

CHAPTER 11

DC

Machines

Introduction

Although DC is not widely used by consumers, DC machines have played a major role in industry over the years.

FE: elevators use DC machines.

DC MACHINE:

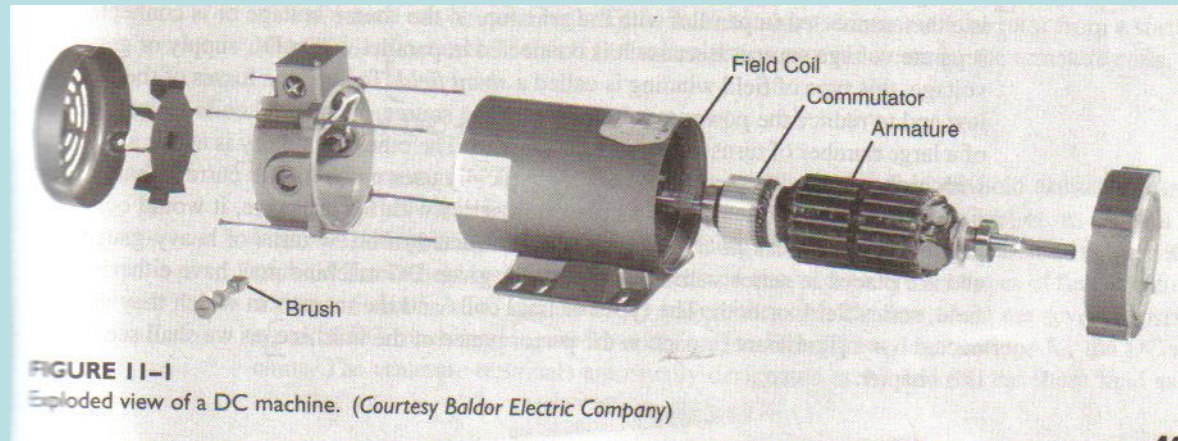


FIGURE 11-1
Exploded view of a DC machine. (Courtesy Baldor Electric Company)

DC machine operates as the result of the interaction of rotating coils with a stationary magnetic field.

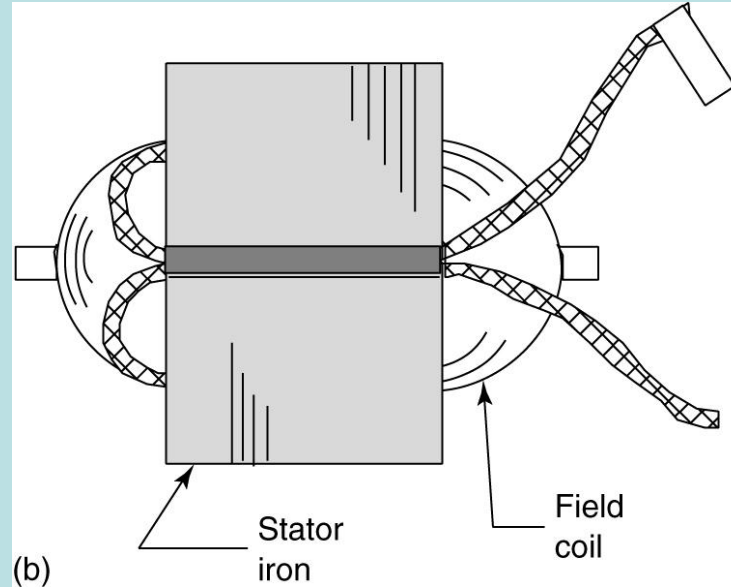
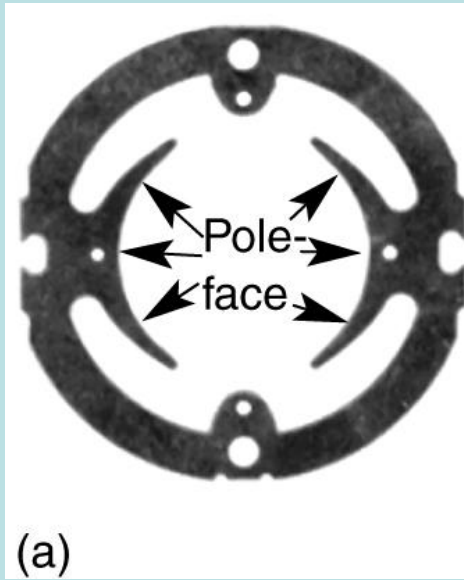
The **rotor** contains the armature winding -a number of coils mounted in lots on the rotor.

The ends of the coils are connected to the **commutator** at the end of the rotor.

Carbon **brushes** ride on the surface of the commutator to make electrical contact from outside the motor to the armature coils.

The stationary magnetic field is provided either by DC coils on the **stator** of the machine or by permanent magnets mounted inside the stator.

The Stator



DC machine stator.
a) Cross section.
b) Side view.

It is simpler and less expensive to make a stator using assembled stack of stamped laminations.

The ear-shaped projections in the center are the pole faces, which actually become a north and south pole during operation of the machine.

The field coils are mounted behind the ear-shaped projections and provide the magnetomotive force (MMF) required to establish the magnetic field.

MMF is the product of current and the number of turns in the coil.

Thus, MMF can be provided with high current and few turns or with low current and many turns.

In the DC machine, there are two types of field windings that are commonly used:

- 1) Shunt field:** - Field windings are connected in parallel with the armature to the source voltage or
- Field windings are connected to a separate voltage source.

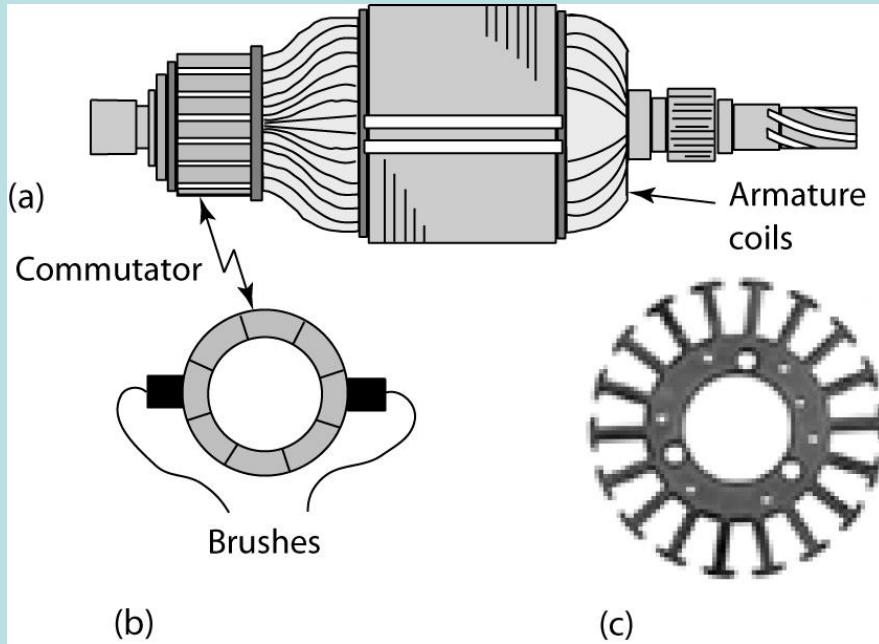
To keep the losses of the machine low and to reduce the power requirements for the source, shunt-field coils are constructed of a large number of turns of smaller-gauge wire.

- 2) Series-field** Field winding are connected in series with the armature.

Series-field windings are constructed of a few turns of heavy-gauge wire and placed in series with the armature.

The types of field coils and the manner in which they are connected has a significant impact on the performance of the machine.

The Rotor



Details of a DC machine armature.

- a. Side view of rotor.
- b. Cross section of commutator.
- c. Cross section of lamination stack.

The armature windings carry AC current, which is rectified by the switching action of the commutator.

The rotor contains the **armature winding**, which is the high-power winding of the DC machine. The **slots** in the lamination provide space for the armature coil.

When operating **as a generator**, the armature winding is rotated through the field, generating a DC voltage that is delivered to the load.

For **motor** operation, the armature winding draws current from the source, and the current in the armature coils interacts with the field to create an electromagnetic torque that drives a mechanical load.

The **commutator** consists of a number of copper segments that are electrically insulated from each other and from the rotor shaft.

The brushes are held by springs against the rotating of the commutator so as to make electrical contact and to provide a connection from the rotating coils to the outside world.

Types of DC Machines

One big advantage of DC machines is the variety of ways that the field excitation can be provided \longleftrightarrow a wide range of operating characteristics

There are two types of **field windings**: **series** and **shunt**.

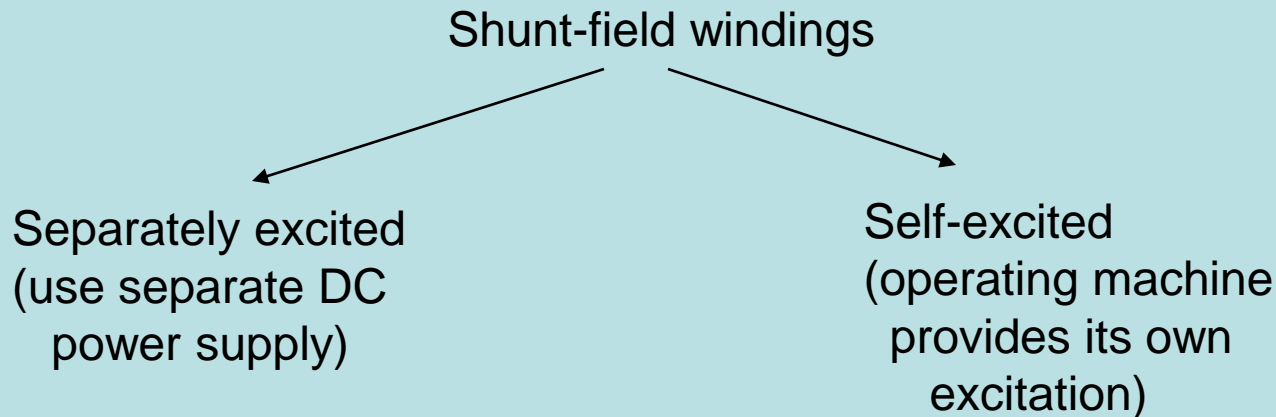
A DC machine may have either or both; therefore there are several possible types of DC machines.

Notation:

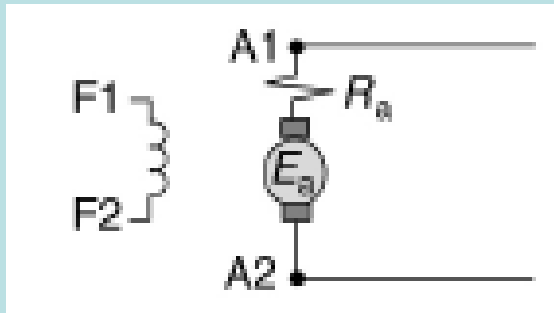
A1-A2: the armature terminals

F1-F2: the shunt field

S1-S2: the series field



Separately excited DC machine.



Notation:

A1-A2: the armature terminals

F1-F2: the shunt field

S1-S2: the series field

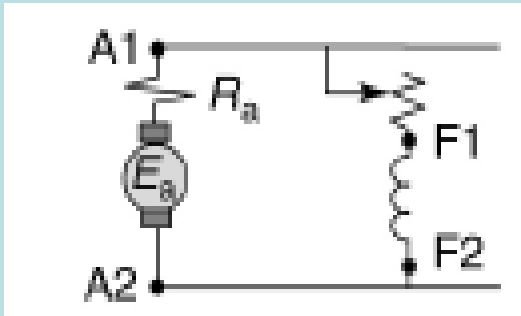
The separately excited field is provided with a **separate DC source**.

Because it requires a separate DC power supply, the separately excited machine is usually used only when a wide range of voltage control (generator) or speed control (motor) is required.

The field current is typically only a few percent of the rated output current, so a small field current controls a much larger armature current.

The separately excited DC generator effectively operates as a power amplifier.

Shunt-motor or self-excited generator



Notation:

A1-A2: the armature terminals

F1-F2: the shunt field

S1-S2: the series field

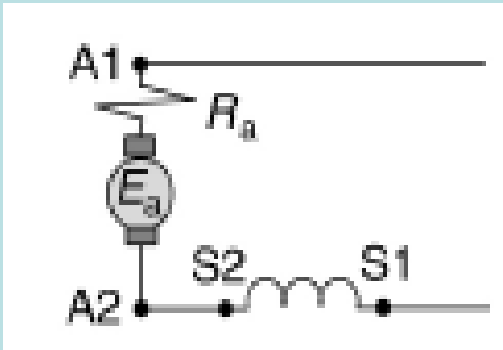
Self-excited means they rely on the generator's own voltage to provide the excitation (note: residual magnetism to provide initial flux is required to build up the initial excitation).

The **shunt connection** is in which the field is connected in parallel, or shunt with the armature.

The voltage from the armature is applied directly to the field in the case of a **generator**, while for **motor** operation the source voltage is applied to the field and armature.

The **rheostat** is used for **changing the field current** without changing the voltage, and therefore provides a measure of **voltage control for the generator** and **speed control for the motor**.

The series machine



Notation:

A1-A2: the armature terminals

F1-F2: the shunt field

S1-S2: the series field

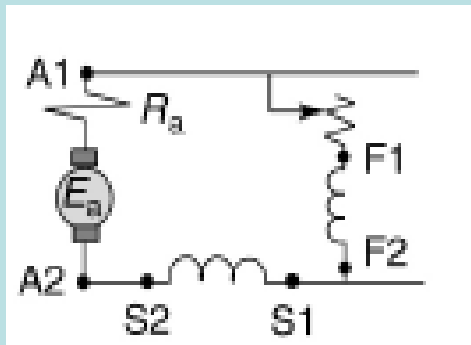
In the **series-connected generator** the field flux is obtained by passing the armature current through the series-field winding.

Because the excitation is provided by the armature current, it varies widely with load, especially for a generator and therefore suffers from very poor voltage regulation.

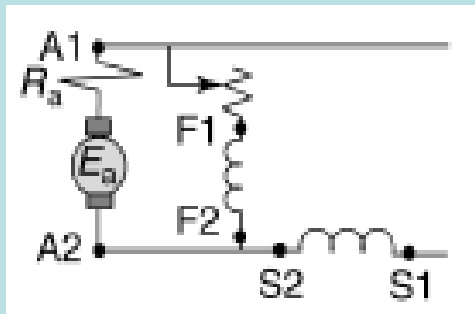
When the **series machine** is connected as a **motor**, the field terminals S1 and S2 have to be reversed, because the current would be going into the machine instead of out of it.

Compound machines

When both series and shunt fields are used on a DC machine, it is called a **compound machine**.



The **long-shunt connection**: the shunt winding is located directly across the terminals of the machine, and the series winding is connected between the shunt and the armature.



The **short-shunt connection**: the shunt winding is directly in parallel with the armature, and the series winding is between the shunt winding and the terminals of the machine.

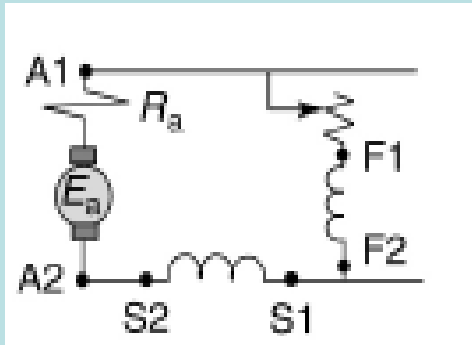
Notation:

A1-A2: the armature terminals

F1-F2: the shunt field

S1-S2: the series field

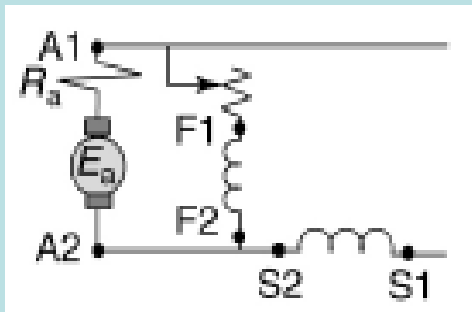
Compound machines



The short or the long shunt can be connected so that the two fields reinforce each other - **cumulative compounding** or so that they are in opposition to each other - **differential compounding**.

For the cumulatively compounded machine, the series winding provides additional flux as the load current increases.

For the generator operation, the current would be from the armature to the terminals so the windings produce flux in the same direction when current enters.



For the motor operation, they would be differentially compounded because the current would be into the motor, and the two fields would be in opposition.

Notation:

A1-A2: the armature terminals

F1-F2: the shunt field

S1-S2: the series field

DC Generators

When operated as a generator, the armature of the DC machine is driven by a prime mover.

When the armature coils move through the flux created by the stator field, a voltage is induced in them.

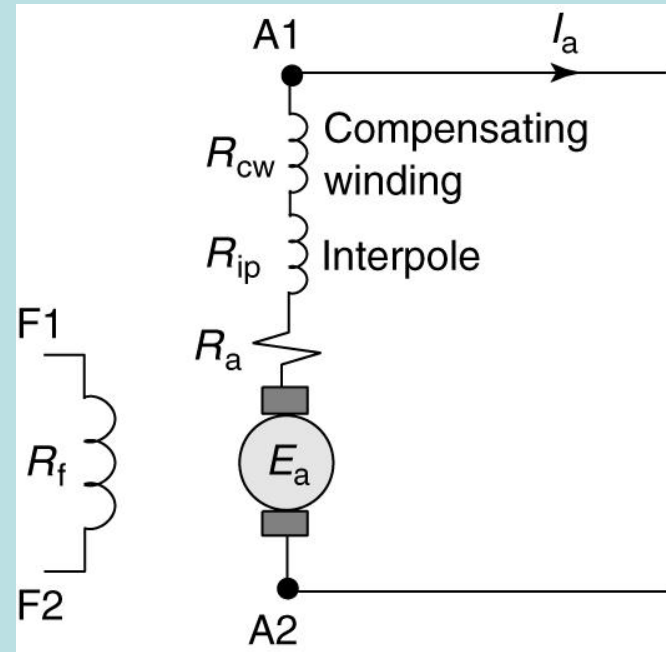
The voltage in a coil is a function of the velocity of the coil through the flux, the length of the coil, and the flux density.

The armature circuit resistance includes the **armature winding** resistance plus the resistance of the **interpoles** and **compensating windings** :

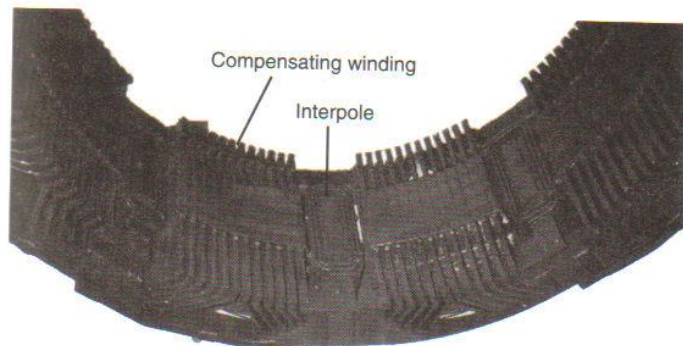
$$R_a = R_{aw} + R_{ip} + R_{cw}$$

The **interpole** is a small, narrow winding, mounted on the back iron of the stator in the neutral zone between the poles to speed up commutation in the generator.

The **compensating winding** mounted on the back iron of the stator and consist of a single heavy turn carrying the armature current in the opposite direction that it is carried in the armature coils in order to cancel the armature reaction. Compensating windings are generally very expensive and are used only in large machines.



Complete equivalent circuit of a separately excited DC generator.



Generated Voltage Equation for a DC Generator:

To understand the operation of the DC generator we need to obtain the relationship for the generated voltage in terms of flux, the physical design of the machine, and its speed.

Faraday's law tell us

$$E_a = N \frac{d\Phi_p}{dt}$$

where N is the # of series conductors.

During each revolution, a conductor cuts the flux of P poles, where P is the number of poles in the machine.

The total amount of flux cut in one revolution:

$$d\Phi = P\Phi_p$$

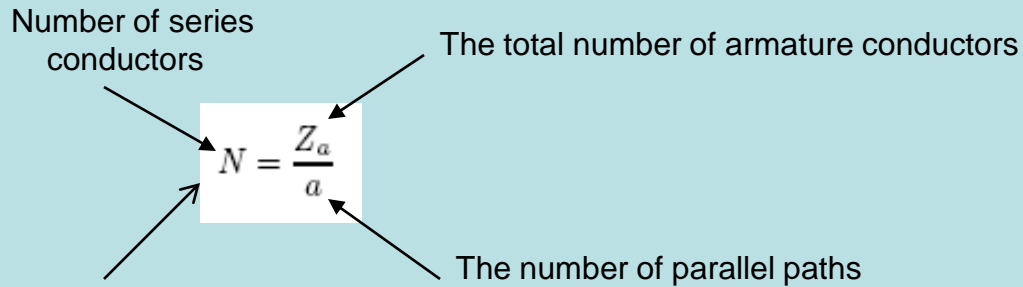
← the flux per pole

If the speed of the machine in RPM is represented by n, then the time to complete one revolution is:

$$\Delta t = \frac{1}{\frac{n}{60}} = \frac{60}{n}$$



$$E_a = NP\Phi_p \frac{n}{60}$$



Depends on the armature winding structure

We need to find the number of series conductors in the armature to calculate the generated voltage

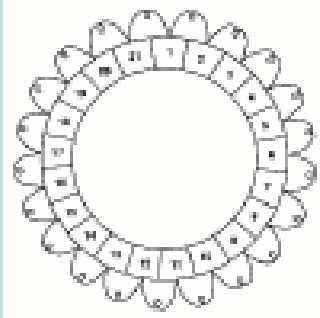
expression for the **generated voltage of a DC generator:**

$$E_a = \left(\frac{PZ_a}{60a}\right)n\Phi_p = K_g n\Phi_p$$

$$K_g = \frac{PZ_a}{60a}$$

K_g is called the generator constant for a DC machine and is a function of the design of the machine-specifically, the number of poles and the type of winding.

Armature Windings



Lap Winding:

The ends of each coil are connected to adjacent commutator segments.

For a lap wound armature the number of parallel paths through the armature is equal to the number of poles.



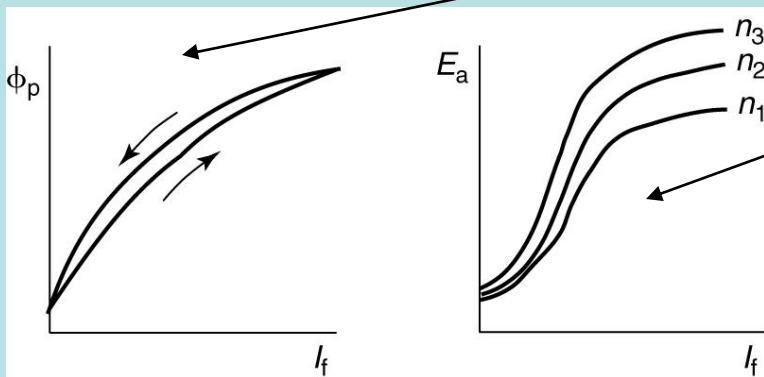
Wave Winding:

The armature coils are connected to commutator segments of commutator on opposite sides of the armature.

The wave winding has only 2 parallel paths between brushes regardless of the number of poles in the machine.

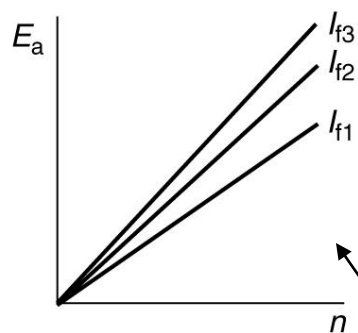
Lap winding provides more parallel paths, which means **higher current and lower voltage**. The **wave winding** has more series coils, yielding **higher voltage and lower current**.

Magnetization Curve of the Separately Excited Machine



(a)

(b)



(c)

$$E_a = \left(\frac{PZ_a}{60a}\right)n\Phi_p = K_g n \Phi_p$$

Because of both hysteresis and saturation effects in the iron of the machine, the flux per pole is a nonlinear function of the field current.

Generated voltage of a DC machine is a linear function of the flux per pole, so it is also nonlinear with respect to the field current.

$$n_1 < n_2 < n_3$$

So if we have two voltages at two different speeds, with the flux per pole held constant:

$$E_{a1} = K_g n_1 \Phi_p$$

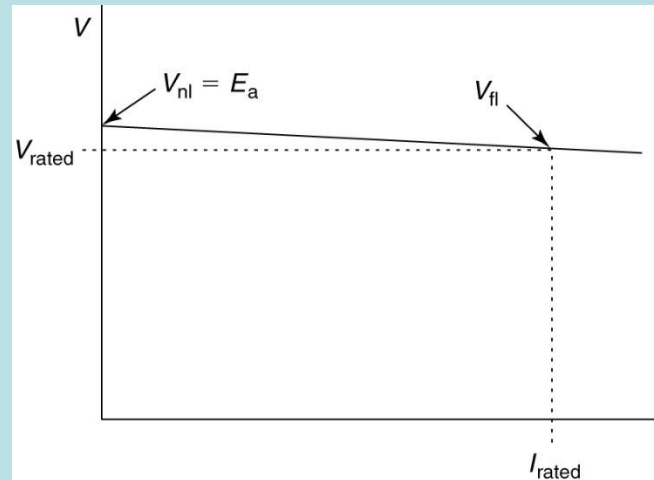
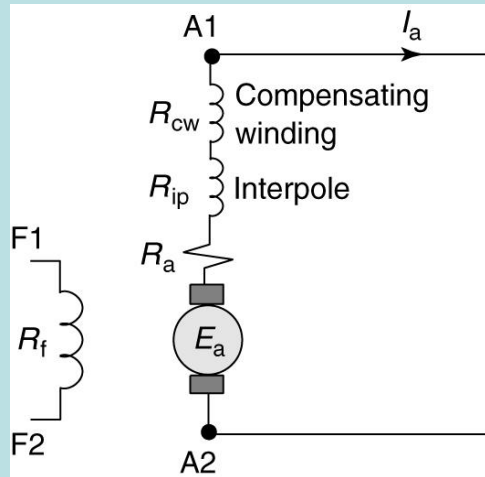
$$E_{a2} = K_g n_2 \Phi_p$$

If we know generated voltage at one speed we can calculate generated voltage using the same DC generator at any other speed using:

$$E_{a2} = E_{a1} \frac{n_2}{n_1}$$

Generated voltage of a DC machine is linear with respect to the rotational speed of the machine

Voltage Regulation



The terminal voltage for the separately excited DC generator:

$$V_t = E_a - I_a R_a$$

1. If the **field voltage** is kept **constant** the field current and the flux per pole will also be constant.
2. If the generator is driven at **constant speed**, the internal voltage, E_a , will be constant .

Under conditions (1 and 2), terminal voltage equation represents a linear relationship between the terminal voltage and the armature current, with a negative slope. As the armature load current increases, the terminal voltage will decrease linearly.

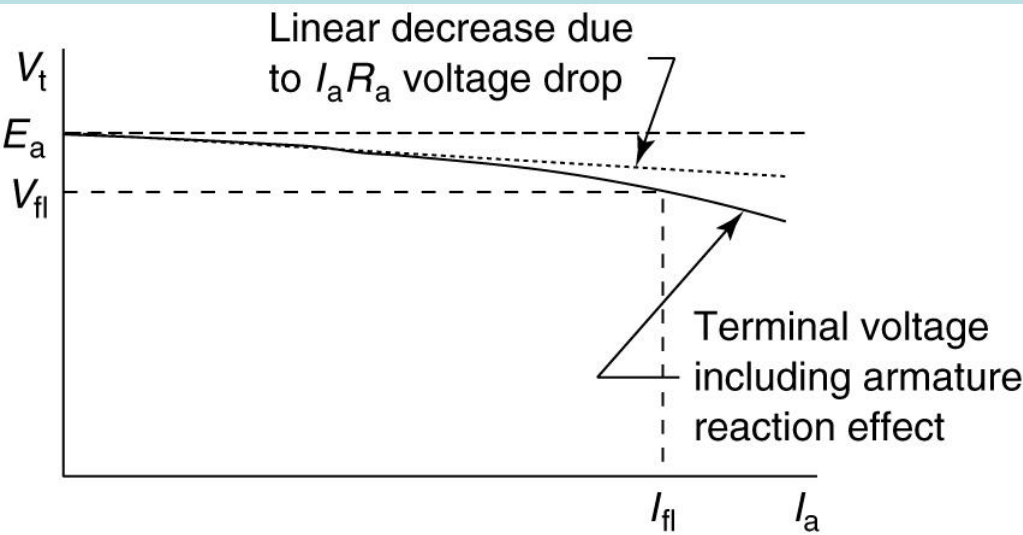
For the separately excited generator, the terminal voltage is above rated voltage at less than full load.

To quantify the variation of terminal voltage with output, we define the **voltage regulation equation**:

$$VR = \frac{V_{nl} - V_{fl}}{V_{fl}} \times 100$$

DC Generator Characteristics

Effect of armature reaction on separately excited generator terminal voltage.



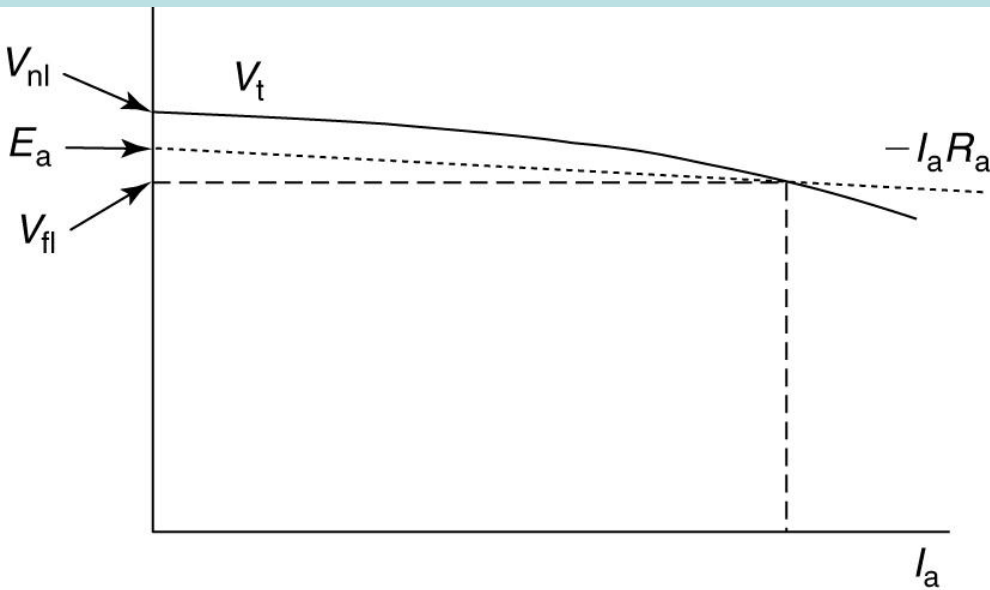
The terminal voltage of the separately excited DC generator would be expected to decrease in a linear manner due to the voltage drop across the armature circuit resistance.

$$VR = \frac{V_{nl} - V_{fl}}{V_{fl}} \times 100$$

As the load current increases armature reaction may lower the flux per pole causing an additional reduction in the terminal voltage. The terminal voltage of a separately excited DC generator will actually decline nonlinearly when the armature current is increased.

In order to properly calculate the VR of the machine, we must account for the armature reaction.

Correcting for armature reaction



1. Redrawing the voltage curves so they converge at the rated voltage.

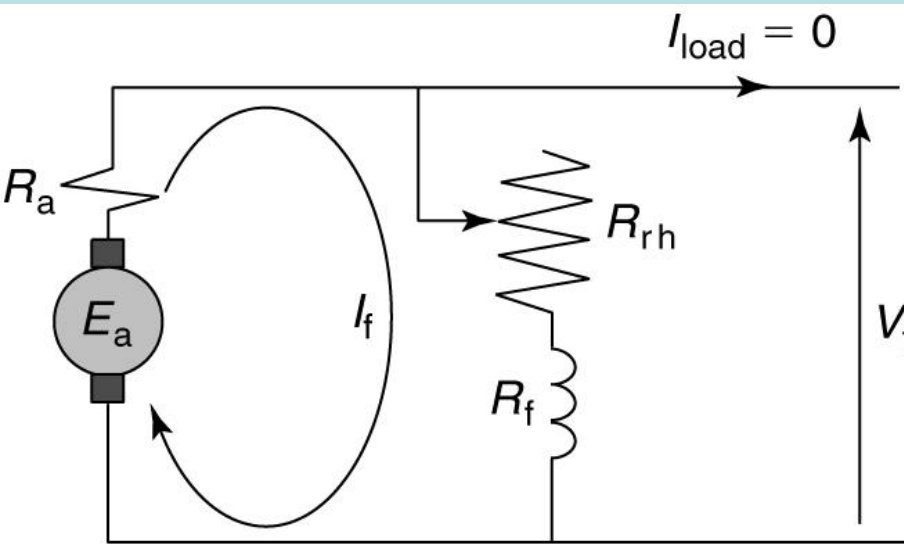
$$V_t = E_a - I_a R_a$$

2. Applying equation to the rated terminal voltage and current, we calculate the generated voltage, E_a , at full load.

We know that the generated voltage will be higher at no load, armature reaction causes the generated voltage to change with the load current.

3. Knowing the terminal voltage at full load, we calculate the generated voltage E_a .
4. Using a magnetization curve for the machine in question, we find the effective MMF, or field current, required to generate this voltage.
5. Divide the MMF required at no load by the efficiency of the given machine to find the actual MMF that must be present at full load to overcome the armature reaction.
6. Using the calculated value of MMF we find the actual no-load voltage.

Self-Excited Generator



Self-excited, shunt DC generator operating at no load

The magnetization curve produced when the machine is separately excited still applies.

Because the load current is zero, the only current in the armature winding is the field current.

Field circuit equation:

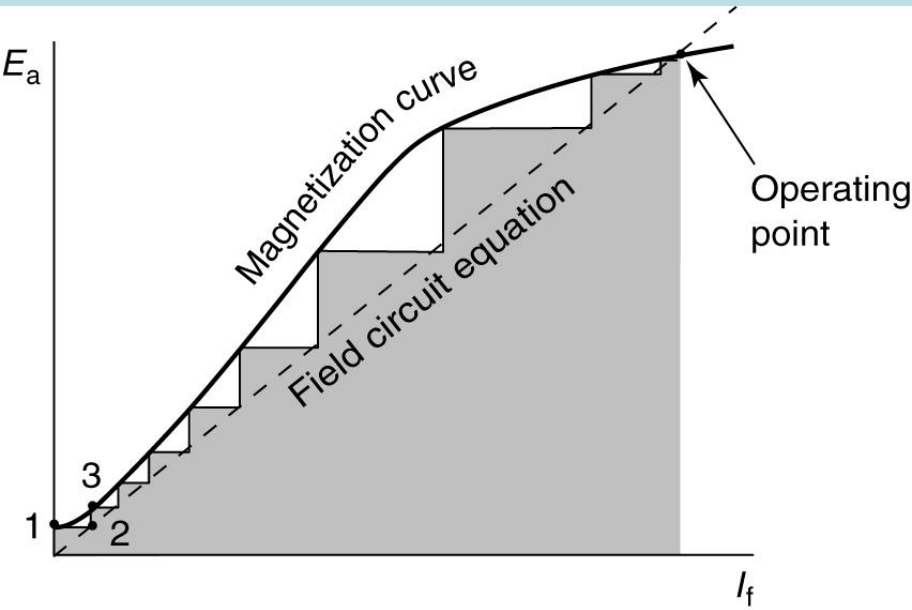
$$E_a = I_f(R_a + R_{rh} + R_f)$$

It represents a linear volt-amp characteristic, which has a slope equal to the sum of the three resistances on the right-hand side of the equation.

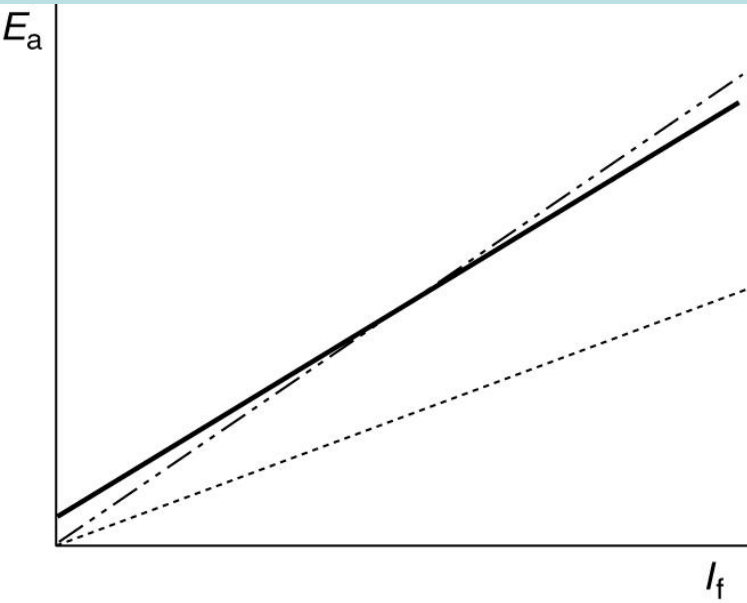
The self-excited generator requires some residual magnetism in the field poles in order to generate voltage.

When the generator is turned, the residual flux induces a small armature voltage (**point 1**). That voltage causes a field current (**point 2**) that increases the voltage more (**point 3**), which causes more field current and so forth.

This process is called **building up**.



Significance of Saturation



Saturation makes the voltage build-up in the self-excited shunt DC generator feasible. Suppose the armature voltage was linear with the field current (solid line).

The field circuit equation also is a straight line.

Case 1:

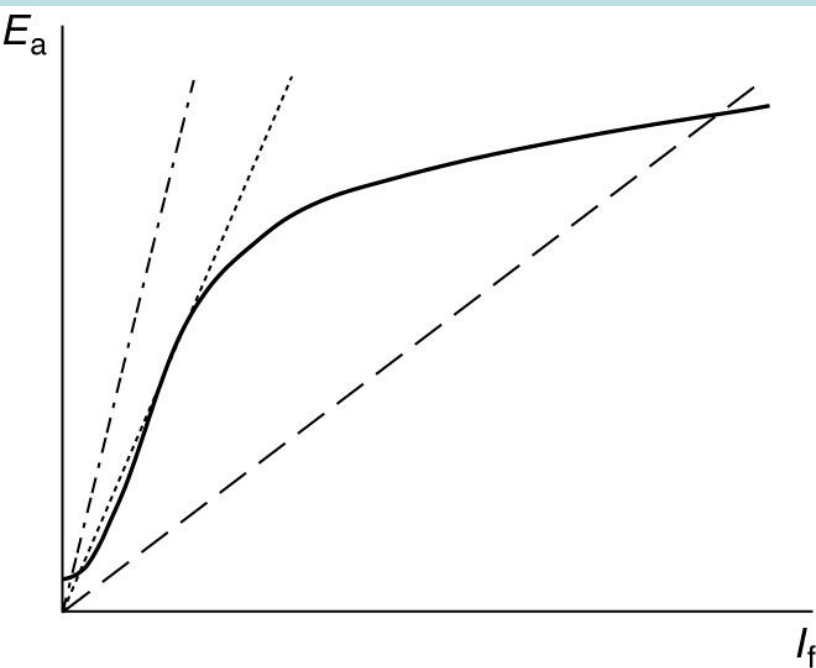
If the resistance of the field circuit is too low (the dotted line), then the two lines won't intersect and there would be no steady-state operating point.

The voltage would build up forever.

Case 2:

If the field circuit resistance is higher (the dot-dash line), then an operating point may occur, but it will change wildly with small changes in the field resistance, because the slopes of the two intersecting lines are similar.

Illustration of the importance of saturation for voltage build-up in the self-excited DC generator.



Saturation provides an upper limit to the amount of resistance in the field circuit.

Case 1:

If the field resistance is too high, the voltage only builds up to a small value (dot-dash line).

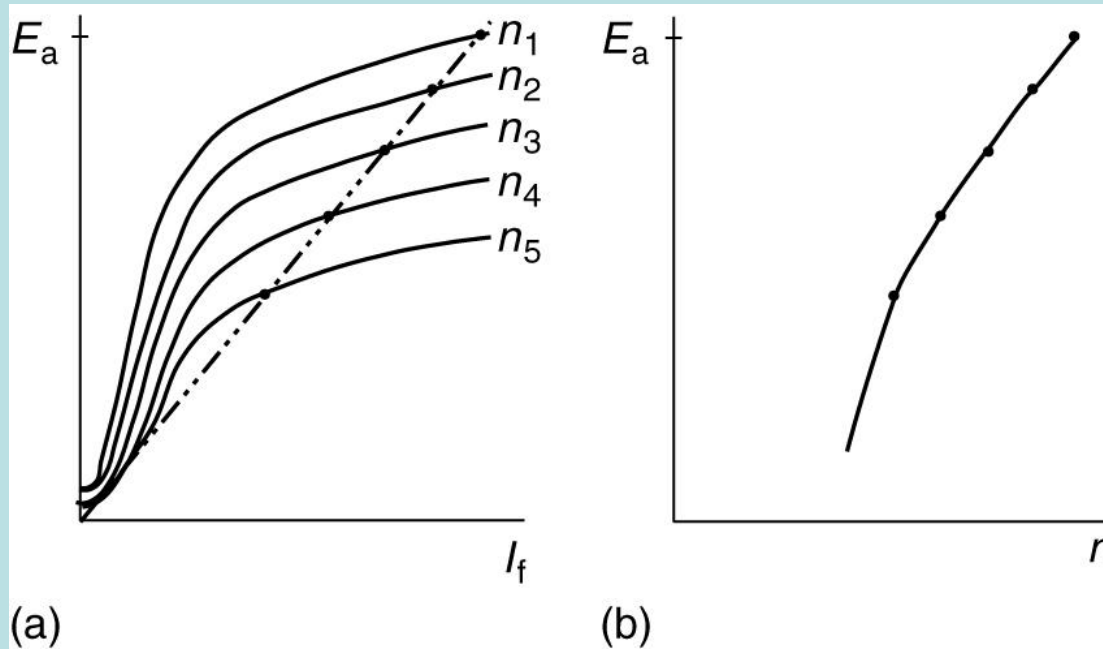
Case 2:

If the field resistance is such that the field circuit voltage requirement line runs close to the linear part of the magnetization curve, then the machine will not provide stable voltage (dotted line).

Case 3:

If the resistance is of a suitable value, then the generator provides required voltage (dashed line).

Variation of the Generated Voltage with Speed.



Performance of self-excited generator as the speed varies.

- Intersection of field circuit line with generated-voltage curves at several speeds.
- Resulting generated voltage as a function of speed.

When the speed changes, we move to a new magnetization curve while moving down the field circuit equation line.

The voltage drops very quickly when the speed is reduced for a self-excited, shunt DC generator. Decreasing the speed decreases the generated voltage, which in turn reduces the field current and flux, thus further reducing the generated voltage.

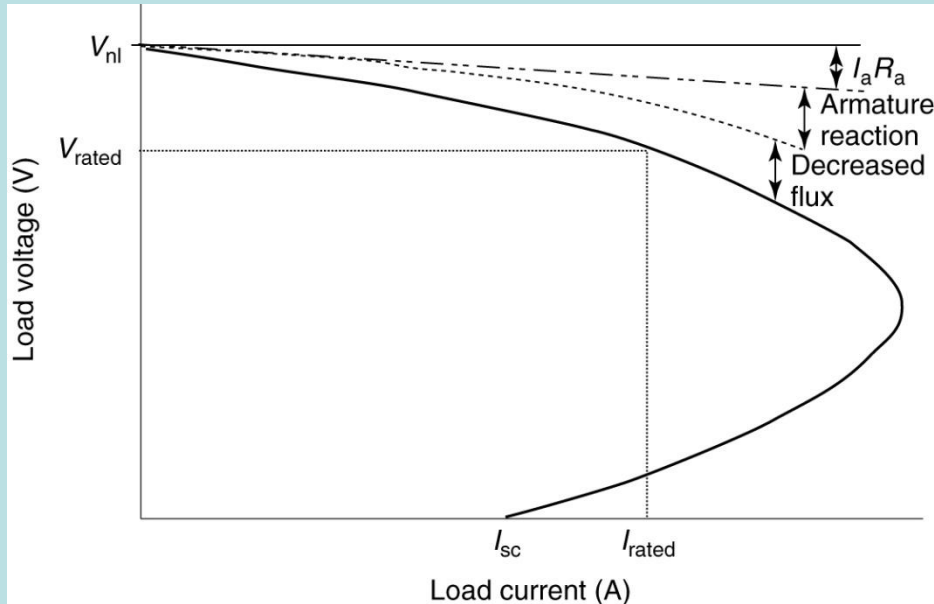
The voltage versus speed plot is nonlinear for the self-excited DC generator.

Conditions for Voltage Build-Up in a Self-Excited DC Generator

- 1. There must be residual magnetism in the machine pole pieces.*
- 2. The field coil must provide flux in the same direction as the residual flux. Otherwise, the machine will build down to zero volts.*
- 3. The field circuit resistance must not be excessive but sufficient.*
- 4. The machine must have sufficient speed (voltage is a function of speed).*

Voltage Under Load

By putting a variable load on the self-excited DC generator, we observe the effect on the terminal voltage of varying the load current.



Volt-amp characteristic of a self-excited DC generator

As the load increases the armature current increases, eventually, the generated voltage decreases so much that the load current starts decreasing and the volt-amp characteristic curve turns around.

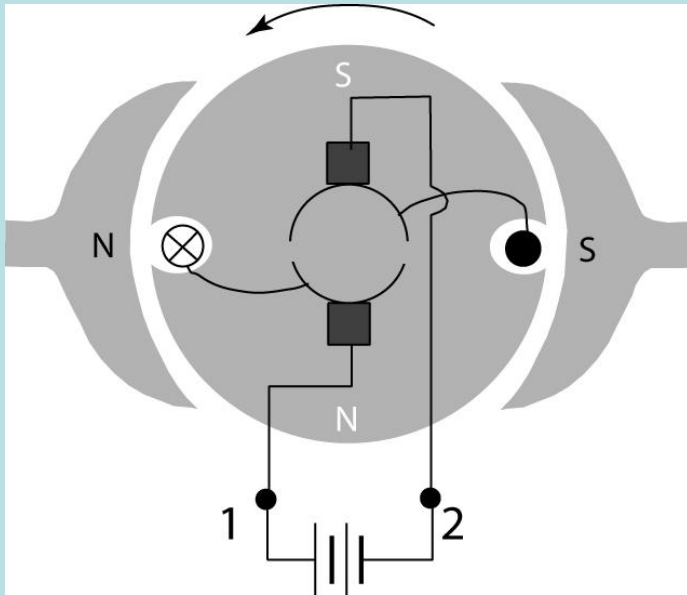
As the load increases, the current and terminal voltage continue to decrease, until the short-circuit current is reached at zero-load voltage.

The point where the curve turns around is called the breakdown point, which usually occurs at about 50% of rated current.

Three effects are observed:

1. As the load increases, the load voltage declines due to the resistance of the armature (dot-dash).
2. As the armature current increases, armature reaction reduces the flux per pole, further reducing the voltage (dotted line) unless it is corrected for implementing compensating windings .
3. Reduction of the field voltage by these two factors causes the field current to decrease, lowering the voltage some more (solid line).

DC MOTORS



Cross section of an elementary DC motor

The armature winding is connected to a DC source that provides current in the direction shown.

The armature current interacts with the magnetic flux produced by the field coils, causing a torque on the rotor. This machine will rotate in the counterclockwise direction.

The **field coils** produce a north pole on the left and a south pole on the right.

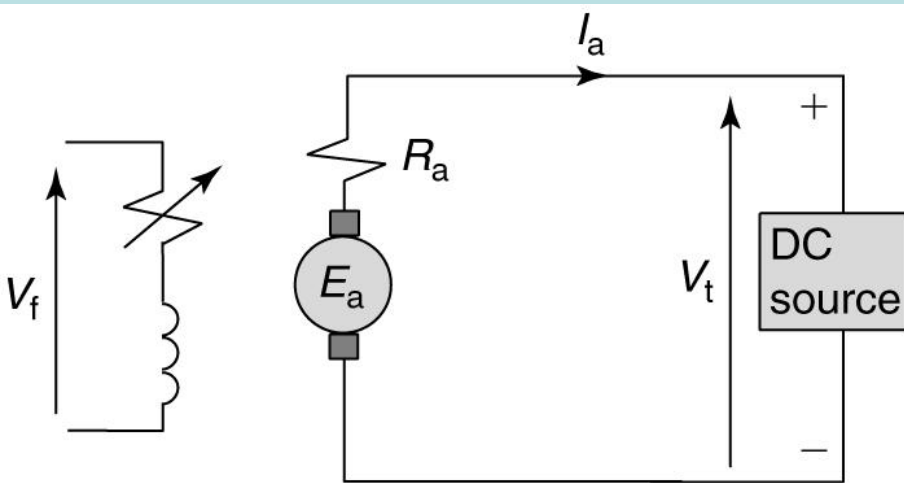
The current in the **armature coil** also produces a magnetic field, with a south pole at the top of the machine and a north pole at the bottom.

An **attractive force** forms between the poles on the armature and those on the field that will rotate the armature.

The force turning the armature is a function of the strength of the magnetic fields on the armature and the stator.

The amount of flux density is related to the current, so the torque should also be related to current.

Torque in the DC Motor



For a motor, the armature current is into the machine and the voltage equation is:

$$V_t = E_a + I_a R_a$$

The power into the machine is the product of the terminal current and voltage:

$$P_{in} = V_t I_a = I_a E_a + I_a^2 R_a$$

Copper losses (the losses due to the resistance of the armature winding):

$$P_{cu} = I_a^2 R_a$$

The portion of the input that is converted to mechanical power (the developed power):

$$P_d = I_a E_a$$

Power to torque relation (related by the speed):

$$T_d = \frac{E_a I_a}{\omega} = \frac{K_a \Phi_p I_a}{\omega}$$

$$\omega = 2\pi f = \frac{2\pi n}{60}$$

$$T_d = \frac{60 K_a \Phi_p I_a}{2\pi} = K_m \Phi_p I_a$$

DC Motor Speed Characteristics

When a load torque is applied to a motor, the motor will slow down due to the load opposing the machine's rotation. Eventually, the machine's electromagnetic torque will equal the load torque and a new steady-state speed will be reached. How the speed changes depends on the type of field connections.

To quantify the variation of speed with load, we define the speed regulation:

$$SR = \frac{n_{nl} - n_f}{n_{nl}}$$

The DC motor may have a shunt field, series field, or compound.

The shunt field may be separately excited or it may be excited from the same supply as the armature.

Shunt Motor

For the motor, the armature current is into machine and the voltage equation is:

$$V_t = E_a + I_a R_a$$

The armature voltage of a motor is:

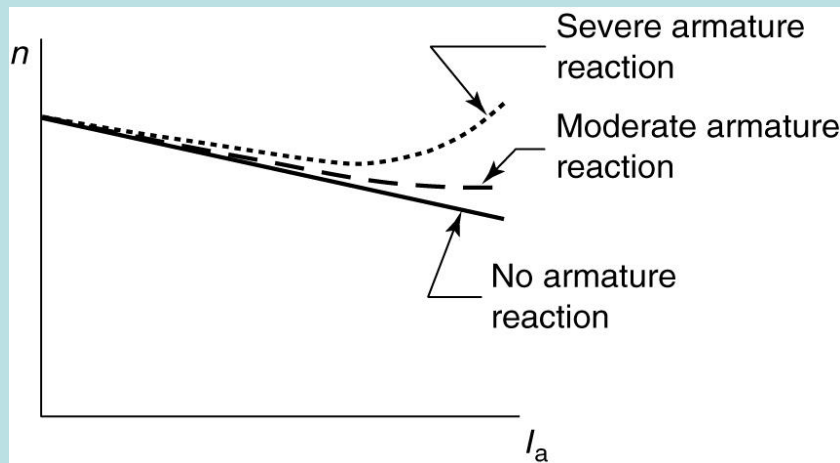
$$E_a = K_g \Phi_p n$$

$$E_a = V_t - I_a R_a = K_g \Phi_p n$$

From the voltage equation we derive the **speed equation**:

$$n = \frac{V_t - I_a R_a}{K_g \Phi_p}$$

For a **shunt machine** connected to a constant voltage, the flux per pole is constant and the speed equation is a linear relation between n and I_a .



Speed characteristic of a shunt DC motor

Recall the electromagnetic torque equation:

$$T_d = K_m \Phi_p I_a$$

If the flux is constant, we would obviously expect the torque to be a linear function of the armature current.

As was the case with the speed, however, the armature reaction has an effect.

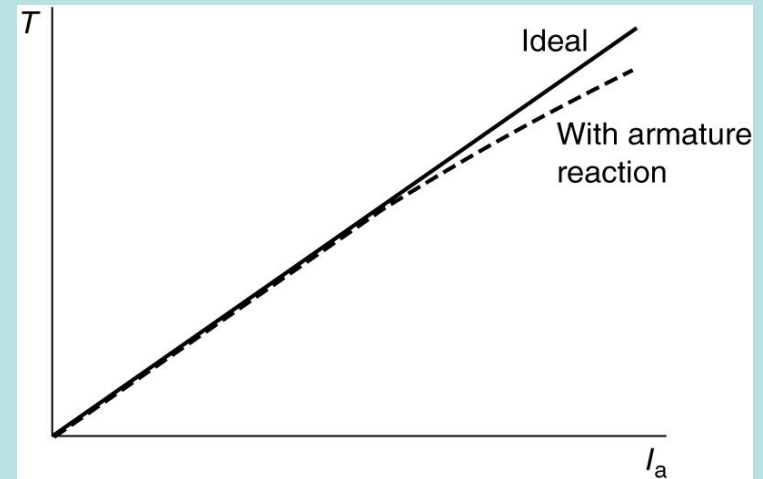
As the armature current increases, the flux per pole decreases causing the electromagnetic torque to be somewhat lower than expected.

Speed-torque relation is given:

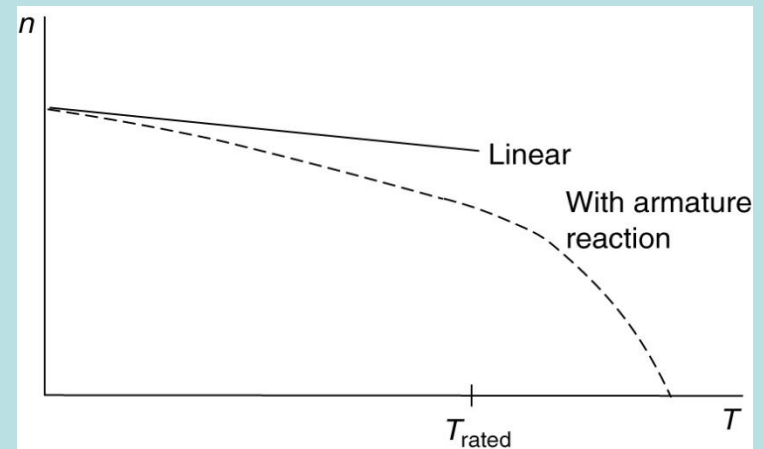
$$n = \frac{V_t - R_a \left(\frac{T_d}{K_m \Phi_p} \right)}{K_f \Phi_p}$$

For a shunt motor with constant terminal voltage and no armature reaction, this is a linear relation between speed and torque.

Armature reaction will cause the speed to falloff faster with torque and limits the torque of the machine at high current.



Torque characteristic of a shunt DC motor



Torque-speed characteristic of a shunt DC motor

SERIES MOTOR

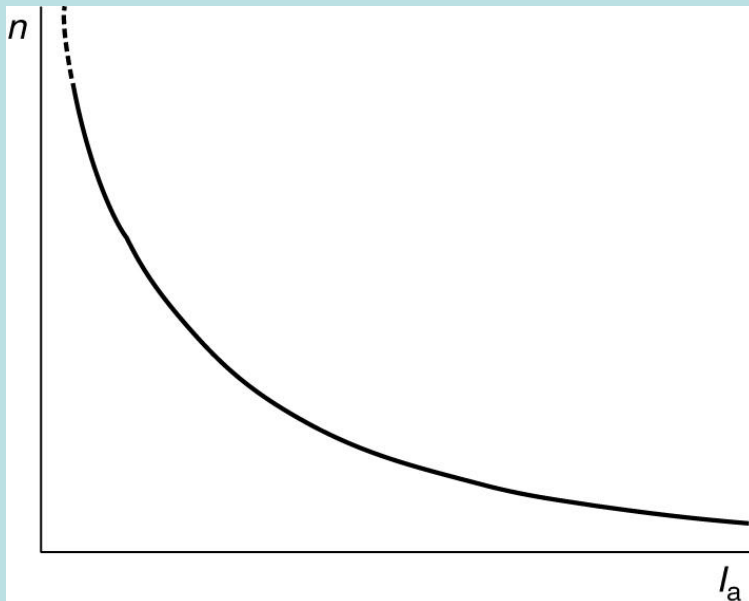
The speed equation for the series DC motor is similar to the shunt, but we must include the series field resistance:

$$n = \frac{V_t - I_a(R_a + R_s)}{K_g \Phi_p}$$

The flux per pole is a function of the armature current.

Splitting above equation into two terms and canceling the armature current from the numerator and denominator of the second term yields:

$$n = \frac{V_t}{K_g K I_a} - \frac{(R_a + R_s)}{K_g \Phi_p}$$



This equation represents an inverse relationship between the speed, n , and the armature current, I_a .

As the current goes to zero, the speed of the machine becomes very large. For this reason, it is essential to always keep a load on a series DC motor, which means the load should be connected directly to the shaft.

As the current gets high, indicating a heavy load on the motor, the speed drops.

Speed characteristic of a series DC motor

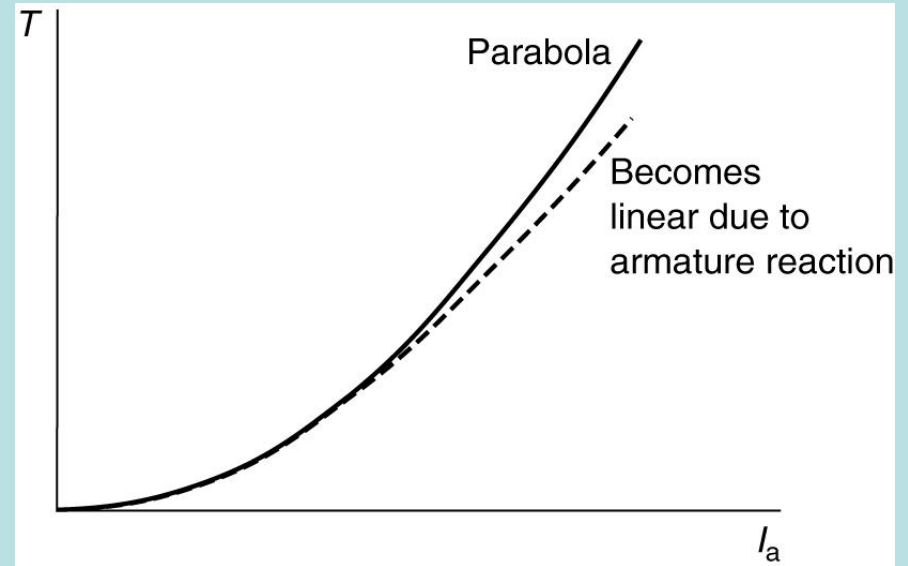
The developed torque in series motor is given:

$$T_d = K_m \Phi_p I_a = K_m K I_a I_a = K' I_a^2$$

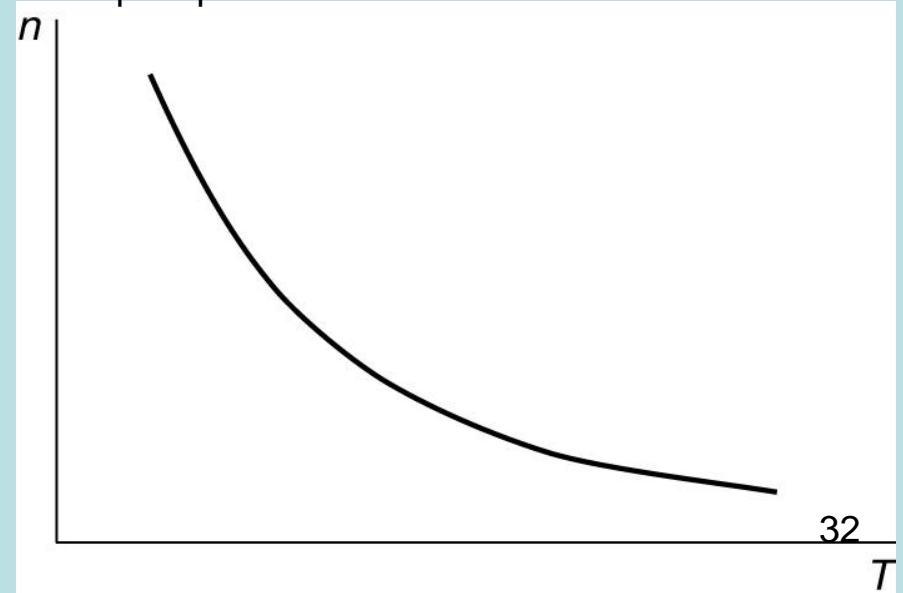
The torque of the series DC motor varies with the square of the armature current (the solid line). As the iron in the machine becomes saturated at high armature current, the flux per pole becomes constant/linear (the dashed line).

The series DC motor produces very high starting torque and will run away at no load. Series motors are useful for driving subway cars where high torque at standstill is required. 400% torque can be provided with only 20% current because of the squared relationship.

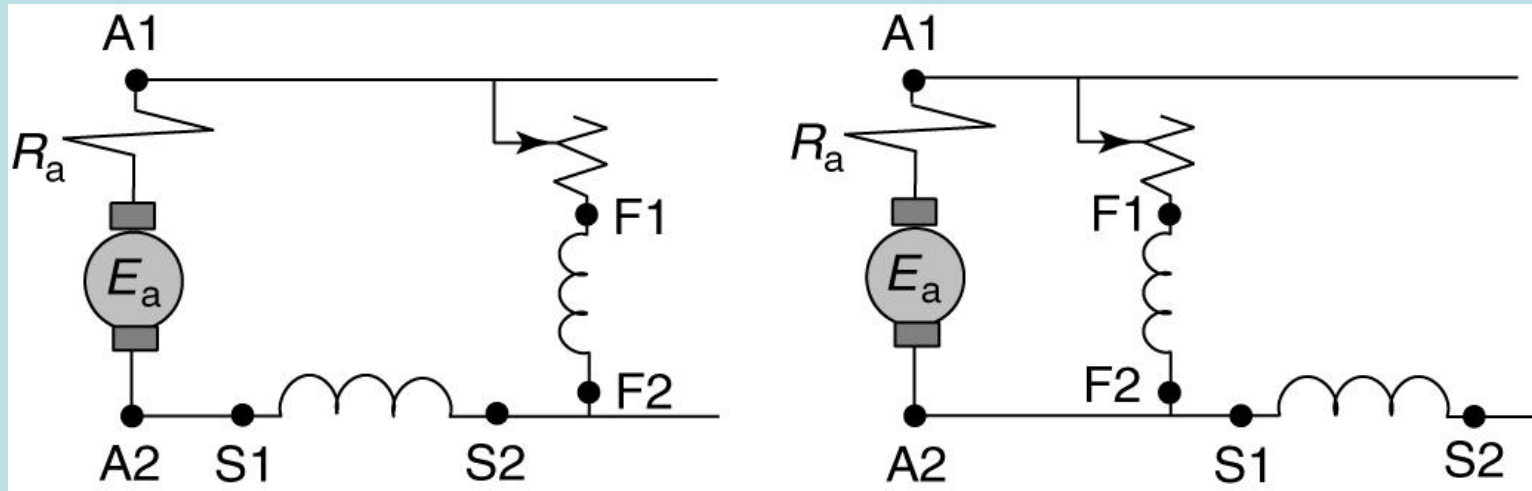
Torque characteristic of a series DC motor.



Torque-speed characteristic of a series DC motor.



Compound DC Motors



Long-shunt compound motor

Short-shunt compound motor

Because compound motors have both series and shunt field windings, the characteristics of the compound motor will exhibit traits of both the series and shunt motor.

The advantages of the series and shunt motors:

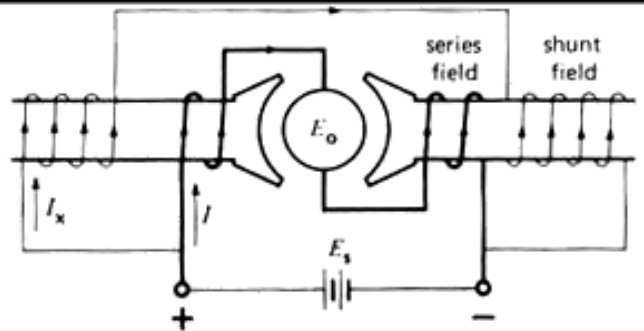
The series motor has greater torque capabilities.

The shunt motor has more constant and controllable speed over various loads.

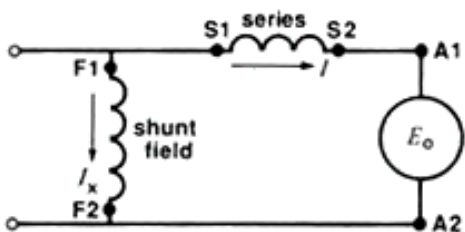
These two desirable characteristics can be found in the same motor by placing both a series field and shunt field winding on the same pole - the compound motor.

The compound motor responds better to heavy load changes than a shunt motor because of the increased current through the series field coils boosting the field strength and providing added torque and speed.

If a shunt coil is added to a series motor at light loads (when a series motor tends to overspeed) the added shunt field flux limits the top speed, eliminating self-destruction.



(a)



(b)

A compound dc motor has a series field and a shunt field.

In a cumulative compound motor, the MMF of the two fields add with the shunt field.

The motor at no-load:

The armature current I in the series winding is low and the MMF of the series field is negligible.

The shunt field is fully excited by current I_x and so the motor behaves like a shunt machine- it does not tend to run away at no-load.

The motor under load:

The MMF of the series field increases.

The MMF of the shunt field remains constant.

The total MMF (and the resulting flux per pole) is greater under load than at no-load.

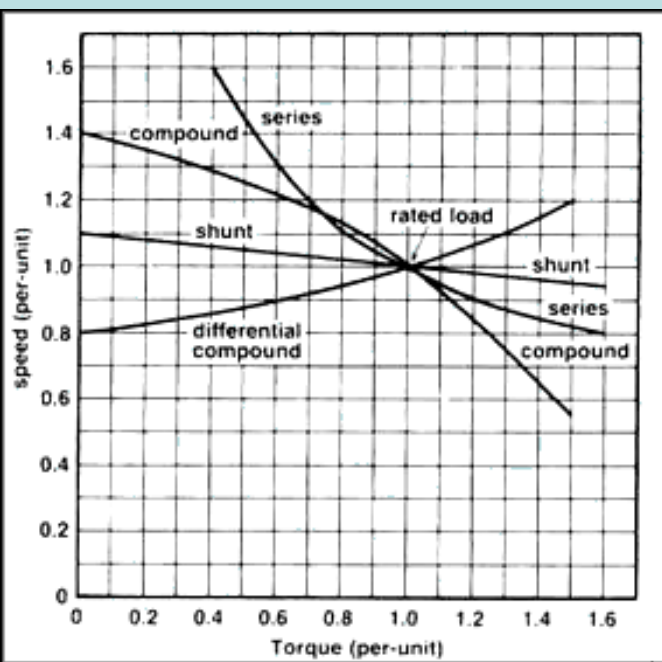
The motor speed falls with increasing load.

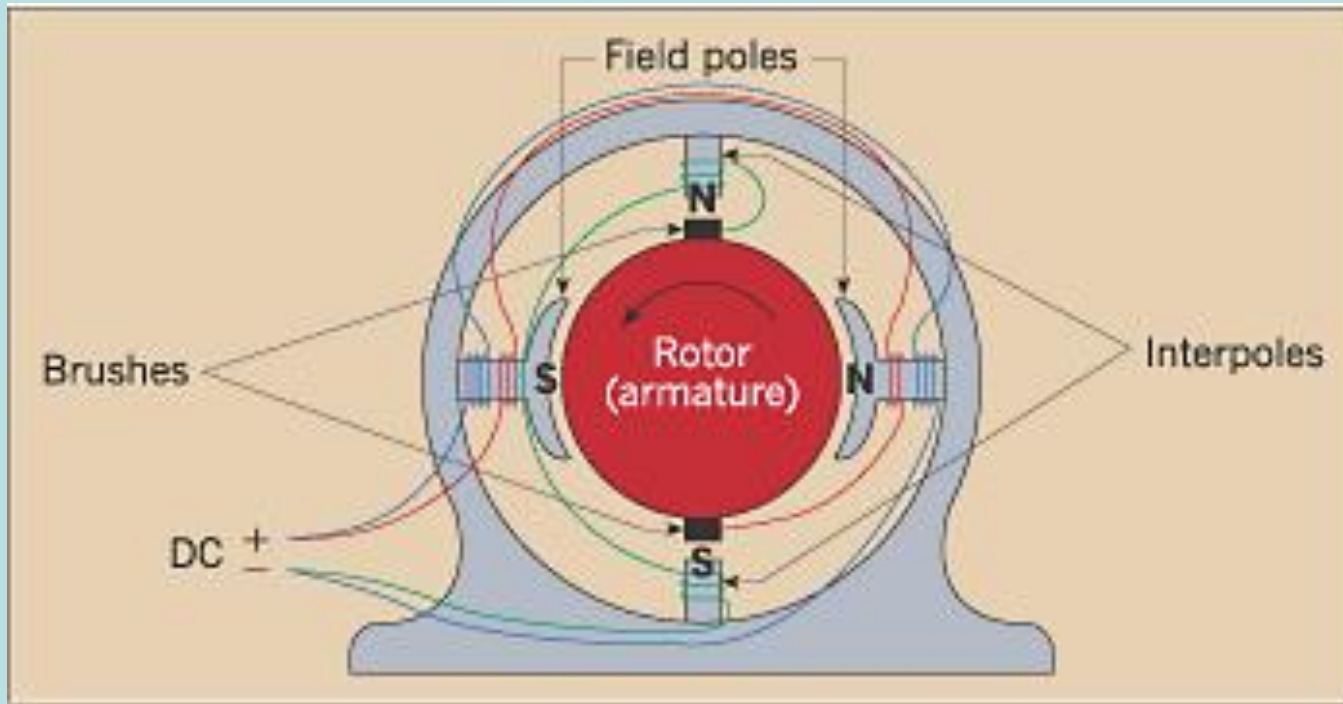
$$n = \frac{V_t - I_a R_a}{K_g \Phi_p}$$

A differential compound motor:

The series field is connected so that it opposes the shunt field. In such a motor, the total MMF (and the resulting flux per pole) decreases with increasing load.

The speed rises as the load increases, and this may lead to instability.





One way of wiring a compound motor with interpoles.

The blue lines indicate the shunt field,

The red lines designate the series field

The green lines show the interpole windings in series with the armature.

DC Motor Speed Control

The major advantage of DC motors has been the ability to control the speed-torque characteristic over a fairly wide range of speeds for a given torque.

From the speed equation for the shunt DC motor

$$n = \frac{V_t - I_a R_a}{K_f \Phi_f}$$

we can see that there are two controllable variables that could be used to change the speed of the machine.

1. Decreasing the flux in the denominator would allow the speed to increase. This is done by increasing the field rheostat resistance.
2. By decreasing the terminal voltage, the speed would drop.
3. Changing the resistance in the armature circuit.

LOSSES AND EFFICIENCY

The losses in a machine affect its cost, rating, and performance. The efficiency of a rotating machine is frequently determined by measuring the losses.

The efficiency is defined the same way as it is for transformers and other types of machines:

$$\eta = \frac{\text{Output Power}}{\text{Input Power}} = \frac{\text{Input} - \text{Losses}}{\text{Input}} = \frac{\text{Output}}{\text{Output} + \text{Losses}}$$

Typical efficiencies of DC motors are shown in the table below. Generators would have similar efficiencies except they would be rated in kW.

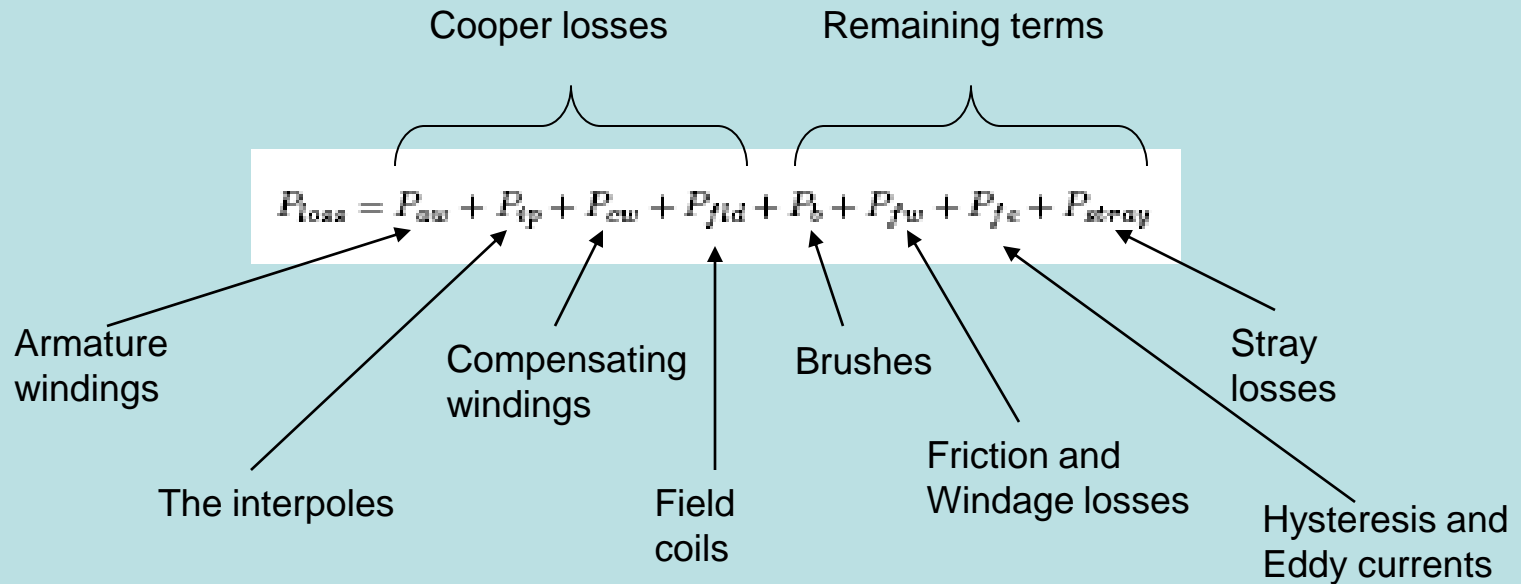
The amount of loss in each category depends on the number of windings, type of core, whether the machine has brushes, etc.

Machine size (HP)	Efficiency (percents)	Typical losses (HP)
5	74	1.76
50	89	6.18
500	93	37.6
5000	97	155

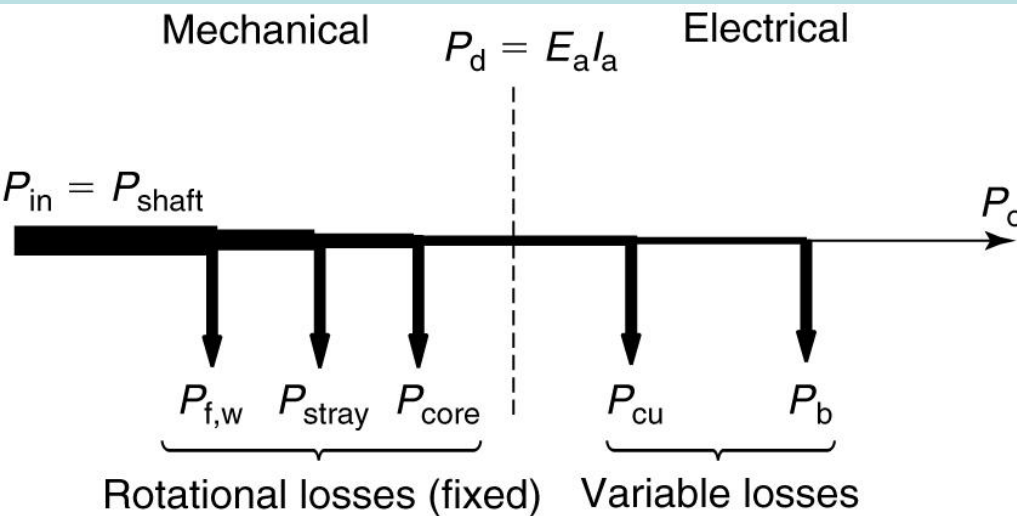
Types of Losses In DC Machines

DC machines have copper, steel, rotational, stray losses, and losses in the brushes.

Whether the machine operates as a motor or as a generator, the losses are:



Generator Power Flow



In the case of a generator, we apply mechanical power to the shaft of the machine and deliver electrical power to a load.

Mechanical power is input at the left side of the diagram.

Rotational losses are mechanical in nature and are thus subtracted from the mechanical power. This includes not only friction and windage but, by convention, stray losses and core losses.

The remaining power is transformed to electrical power, and this quantity is called the ***developed power***.

The ***developed power*** is calculated as:

$$P_d = E_a I_a$$

The power in the controlled voltage source of the equivalent circuit represents the developed power.

From the developed power we subtract the copper losses in the machine windings, as well as the loss due to the brushes.

Note that in a self-excited generator, the field losses would be included in the copper losses, whereas in a separately excited generator the field losses would constitute a separate electrical input that would also be added to the losses of the machine. ***The final result is the electrical output of the generator.***

■ IMPORTANT NUMBERED EQUATIONS FROM THIS CHAPTER

$$e = \mathbf{B} \times \ell \times v \quad (11-1)$$

$$\theta_e = \frac{P}{2} \times \theta_m \quad (11-4)$$

$$R_a = R_{aw} + R_{ip} + R_{cw} \quad (11-7)$$

$$E_a = \left(\frac{PZ_a}{60a} \right) n \Phi_p = K_g \Phi_p n \quad (11-13)$$

$$K_g = \frac{PZ_a}{60a} \quad (11-14)$$

$$V_t = E_a - I_a R_a \quad (11-18)$$

$$VR = \frac{V_{nl} - V_{fl}}{V_{fl}} \times 100\% \quad (11-19)$$

$$E_a = I_f(R_a + R_{rh} + R_f) \quad (11-2)$$

$$V_t = E_a + I_a R_a \quad (11-3)$$

$$P_{in} = V_t I_a = R_a I_a^2 + E_a I_a \quad (11-5)$$

$$P_{cu} = R_a I_a^2 \quad (11-6)$$

$$P_d = E_a I_a \quad (11-10)$$

$$T_d = \frac{E_a I_a}{\omega} = \frac{K_a \Phi_p n I_a}{\omega} \quad (11-11)$$

$$T_d = \frac{(60) K_a \Phi_p I_a}{2\pi} = K_m \Phi_p I_a \quad (11-12)$$

$$SR = \frac{n_{nl} - n_{fl}}{n_{nl}} \quad (11-17)$$

$$E_a = V_t - I_a R_a = K_g \Phi_p n \quad (11-30)$$

$$n = \frac{V_t - I_a R_a}{K_g \Phi_p} \quad (11-31)$$

$$n = \frac{V_t - R_a \left(\frac{T_d}{K_m \Phi_p} \right)}{K_g \Phi_p} \quad (11-32)$$

$$n = \frac{V_t - I_a (R_a + R_s)}{K_g \Phi_p} \quad (11-33)$$

$$n = \frac{V_t}{K_g K I_a} - \frac{(R_a + R_s)}{K_g K} \quad (11-34)$$

$$T_d = K_m \Phi_p I_a = K_m K I_a I_a = K' I_a^2 \quad (11-35)$$

$$P = \omega T \quad (11-36)$$

$$\eta = \frac{\text{Output power}}{\text{Input power}} = \frac{\text{Input} - \text{Losses}}{\text{Input}} \\ = \frac{\text{Output}}{\text{Output} + \text{Losses}} \quad (11-37)$$

$$P_{\text{loss}} = P_{aw} + P_{ip} + P_{cw} + P_{fld} \\ + P_b + P_{fw} + P_{fe} + P_{\text{stray}} \quad (11-38)$$

$$P_d = E_a I_a \quad (11-39)$$