The Stream of Consciousness Chart.

* Model Classification

* We start with Declarative State Based Models & Stochastic Chains & Processes.

* We choose to study the Queueing System
  - Arrival Process, Service rates, → Stochastic Process.

steady state $\rho = \frac{\lambda}{\mu}$,

Little's Law $L = \lambda W$.

  - We calculated simple counters
    ↳ HW 2 + HW3 Problem 1 (Simple formulation of Q'ing Systems).

* Concentrate on Stochastic nature of Arrival Process
  - Assumptions about Arrival process.
    ↳ Derived that the arrival process is a Poisson Process.

$p_n(t) = \frac{(\lambda t)^n e^{-\lambda t}}{n!}$

    ↳ Derived that given a Poisson Process the inter-arrival times are Exponentially distributed
    (HW3 - Problem 2 Discrete Counterpart).

$P(T \geq t) = p_0(t) = e^{-\lambda t}$

$\therefore$ CDF $= P(T \leq t) = 1 - e^{-\lambda t}$

$\therefore$ pdf of IA times $= \lambda e^{-\lambda t}$.

    ↳ Discussed the consequences of the very important Memoryless Property of Exp. distributions

$P(T \leq t_1 \mid T \geq t_0) = P(0 \leq T \leq t_1 - t_0)$

* Understanding Stochastic Chains & Process
  - Distinction between Discrete / Continuous Parameter
    Discrete / Continuous State Space.

Markov Processes & Chains

— Definition $P_{i,j} = Pr\{X_{n+1} = j \mid X_n = i\}$. $\{X(t), t = 0, 1, 2 \ldots\}$

↳ Stationary & temporally Homogeneous processes.

— One Step transition matrix.

$$\vec{P} = |P_{ij}|, \quad P_{ij} \geq 0, \quad \sum_{j=0}^{\infty} P_{ij} = 1$$

Derivation uses
- Conditional probability
- Total Probability
- Induction

↳ All you need is $\vec{P}$ & distribution of initial state P_{i_0}

↳ Calculation of $Pr\{X_0 = i_0, X_1 = i_1 \ldots X_n = i_n\}$.

~~vs~~ and $Pr\{X_n = i_n \mid X_0 = i_0, \ldots X_{n} = i_{n-1}\}$

(Classwork problems)

— n-Step transition probabilities ; $P_{ij}^{(n)} = Pr\{X_{m+n} = j \mid X_m = i\}$.

$$P_{ij}^{(n)} = \sum_{k=0}^{\infty} P_{ik} P_{kj}^{(n-1)}$$

Which leads to $p_k = \sum_{k=0}^{\infty} p_j P_{jk}^{(n)}$.

$$\vec{P}^{(m)} = \vec{P}^{(m-k)} \vec{P}^{(k)} \qquad \vec{P}^{(m)} = \vec{P}^{m}.$$

— $\vec{\pi}^{(m)} = \{\pi_j^{(m)}\}$. → Probabity of being in state j after m transitions irrespective of starting point.

$$\vec{\pi}^{(m)} = \vec{\pi}^{(0)} \vec{P}^{m}.$$

If $\vec{Q} = \vec{P} - \vec{I}$ + $\vec{\pi}^{(m)} = \vec{\pi}^{(m-1)} \vec{P}$ | then $\pi^{(m)} - \pi^{(m-1)} = \pi^{(m-1)} \vec{Q}$

* "Markov Chains & Processes

- Continuous Time M.P. $X(t),\ t \in T;\ T = \{0 \le t \le \infty\}$

$$p_{ij}(u,s) = \sum_{r} p_{ir}(u,t)\ p_{rj}(t,s).$$

or $\vec{P}(u,s) = \vec{P}(u,t)\,\vec{P}(t,s).$

Desire $p_j(t+\Delta t)$ when $u = 0$, $s = t+\Delta t$.

$$p_j(t+\Delta t) = \sum_{r} p_r(t)\ p_{rj}(t+\Delta t).$$

↳ For ~~start~~ arrival process this leads to the Poisson Process derivation

- Definition of the Infinitesimal Generator Matrix $\vec{Q}$

$$\frac{dp_j}{dt} = -q_j\,p_j(t) + \sum_{r \ne j} p_r(t)\,q_{rj}$$

Where $p_{ij}(t+\Delta t) = q_{ij}(t)(\Delta t) + o(\Delta t).$

Prob. of a change in state $= 1 - p_{ii}(t+\Delta t)$
$= q_i(t)(\Delta t) + o(\Delta t)$

$$\vec{\pi}^{(m)} - \vec{\pi}^{(m-1)} = \vec{\pi}^{(m-1)}\vec{Q} \iff \vec{p}\,'(t) = \vec{p}(t)\,\underline{\underline{\vec{Q}}}.$$

Kolmogorov Fwd Eq.

- Applications to Q-Theory.
  - Discrete & Cont. Formulation
  - Steady State derivation $\rho < 1$

Classwork Problems: