0.000 0.418 0.418  
 speed (b) 0.138 0.138 1.002 0.316 0.086 0.086  
 visual ~   
 speed (c) 0.378 0.093 4.046 0.000 0.471 0.471  
  
Variances:  
 Estimate Std.Err z-value P(>|z|) Std.lv Std.all  
 .x2 1.134 0.102 11.146 0.000 1.134 0.821  
 .x1 0.549 0.114 4.833 0.000 0.549 0.404  
 .x3 0.844 0.091 9.317 0.000 0.844 0.662  
 .x4 0.371 0.048 7.779 0.000 0.371 0.275  
 .x5 0.446 0.058 7.642 0.000 0.446 0.269  
 .x6 0.356 0.043 8.277 0.000 0.356 0.298  
 .x7 0.799 0.081 9.823 0.000 0.799 0.676  
 .x8 0.488 0.074 6.573 0.000 0.488 0.477  
 .x9 0.566 0.071 8.003 0.000 0.566 0.558  
 .visual 0.193 0.061 3.150 0.002 0.779 0.779  
 [ reached getOption("max.print") -- omitted 2 rows ]  
  
Defined Parameters:  
 Estimate Std.Err z-value P(>|z|) Std.lv Std.all  
 indirect 0.115 0.103 1.107 0.268 0.036 0.036  
 total 0.493 0.127 3.877 0.000 0.507 0.507

semPaths(fit\_sem3a, what = "col", whatLabels = "std", style = "mx", rotation = 1,   
 layout = "tree", nCharNodes = 7, shapeMan = "rectangle", sizeMan = 8, sizeMan2 = 5)



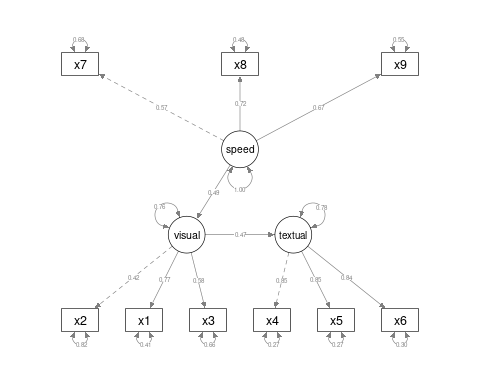
anova(fit\_sem3a, fit\_sem2)

Chi-Squared Difference Test  
  
 Df AIC BIC Chisq Chisq diff Df diff Pr(>Chisq)  
fit\_sem3a 24 7517.5 7595.3 85.305   
fit\_sem2 24 7517.5 7595.3 85.305 0 0

HS.sem3 <- " visual =~ x2 + x1 + x3  
 textual =~ x4 + x5 + x6  
 speed =~ x7 + x8 + x9   
   
 textual ~ visual   
 visual ~ speed  
   
"  
  
fit\_sem3 <- sem(HS.sem3, data = HolzingerSwineford1939)  
summary(fit\_sem3, fit.measures = T, standardized = T)

lavaan 0.6-8 ended normally after 32 iterations  
  
 Estimator ML  
 Optimization method NLMINB  
 Number of model parameters 20  
   
 Number of observations 301  
   
Model Test User Model:  
   
 Test statistic 86.310  
 Degrees of freedom 25  
 P-value (Chi-square) 0.000  
  
Model Test Baseline Model:  
  
 Test statistic 918.852  
 Degrees of freedom 36  
 P-value 0.000  
  
User Model versus Baseline Model:  
  
 Comparative Fit Index (CFI) 0.931  
 Tucker-Lewis Index (TLI) 0.900  
  
Loglikelihood and Information Criteria:  
  
 Loglikelihood user model (H0) -3738.247  
 Loglikelihood unrestricted model (H1) -3695.092  
   
 Akaike (AIC) 7516.494  
 Bayesian (BIC) 7590.637  
 Sample-size adjusted Bayesian (BIC) 7527.208  
  
Root Mean Square Error of Approximation:  
  
 RMSEA 0.090  
 90 Percent confidence interval - lower 0.070  
 90 Percent confidence interval - upper 0.111  
 P-value RMSEA <= 0.05 0.001  
  
Standardized Root Mean Square Residual:  
  
 SRMR 0.067  
  
Parameter Estimates:  
  
 Standard errors Standard  
 Information Expected  
 Information saturated (h1) model Structured  
  
Latent Variables:  
 Estimate Std.Err z-value P(>|z|) Std.lv Std.all  
 visual =~   
 x2 1.000 0.496 0.422  
 x1 1.801 0.321 5.613 0.000 0.894 0.767  
 x3 1.317 0.240 5.493 0.000 0.654 0.579  
 textual =~   
 x4 1.000 0.990 0.852  
 x5 1.112 0.065 17.005 0.000 1.101 0.854  
 x6 0.926 0.055 16.710 0.000 0.917 0.838  
 speed =~   
 x7 1.000 0.615 0.565  
 [ reached getOption("max.print") -- omitted 2 rows ]  
  
Regressions:  
 Estimate Std.Err z-value P(>|z|) Std.lv Std.all  
 textual ~   
 visual 0.939 0.196 4.790 0.000 0.471 0.471  
 visual ~   
 speed 0.394 0.095 4.158 0.000 0.488 0.488  
  
Variances:  
 Estimate Std.Err z-value P(>|z|) Std.lv Std.all  
 .x2 1.135 0.102 11.171 0.000 1.135 0.822  
 .x1 0.559 0.108 5.162 0.000 0.559 0.411  
 .x3 0.848 0.090 9.443 0.000 0.848 0.665  
 .x4 0.370 0.048 7.759 0.000 0.370 0.274  
 .x5 0.448 0.058 7.664 0.000 0.448 0.270  
 .x6 0.356 0.043 8.265 0.000 0.356 0.297  
 .x7 0.805 0.082 9.878 0.000 0.805 0.681  
 .x8 0.487 0.074 6.534 0.000 0.487 0.476  
 .x9 0.563 0.071 7.936 0.000 0.563 0.555  
 .visual 0.188 0.060 3.140 0.002 0.762 0.762  
 [ reached getOption("max.print") -- omitted 2 rows ]

semPaths(fit\_sem3, what = "col", whatLabels = "std", style = "mx", rotation = 1,   
 layout = "tree", nCharNodes = 7, shapeMan = "rectangle", sizeMan = 8, sizeMan2 = 5)



anova(fit\_sem3, fit\_sem2)

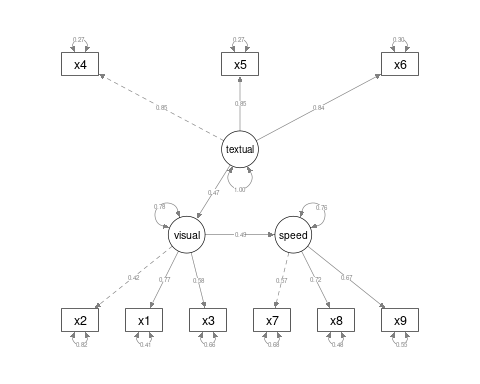
Chi-Squared Difference Test  
  
 Df AIC BIC Chisq Chisq diff Df diff Pr(>Chisq)  
fit\_sem2 24 7517.5 7595.3 85.305   
fit\_sem3 25 7516.5 7590.6 86.310 1.0046 1 0.3162

## Let’s reverse the causal chain

HS.sem4 <- " visual =~ x2 + x1 + x3  
 textual =~ x4 + x5 + x6  
 speed =~ x7 + x8 + x9   
   
 speed ~ visual  
 visual ~ textual  
  
"  
  
fit\_sem4 <- sem(HS.sem4, data = HolzingerSwineford1939)  
summary(fit\_sem4, fit.measures = T, standardized = T)

lavaan 0.6-8 ended normally after 34 iterations  
  
 Estimator ML  
 Optimization method NLMINB  
 Number of model parameters 20  
   
 Number of observations 301  
   
Model Test User Model:  
   
 Test statistic 86.310  
 Degrees of freedom 25  
 P-value (Chi-square) 0.000  
  
Model Test Baseline Model:  
  
 Test statistic 918.852  
 Degrees of freedom 36  
 P-value 0.000  
  
User Model versus Baseline Model:  
  
 Comparative Fit Index (CFI) 0.931  
 Tucker-Lewis Index (TLI) 0.900  
  
Loglikelihood and Information Criteria:  
  
 Loglikelihood user model (H0) -3738.247  
 Loglikelihood unrestricted model (H1) -3695.092  
   
 Akaike (AIC) 7516.494  
 Bayesian (BIC) 7590.637  
 Sample-size adjusted Bayesian (BIC) 7527.208  
  
Root Mean Square Error of Approximation:  
  
 RMSEA 0.090  
 90 Percent confidence interval - lower 0.070  
 90 Percent confidence interval - upper 0.111  
 P-value RMSEA <= 0.05 0.001  
  
Standardized Root Mean Square Residual:  
  
 SRMR 0.067  
  
Parameter Estimates:  
  
 Standard errors Standard  
 Information Expected  
 Information saturated (h1) model Structured  
  
Latent Variables:  
 Estimate Std.Err z-value P(>|z|) Std.lv Std.all  
 visual =~   
 x2 1.000 0.496 0.422  
 x1 1.801 0.321 5.613 0.000 0.894 0.767  
 x3 1.317 0.240 5.493 0.000 0.654 0.579  
 textual =~   
 x4 1.000 0.990 0.852  
 x5 1.112 0.065 17.005 0.000 1.101 0.854  
 x6 0.926 0.055 16.710 0.000 0.917 0.838  
 speed =~   
 x7 1.000 0.615 0.565  
 [ reached getOption("max.print") -- omitted 2 rows ]  
  
Regressions:  
 Estimate Std.Err z-value P(>|z|) Std.lv Std.all  
 speed ~   
 visual 0.604 0.143 4.212 0.000 0.488 0.488  
 visual ~   
 textual 0.236 0.050 4.739 0.000 0.471 0.471  
  
Variances:  
 Estimate Std.Err z-value P(>|z|) Std.lv Std.all  
 .x2 1.135 0.102 11.171 0.000 1.135 0.822  
 .x1 0.559 0.108 5.162 0.000 0.559 0.411  
 .x3 0.848 0.090 9.443 0.000 0.848 0.665  
 .x4 0.370 0.048 7.759 0.000 0.370 0.274  
 .x5 0.448 0.058 7.664 0.000 0.448 0.270  
 .x6 0.356 0.043 8.265 0.000 0.356 0.297  
 .x7 0.805 0.082 9.878 0.000 0.805 0.681  
 .x8 0.487 0.074 6.534 0.000 0.487 0.476  
 .x9 0.563 0.071 7.936 0.000 0.563 0.555  
 .visual 0.192 0.060 3.176 0.001 0.778 0.778  
 [ reached getOption("max.print") -- omitted 2 rows ]

semPaths(fit\_sem4, what = "col", whatLabels = "std", style = "mx", rotation = 1,   
 layout = "tree", nCharNodes = 7, shapeMan = "rectangle", sizeMan = 8, sizeMan2 = 5)



anova(fit\_sem4, fit\_sem3)

Chi-Squared Difference Test  
  
 Df AIC BIC Chisq Chisq diff Df diff Pr(>Chisq)  
fit\_sem4 25 7516.5 7590.6 86.31   
fit\_sem3 25 7516.5 7590.6 86.31 3.739e-09 0