CEE 5390 - Modeling and Simulation in CEE

Week 2: Law 1.4

January 23, 2008

Modeling a Queueing System

Problem Statement: Consider a single server queueing system which consists of a server fulfilling a specific service to arriving customers who are waiting in a queue. The customer inter-arrival times are *independent and identically distributed* (IID) random variables and the customer service times are also IID random variables. A customer who arrives when the server is busy has to join the queue and follow a specific *queue discipline*. Hence the chief components of a QS are:

- Arrival process/time
- Service mechanism/ service time
- Queue discipline → First In First Out (FIFO) and Last In First Out (LIFO)

Representation

The System State consists of the following:

- Server status (state)
- Number of customers in the queue (state)
- A list of arrival times (state)
- Time of last event (state)
- Time representation (simulation clock)
- Next scheduled event: arrival and departure (event list)
- Statistical counters

Inter arrival time is denoted by A_i which is the time between arrivals $(i-1)$ and i. Values of A_i are IID from a distribution function F_A

Service times are denoted by S_i for the $i-th$ customer. Values of S_i are IID from a distribution function F_S.

e_0 is the very first null event on the time line and is a null event.

The $i-th$ customer arrives at the time point t_i

The length of time that a customer has to wait in line before getting serviced is called a delay and denoted by D_i.

Hence, the following relationship can be derived:

$$C_i = t_i + D_i + S_i \tag{1}$$

where C_i is the departure time point of the $i-th$ customer. Also, the average customer arrival rate can be expressed as:

$$\lambda = 1/E(A) \tag{2}$$

where $E(A)$ is the *Expected* value of the arrival rate A.

Measuring System Performance

The following parameters can be used to measure performance in the queueing system. (Note: the ˆ hat on a quantity is an estimator of the quantity)

- Expected average delay:

$$\hat{d}(n) = \frac{\sum_{i=1}^{n} D_i}{n} \tag{3}$$

- Expected average number of customers in the queue but not being served:

$$\hat{q}(n) = \frac{\sum_{i=0}^{\infty} iT_i}{T(n)} = \frac{\int_0^{T(n)} Q(t)dt}{T(n)} \tag{4}$$

  where $Q(t)$ is the number of customers in a queue for time point t and $T(n)$ is the time required to observe n delays in the queue. So:

$$0 < t \leq T(n), Q(t) \geq 0 \tag{5}$$

- Expected server utilization:

$$\hat{u}(n) = \frac{\int_0^{T(n)} B(t)dt}{T(n)} \tag{6}$$

  where $\hat{u}(n)$ is the expected proportion of time (in between 0 and $T(n)$) when the server is busy. Hence:

$$0 \geq \hat{u}(n) \leq 1 \tag{7}$$

  and $B(t)$ is a unit step function defined as:

$$B(t) = \begin{cases} 1; & \text{when server is busy} \\ 0; & \text{when server is free} \end{cases} \tag{8}$$

Please note that both $\hat{q}(n)$ and $\hat{u}(n)$ are continuous time averages while $\hat{d}(n)$ is a discrete estimator.

Discussion Points

Please consider the following:

- Time representation
- Discrete and continuous time statistics
- Developing a flowchart for this simulation
- Steady state measures of the statistics
- Termination point of the simulation

From the above figure calculate $\hat{d}(1), \hat{q}(1)$ and $\hat{u}(1)$

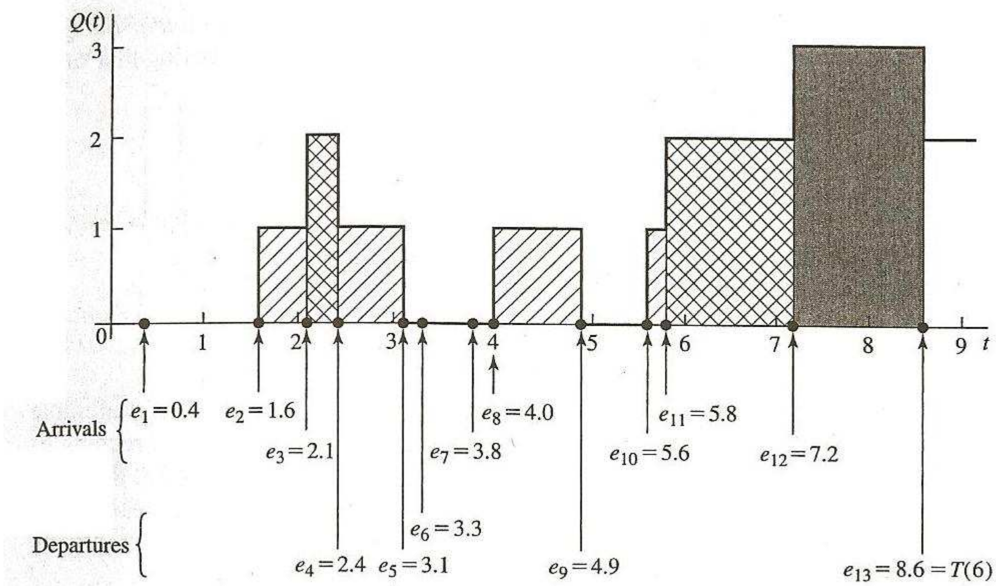

Figure 1: $Q(t)$ arrival and departure events and times in the QS (Law fig1.5)